

Unit 5

Exponents & Square Roots

5-1 Exponents and Bases

A series of horizontal dotted lines for writing.

5-1 Exponents and Bases

Fill in the following chart. The more of these common powers you know, the better. (The shaded cells are powers you should know!)

		P o w e r									
		1	2	3	4	5	6	7	8	9	10
B a s e	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										

I. Write as repeated multiplication and evaluate:

1. $(-2)^7$ _____

2. $-(2)^7$ _____

3. $(-3)^4$ _____

4. $-(3)^4$ _____

5. $\left(\frac{1}{2}\right)^3$ _____

6. $\frac{(1)^3}{2}$ _____

7. $\left(\frac{2}{3}\right)^3$ _____

8. $\frac{(2)^3}{3}$ _____

9. $\frac{2}{(3)^3}$ _____

10. $\frac{(2)^3}{(3)^3}$ _____

II. Write in exponential form:

1. eight cubed: _____ 2. The sixth power of 2: _____

3. $(-2)(-2)(-2)(-2)(-2)$: _____ 4. $-(10)(10)(10)(10)$: _____

III. Evaluate:

1. $7^2 =$ _____ 2. $12^2 =$ _____ 3. $3^4 =$ _____ 4. $4^3 =$ _____

5. $\left(\frac{2}{5}\right)^2 =$ _____ 6. $\left(1\frac{3}{4}\right)^2 =$ _____ 7. $\left(\frac{1}{2}\right)^3 =$ _____ 8. $\left(-\frac{1}{9}\right)^2 =$ _____

9. $(0.8)^2 = \underline{\hspace{2cm}}$ 10. $(1.2)^2 = \underline{\hspace{2cm}}$ 11. $(0.13)^2 = \underline{\hspace{2cm}}$ 12. $(-2)^4 = \underline{\hspace{2cm}}$

13. $-2^4 = \underline{\hspace{2cm}}$ 14. $(-2)^3 = \underline{\hspace{2cm}}$ 15. $-2^3 = \underline{\hspace{2cm}}$ 16. $(-1)^7 = \underline{\hspace{2cm}}$

17. $(-1)^{2020} = \underline{\hspace{2cm}}$ 18. $2^3 \times 3^2 = \underline{\hspace{2cm}}$ 19. $2^3 + 2^5 = \underline{\hspace{2cm}}$

20. $5^2 - 2^5 = \underline{\hspace{2cm}}$ 21. $4^2 \div 2^4 = \underline{\hspace{2cm}}$ 22. $2^4 - 2^2 = \underline{\hspace{2cm}}$

23. $(-3 \times 4)^2 = \underline{\hspace{2cm}}$ 24. $3(4)^2 = \underline{\hspace{2cm}}$ 25. $-3(4)^2 = \underline{\hspace{2cm}}$

26. $(3 \cdot 4)^2 = \underline{\hspace{2cm}}$ 27. $-(-3 \cdot 4)^2 = \underline{\hspace{2cm}}$ 28. $2(3 - 4)^2 = \underline{\hspace{2cm}}$

29. $-1^2 - 1^3 = \underline{\hspace{2cm}}$ 30. $-(3^2 - 3)^2 = \underline{\hspace{2cm}}$

IV. Show the substitution for $m = 4$ and $n = -3$, then evaluate:

1. m^2

2. $3n^2$

3. $(3m)^2$

4. $-m^3$

5. $(n - m)^3$

6. $m^3 - n^3$

V. Problems and Thought-Provokers

1. $1 + 3 =$

$1 + 3 + 5 =$

$1 + 3 + 5 + 7 =$

$1 + 3 + 5 + 7 + 9 =$

$1 + 3 + 5 + 7 + 9 + 11 =$

What is the pattern to these sums of consecutive odd numbers?

Make a conjecture about the sums of consecutive odd numbers:

Explain why this happens. Be creative

2. The same number is subtracted from each of 71 and 58 resulting in two perfect squares. What is the number?

3. If $2^x + 3^y = 43$, where x and y are integers, then the value of $(x + y)$ is:

4. If each of the numbers 1, 2, 3, 4 is substituted, in some order, for p , q , r and s , determine the greatest possible value for $p^q + r^s$.

