UNIT 1

NUMBER SENSE

1-1 Number Systems

Provide a definition of each of the following number systems with examples:

Natural Numbers ____

____

____

Whole Numbers____

____

____

Integers____

____

____

Rational Numbers____

____

____

Irrational Numbers____

____

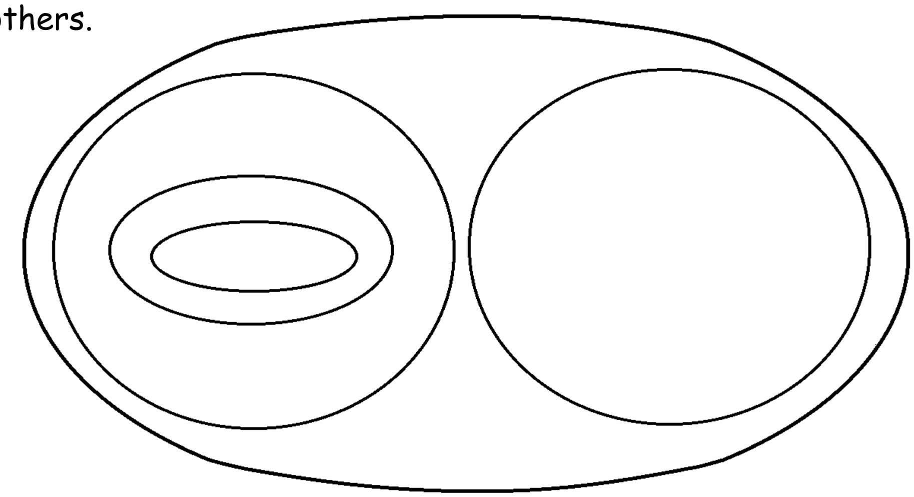
____

Real Numbers____

____

____

1. In the following diagram, show the relationships among the number systems, as some are entirely contained within others.



2. Determine whether each statement is true or false:

a) Every whole number is an integer

b) Every integer is a natural number

c) Every integer is a whole number

d) Irrational numbers are non-repeating, non-terminating decimals.

e) Irrational numbers are real numbers.

f) Every rational number can be written as a fraction.

g) Every integer can be written as a fraction, therefore every integer is a rational number.

h) Every rational number is a whole number.

i) Every natural number is a whole number.

j) There are more rational numbers than irrational numbers.

k) There are an infinite number of real numbers between any two different real numbers

l) The sum of a rational number and an irrational number is always an irrational number.

m) The product of two irrational numbers is always an irrational number.

3. State if the following are Natural, Whole, Integer, Rational, Irrational or Real Numbers. (some may belong to as many as five of these sets).

a) 5

b) –5

c) $\frac{2}{3}$

d) $\sqrt{10}$

e) $\sqrt{-2}$

f) π

g) $0.\overline{1738}$

4. List the numbers in the following set that belong to each set of numbers:

$$\left\{-4,\ -\frac{3}{4},\ 0,\ \sqrt{3},\ \frac{1}{4},\ 5,\ 7.3,\ 1.2583...,\ 5.\bar{3}\right\}$$

a) Natural Numbers ____________________________________

b) Whole Numbers ____________________________________

c) Integers ____________________________________

d) Rational Numbers ____________________________________

e) Irrational Numbers ____________________________________

f) Real Numbers ____________________________________

1-2 Divisibility Rules

Divisible by:	Rule
2	The number is even (it ends in 0, 2, 4, 6 or 8)
3	The sum of its digits is divisible by 3 (eg 741: 7 + 4 + 1 = 12)
4	The last two digits are divisible by 4 (eg 932: 32 is divisible by 4)
5	The number ends in 5 or 0
6	The number is divisible by 2 and 3
8	The last three digits are divisible by 8 (eg 5256: 256 is div by 8)
9	The sum of its digits is divisible by 9 (eg 738: 7 + 3 + 8 = 18)
10	The number ends in 0

1. Using the divisibility rules, tell whether the following numbers are divisible by each of 2, 3, 4, 5, 6, 8, 9, or 10:

a) 9840

b) 13579

c) 489458295

d) 85318075248

2. The number 79732@, where @ represents a digit from 0 – 9, is divisible by 6. What digit(s) could @ represent?

3. The number 479@213, where @ represents a digit from 0 – 9, is divisible by 9. What digit(s) could @ represent?

4. The number 7409258094@, where @ represents a digit from 0 – 9, is divisible by 2 and 9. What digit(s) could @ represent?