5. In the following equations, the letters a , b , and c represent different numbers.

$$a^3 = 1 + 7,$$

$$3^3 = 1 + 7 + b,$$

$$4^3 = 1 + 7 + c$$

Find the numerical value of $a + b + c$:

6. When $(5^3)(7^{52})$ is evaluated, the unit's digit is:

***VI. Just for fun... You may use a calculator for the following questions if you wish.**

1. $2^5 \cdot 9^2$ _____

2. $3^3 + 4^4 + 3^3 + 5^5$ _____

3. $6 + 6 + 6 + (6 \times 6 \times 6) + (6 \times 6 \times 6) + (6 \times 6 \times 6)$ _____

4. $(3 + 4 + 0 + 1 + 2 + 2 + 2 + 4)^6$ _____

5. $12^2 + 33^2 =$ _____

6. What is the smallest integer greater than 1 that is both a perfect square and a perfect cube?

What is the next smallest number to have the same property?

7. $15 \times 15 =$ _____ $25 \times 25 =$ _____ $35 \times 35 =$ _____

$45 \times 45 =$ _____ $55 \times 55 =$ _____ $65 \times 65 =$ _____

Can you find a pattern in the squares of these types of numbers? See if it works for additional examples.

5-2 Perfect Squares and Cubes

A series of horizontal dotted lines for writing.

5-2 Perfect Squares and Cubes

I. Calculate the first 25 perfect natural number squares:

1^2 _____ 2^2 _____ 3^2 _____ 4^2 _____ 5^2 _____

6^2 _____ 7^2 _____ 8^2 _____ 9^2 _____ 10^2 _____

11^2 _____ 12^2 _____ 13^2 _____ 14^2 _____ 15^2 _____

16^2 _____ 17^2 _____ 18^2 _____ 19^2 _____ 20^2 _____

21^2 _____ 22^2 _____ 23^2 _____ 24^2 _____ 25^2 _____

II. Calculate the first 12 perfect natural number cubes:

1^3 _____ 2^3 _____ 3^3 _____ 4^3 _____ 5^3 _____

6^3 _____ 7^3 _____ 8^3 _____ 9^3 _____ 10^3 _____

11^3 _____ 12^3 _____

III. Circle the numbers that are perfect squares. Put a triangle around the numbers that are perfect cubes.

1 -1 16 8 $1\frac{7}{9}$ 25 0.64 32 $\frac{8}{18}$ 18

$\frac{27}{64}$

48 $\frac{8}{27}$ 121 -27 $\frac{1}{2}$ $\frac{4}{125}$ 1.44 343000 -100

**IV. Determine a number such that its square equals the following perfect squares:
(e.g. $81 = 9 \times 9$)**

a) 100 _____ b) 441 _____ c) 225 _____ d) 361 _____

e) 529 _____ f) 2 890 000 _____ g) 25 000 000 _____

h) 72 900 _____ i) 9 610 000 _____ j) 0.000 004 _____

**V. Determine a number such that its cube equals the following perfect cubes:
(e.g. $216 = 6 \times 6 \times 6$)**

a) 27 _____ b) 1000 _____ c) 343 _____ d) 1728 _____

e) 3375 _____ f) -8000 _____ g) 125 000 000 _____

h) 216 000 _____ i) 64 000 000 000 _____

j) 0.000 008 _____

VI. Find a simple rational number (other than the given number) that will multiply with each of the following to create a perfect square not equal to 1.

$$\left(\begin{array}{l} \text{e.g. for } \frac{4}{3}: \frac{4}{3} \times \frac{1}{3} = \frac{4}{9}, \text{ a perfect square} \\ \text{or } \frac{4}{3} \times \frac{3}{1} = \frac{12}{3} = 4, \text{ a perfect square} \end{array} \right)$$

a) 18 _____

b) 12 _____

c) 10 _____

d) $\frac{2}{3}$ _____

e) $\frac{8}{5}$ _____

f) $\frac{1}{6}$ _____

g) $-\frac{5}{4}$ _____

h) $-\frac{11}{8}$ _____

VII. Find a simple rational number (other than the given number squared) that will multiply with each of the following to create a perfect cube not equal to 1.

$$\left(\begin{array}{l} \text{e.g. for } \frac{4}{3}: \frac{4}{3} \times \frac{2}{9} = \frac{8}{27}, \text{ a perfect cube} \\ \text{or } \frac{4}{3} \times \frac{6}{1} = \frac{24}{3} = 8, \text{ a perfect cube} \end{array} \right)$$

a) 16 _____

b) 32 _____

c) 10 _____

d) $\frac{49}{27}$ _____

e) $\frac{8}{25}$ _____

f) $\frac{9}{16}$ _____

g) $-\frac{5}{6}$ _____

h) $-\frac{11}{8}$ _____

A series of horizontal dotted lines for writing, spanning the width of the page.

5-3 Exponent Laws

I. Write the following as a single power. Do not evaluate!

1. $(9^3)(9^5) = \underline{\hspace{2cm}}$

2. $7^6 \cdot 7^6 = \underline{\hspace{2cm}}$

3. $(4^{13})(4)(4^5) = \underline{\hspace{2cm}}$

4. $(x^7)(x^{11}) = \underline{\hspace{2cm}}$

5. $w(w^3)(w^{50}) = \underline{\hspace{2cm}}$

6. $(-6)^8(-6) = \underline{\hspace{2cm}}$

7. $24^8 \div 24^3 = \underline{\hspace{2cm}}$

8. $\frac{3^{13}}{3^8} = \underline{\hspace{2cm}}$

9. $t^{23} \div t = \underline{\hspace{2cm}}$

10. $5^9 \div 5^9 = \underline{\hspace{2cm}}$

11. $2^5 \div 2^2 = \underline{\hspace{2cm}}$

12. $\frac{(8^{14})(8^5)}{8^{11}} = \underline{\hspace{2cm}}$

13. $\frac{11^4(11^3 \cdot 11^7)}{11^8(11)} = \underline{\hspace{2cm}}$

II. The Power Zero

1. $(56)^0 =$ _____

2. $(-11)^0 =$ _____

3. $-11^0 =$ _____

4. $\left(-\frac{25}{4}\right)^0 =$ _____

5. $-8(92mn)^0 =$ _____

6. $3^0 =$ _____

7. $4(3)^0 =$ _____

8. $0^3 - 3^0 =$ _____

III. Power of a Power

1. $(2^3)^2 =$ _____

2. $(2^2)^3 =$ _____

3. $(7^3)^2 =$ _____

4. $[(-3)^7]^8 =$ _____

5. $(12^7 \times 12^4)^3 =$ _____

6. $\left(\frac{2^5}{2^3}\right)^3 =$ _____

7. $(n^3)^4 =$ _____

8. $\left(\frac{n^6}{n^2}\right)^3 =$ _____

9. $(-x^3)^6 =$ _____

10. $(abc)^3 = \underline{\hspace{2cm}}$

11. $(3m^2)^4 = \underline{\hspace{2cm}}$

12. $(2xy)^5 = \underline{\hspace{2cm}}$

13. $\left(\frac{3^5 \cdot 3^{12}}{3^2}\right)^3 = \underline{\hspace{2cm}}$

14. Would it be correct to use the exponent laws on the following questions? If so, use them to write the answer as a single power. If not, evaluate the answer.

a) $3^2 \times 3^3 = \underline{\hspace{2cm}}$

b) $2^4 + 2^5 = \underline{\hspace{2cm}}$

c) $3^4 - 3^3 = \underline{\hspace{2cm}}$

d) $2^8 \div 2^5 = \underline{\hspace{2cm}}$

e) $(2^2 + 2^1)^3 = \underline{\hspace{2cm}}$

f) $(2^6 \times 2^3)^4 = \underline{\hspace{2cm}}$

A series of horizontal dotted lines for writing.