5. Find the smallest number greater than 200, which is divisible by 5, 6, 8 and 9.

6. True or False: (If false, give an example showing it is false)

   a) If a number is divisible by 3, then it is also divisible by 9. ______

   b) If a number is divisible by 9, then it is also divisible by 3. ______

c) If a number is divisible by 2 and 5, then it ends in a zero. ______

d) If a number is divisible by 4 then it must be even. _______

e) If a number is divisible by 4, then it must be divisible by 6._______

f) If a number is divisible by 2 and 4, then it must be divisible by 8._______

g) If you write down 4 consecutive numbers, one of them must be divisible by 4.

____

1-3 Factors and Multiples

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1. List all the factors of each of the following.
(Are you confident you've found them all?)

a) 15

b) 28

c) 42

d) 78

e) 63

f) 120

2. List all the factors of 60 and 96. Then list the common factors.

3. List the first ten multiples of each of the following:

a) 2

b) 3

c) 6

d) 9

e) 12

f) 18

4. List all the common multiples of 12 and 18 found from the lists in last question.

5. Find the largest multiple of 7 less than (a) 100, (b) 200, (c) 500, and (d) 1000. [Calc]

6. A certain number of students went on a field trip. They could be split equally into groups of 7, or groups of 6, but if they were split into groups of 5, then one group was a student short. What is the minimum number of students that went on the trip?

1-4 Prime and Composite Numbers

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How do we know if a number is prime?

A man called Eratosthenes (pronounced air-uh-TOS-thuh-neez) found out twenty-two centuries ago. He was a Greek mathematician, astronomer, athlete, poet, and librarian at the University of Alexandria, Egypt around 240 B.C.E.

Eratosthenes' method to tell if a number is prime was to imagine that a giant kitchen sieve contains all the whole numbers. When the sieve is shaken the first time the first prime number "2" stays in the sieve and all the multiples of 2 (4, 6, 8, ...) fall out through the holes in the sieve. The sieve is shaken again. This time the second prime number "3" also stays inside the sieve and the multiples of 3 (9, 15, 21, ...) fall through the sieve. The third time the sieve is shaken, the third prime number "5" is held back and lets through all the multiples of 5 that have not already fallen out. Each time the sieve is shaken the next prime number is held back and its multiples fall through the mesh. The only numbers left in the sieve are the prime numbers.

Apply this method to find all the prime numbers up to 100. When you are done, circle or highlight the prime numbers.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Is the number one prime, composite or neither?

2. If a number is prime, can it also be composite?

3. State whether each of the following statements is true or false. If it is false give a counter-example.

e.g. The statement "All odd numbers are prime numbers" is false.
A counter-example is 9.

a) All prime numbers are odd.

b) If a natural number can be expressed as a product of exactly two factors, then it is prime.

c) If a natural number is divisible by 1 and itself, then the number is prime.

d) All composite numbers can be expressed as a product of prime numbers.

e) If a natural number is not a composite number than it is prime.

f) If a composite number is divisible by a second composite number, then the first composite number is divisible by each of the prime factors of the second composite number.

4. Tell whether the following statements are Always True, Sometimes True, or Never True. Give evidence for your decision.

a) The sum of two prime numbers is prime.

b) The product of two prime numbers is prime.

c) The sum of two composite numbers is composite.

d) The product of two composite numbers is composite.

5. Do you think there are more primes between 1 and 100 or between 101 and 200? Explain your choice.

6. For each of the following, make a conjecture (prediction) then test it out with some examples. Once you're confident your conjecture is true then try to explain/prove why it is should be true.

a) Odd + Odd = __________

b) Even + Even = __________

c) Odd + Even = __________

d) Odd x Odd = __________

e) Even x Even = ___________

f) Odd x Even = ______________

1-5 Prime Factorization

1-5 Prime Factorization

Recall that 2x2x2x7x7 can be written in a more compact way as $2^3 \times 7^2$ or $(2^3)(7^2)$

The Prime Numbers from 1 to 50:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

1. Prime Factorize the following numbers. Show your factor tree for d) through l)

a) 10 = ______________ b) 14 = ______________ c) 31 = ______________

d) 36 = ______________ e) 80 = ______________ f) 48 = ______________

g) 72 = ______________ h) 256 = ______________ i) 124 = ______________

j) 105 = ____________ k) 429 = ____________ l) 455 = ___________

m) 1092 = ___________ n) 252 = ___________ o) 1296 = __________

2. Prime Factorization is sometimes called the DNA or the fingerprint of a number. Why do you think this might be?

1-6 GCF and LCM

Note: This section will not be taught or tested, but you are free to work through it if you want.