5-4 Roots

I. Find the perfect square root if it exists:

1. 25 _____

2. 29 _____

3. 80 _____

4. 81 _____

5. 169 _____

6. 99 _____

7. 1600 _____

8. 900 _____

9. $\frac{81}{400}$ _____

10. 0.64 _____

II. Find the perfect cube root if it exists:

1. 8 _____

2. 9 _____

3. 64 _____

4. 81 _____

5. 100 _____

6. 216 _____

7. 1000 _____

8. 144 _____

9. 625 _____

10. 729 _____

11. $\frac{81}{24}$ _____

12. 0.64 _____

III. Evaluate the following. Estimate only when necessary:

1. $\sqrt{49} =$ _____

2. $\sqrt{144} =$ _____

3. $\sqrt{81} =$ _____

4. $\sqrt{0.25} =$ _____

5. $\sqrt{0.09} =$ _____

6. $\sqrt{12100} =$ _____

7. $\sqrt{490000} =$ _____

8. $\sqrt{0.0144} =$ _____

9. Why is $\sqrt{0.00144}$ different from #8?

10. $\sqrt{0.0121} =$ _____

11. $\sqrt{\frac{4}{9}} =$ _____

12. $\sqrt{5\frac{4}{9}} =$ _____

13. $\sqrt{\frac{50}{72}} =$ _____

14. $\sqrt{30} =$ _____

15. $\sqrt{78} =$ _____

16. $\sqrt{649^2} =$ _____

17. $\frac{\sqrt{64}}{16} =$ _____

18. $\sqrt{13^2 - 5^2} =$ _____

19. $\sqrt{13^2} - \sqrt{5^2} =$ _____

20. $\sqrt{-36} =$ _____

21. $-\sqrt{64} =$ _____

22. $2\sqrt{3^2 + 4^2} = \underline{\hspace{2cm}}$

23. $\sqrt{3(-2)^2 - (-4)} = \underline{\hspace{2cm}}$

24. $\sqrt{81} + \sqrt{52} = \underline{\hspace{2cm}}$

25. $(\sqrt{36})(\sqrt{4}) = \underline{\hspace{2cm}}$

26. $\sqrt{36} + \sqrt{4} = \underline{\hspace{2cm}}$

27. $\sqrt{9} \cdot \sqrt{9} = \underline{\hspace{2cm}}$

28. $\sqrt{20} \cdot \sqrt{20} = \underline{\hspace{2cm}}$

29. $\sqrt{(2^4 + 3^2)^0} = \underline{\hspace{2cm}}$

30. $\sqrt{16+9} = \underline{\hspace{2cm}}$

31. $\sqrt{16-9} = \underline{\hspace{2cm}}$

32. $\sqrt{16 \times 9} = \underline{\hspace{2cm}}$

33. $\sqrt{16 \div 9} = \underline{\hspace{2cm}}$

34. $\sqrt{16} + \sqrt{9} = \underline{\hspace{2cm}}$

35. $\sqrt{16} - \sqrt{9} = \underline{\hspace{2cm}}$

36. $\sqrt{16} \times \sqrt{9} = \underline{\hspace{2cm}}$

37. $\sqrt{16} \div \sqrt{9} = \underline{\hspace{2cm}}$

IV. State if the square root of the number is rational or irrational.

1a) 121 _____

b) 60 _____

c) 729 _____

d) 750 _____

e) $\frac{81}{5}$ _____

f) $\frac{8}{18}$ _____

g) 1.6 _____

h) $\frac{24}{54}$ _____

i) 0.9 _____

j) 0.09 _____

V. Problems and Thought Provokers

1. What number(s) do not have a square root? Explain why not.

2. What numbers have a whole number as their square root? List all of these numbers up to 150. You should be mentally familiar with these numbers as they occur very frequently in math.

3. If a square has an area of 64 m^2 , what is its perimeter?

4. Four students were trying to figure out the square root of 42.

Hewey said, "the square root of 36 is 6, and the square root of 49 is 7, so the square root of 42 is most likely 6.5."

Dewey said "when I punch it into the calculator, it says 6.407406."

Louie said, "I used a graphing calculator and the square root of 42 is 6.4070698."

Chloe said, "the square root of 42 is $\sqrt{42}$."

Which student gave the most accurate answer? Which student gave the best answer?

What answer would you give? Why?

5. Are there any numbers whose square root is the same as the number itself? Which number(s) are they?

6. For each statement below, tell whether it is Always, Sometimes or Never true. Explain why or provide an example why not.

a) The square root of a positive number is larger than the number.

b) The square of an even number is also even.

c) The square root of an even number is an even number.

d) The square root of a prime number is a whole number.

*7. Think about what "square root" means. What do you think "cube root" might mean. Give an example.

*8. What do you think $\sqrt{x^{16}}$ would simplify to? How can you prove your answer is correct?

*9. This chapter is called "Exponents and Square Roots", but really it could easily just be called "Exponents". Why?

Hint: $(\sqrt{9})^2 = 9 = 9^1$, thus if we think of square root as an exponent then $(9^?)^2 = 9^1$ and so by our exponent law $9^{2 \times ?} = 9^1$ and thus $2 \times ? = 1$ so $? = \underline{\quad}$. Neat eh?

5-5 The Pythagorean Theorem

A series of horizontal dotted lines for writing.

5-5 The Pythagorean Theorem

1. Pythagorean Triples are sets of 3 whole number side lengths that form a right triangle. For example (3,4,5) is a Pythagorean Triple because $(3^2 + 4^2 = 5^2)$.

To make Pythagorean Triples, substitute any two positive whole numbers for m and n in the following expressions: ($m > n$)

$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2$$

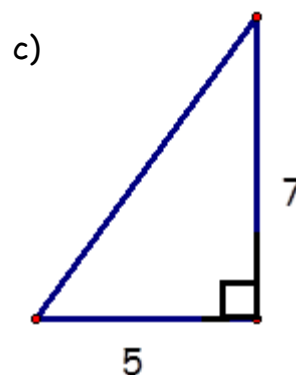
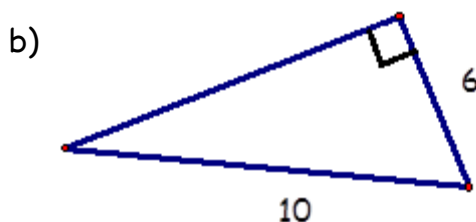
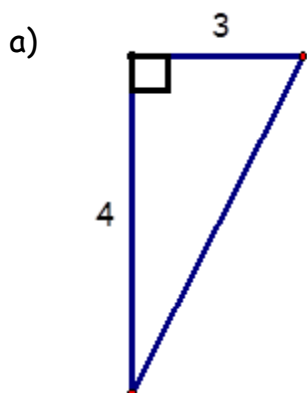
e.g. if $m = 3$ and $n = 1$, then $a = 8$, $b = 6$, and $c = 10$

Find 6 more Pythagorean Triples:

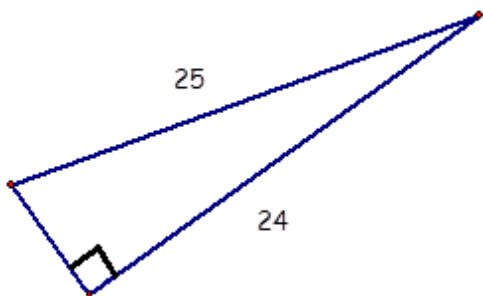
- | | |
|----|----|
| 1) | 2) |
| 3) | 4) |
| 5) | 6) |

For all questions with irrational answers, give your answer in exact form (ie. $\sqrt{45}$), then estimate the answer to one decimal place where necessary. Be sure to show the setup of the Pythagorean Theorem for each question.

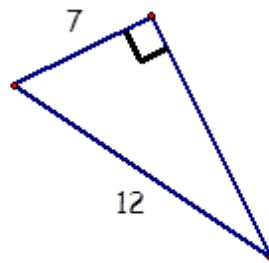
2. Find the length of the missing side:



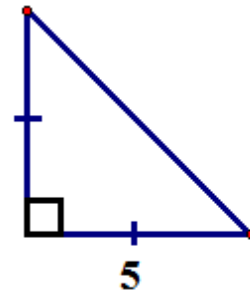
d)



e)

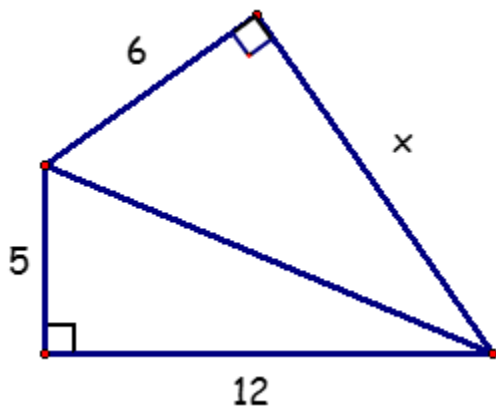


f)

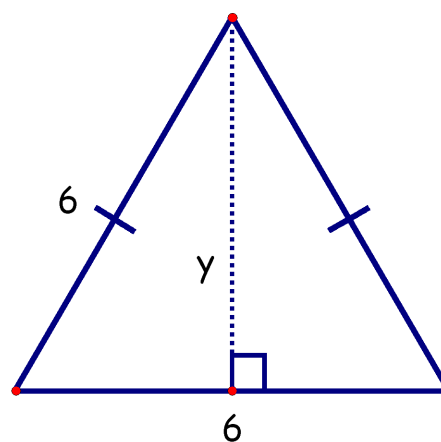


3. Solve for the indicated side (more than one-step may be required).

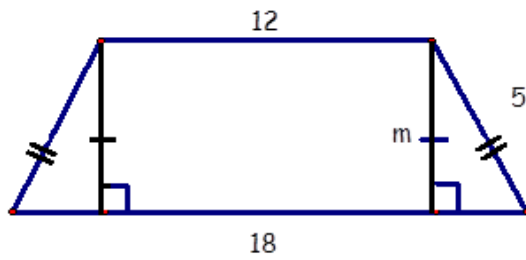
a)



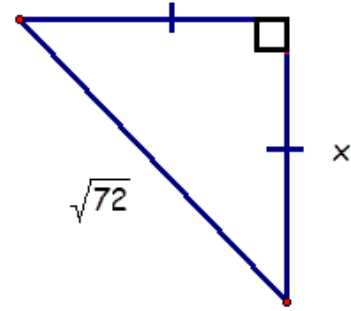
b)



c)



d)



4. Applications (Be sure to draw a diagram for each question!)

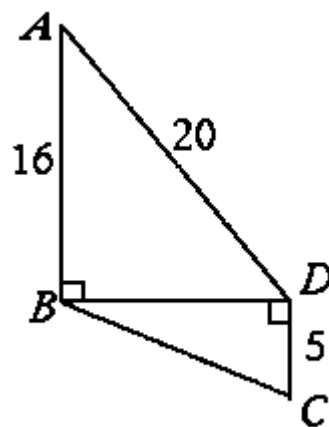
a) A rectangle has a base of 6 cm and a diagonal of 10 cm, what is the height of the rectangle?

b) An 8 m ladder is placed against a wall such that the base of the ladder is 3 m from the base of the wall. How far up the wall does the ladder reach?

c) David Beckham is taking a corner kick from the south-west corner of a soccer field which measures 35 m by 120 m. If he wants to run to the north-east corner, how much shorter will his path there be if he runs directly there as opposed to running along the sideline (edge) of the field?

d) Two cars start at the same point and leave at the same time. One car drives east at 30 m/s, while the other car drives north at 40 m/s. How far apart will they be after 10 s?

e) Juan walks from A to D to C to B and then back to A. How far has he walked?



f) The homes of Fred (F), Sandy (S), Robert (R), and Guy (G) are marked on the rectangular grid with straight lines joining them as shown. $FS = 5$ km, $SG = 9$ km, and $SR = 12$ km. Fred is considering four routes to visit each of his friends:

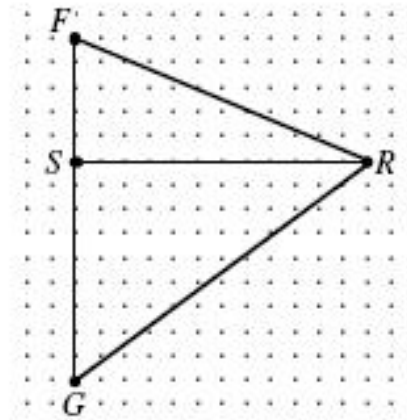
i) $F \rightarrow R \rightarrow S \rightarrow G$

ii) $F \rightarrow S \rightarrow G \rightarrow R$

iii) $F \rightarrow R \rightarrow G \rightarrow S$

iv) $F \rightarrow S \rightarrow R \rightarrow G$

What is the difference between the longest and shortest trip?



5. Will the following side lengths form a right triangle? Justify your answer.

a) 5,6,7

b) 8,15,17

c) 6, 12, 18

5. Two sides of a right triangle are 5 and 7. Find all possible lengths for the third side.