I. Greatest Common Factor

1. Find the GCF of the following numbers:

a) 10, 16

b) 12, 18

c) 15, 60

d) 20, 21

e) 18, 42

f) 21, 56

g) 72, 96

h) 72, 132

i) 144, 360

j) 24, 36, 40

k) $2^3 \times 3^4 \times 5 \times 7^2,\ 2^2 \times 3^2 \times 5^3 \times 7 \times 11$
(do not multiply out GCF or the numbers)

2. The GCF of two numbers is 12, and the sum of the numbers is 72. Find the two numbers.

3. Suppose that GCF(48, A) = 6. Find at least 3 possible values of A. Justify your answer.

4. Suppose GCF(48, A) = 5. What are the possible value(s) of A, if any? Justify your answer.

5. Will there always be a GCF of two numbers? Explain.

II. Lowest Common Multiple

(Note: Some of these numbers are repeats from the GCF section. Use that to your advantage.)

1. Find the LCM of the following numbers:

a) 5, 20

b) 14, 25

c) 12, 18

d) 10, 16

e) 18, 42

f) 72, 108

g) 240, 450

h) 8, 16, 24

i) 18, 20, 32

j) $2^3 \times 3^4 \times 5 \times 7^2,\ 2^2 \times 3^2 \times 5^3 \times 7 \times 11$ (do not multiply out LCM or the numbers)

2. Suppose LCM(6, A) = 36. What are the possible value(s) of A, if any? Justify your answer

3. Suppose LCM(18, A) = 72. Find 3 possible values of A. Justify your answer.

4. If you just multiply two numbers together, you will always get a common multiple, but not necessarily the lowest. What kind of numbers would give you the LCM when you multiply them together? Give some examples.

5. Look at questions 1 and 6, you will notice you worked with 3 pairs of numbers that were the same in both question (10, 16), (12, 18), and (18, 42). For each pair of numbers find the product of their GCF and LCM, and then find the product of the two numbers. What do you notice? Make a conjecture (hypothesis).

(10, 16): GCF______ x LCM ________ = ___________ 10 x 16 = ___________

(12, 18): GCF______ x LCM ________ = ____________ 12 x 18 = ___________

(18, 42): GCF______ x LCM ________ = ___________ 18 x 42 = ___________

Conjecture:

Try to explain why this happens:

1-7 Integers: Addition and Subtraction

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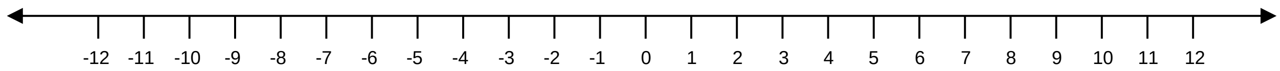


I. Integer Addition

1. $7 + (-2) =$ _______

2. $(-8) + 16 =$ _______

3. $20 + (-13) =$ _______

4. $-8 + (-2) =$ _______

5. $12 + (-18) =$ _______

6. $(-7) + (-7) =$ _______

7. $(-7) + 2 =$ _______

8. $0 + (-36) =$ _______

9. $29 + (-33) =$ _______

10. $17 + (-6) =$ _______

11. $-3 + (-2) =$ _______

12. $(-4) + 6 =$ _______

13. $-5 + (-5) =$ _______

14. $(-12) + (-18) =$ _______

15. $-4 +$ _____ $= 8$

16. _____ $+ (-3) = -9$

For each of these statements, tell whether the answer will be definitely **positive**, **negative** or **dependent** (on the size of the numbers). Give 2 examples to support your answer for each question.

17. A positive plus a positive is _________:

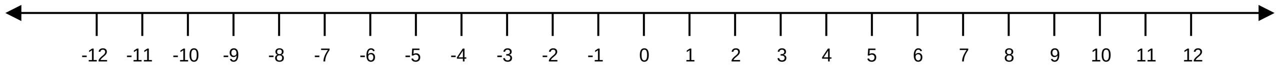
18. A positive plus a negative is _________:

19. A negative plus a positive is __________:

20. A negative plus a negative is _________:

21. Explain what you are thinking to do a question such as $-8+3$:

II. Integer Subtraction



1. 7 – (–2) = _______

2. –8 – 16 = _______

3. 20 – 13 = _______

4. –8 – (–2) = _______

5. 12 – 18 = _______

6. (–7) – (–7) = _______

7. –2 – 7 = _______

8. 0 – (–36) = _______

9. 29 – 33 = _______

10. –5 – 5 = _______

11. 17 – (–6) = _______

12. –3 – (–2) = _______

13. –4 – 6 = _______

14. (–12) – (–18) = _______

15. –3 – _____ = –9

16. 6 – _____ = 13

17. 9 – _____ = –2

18. _____– (– 4) = 19

For each of these statements, tell whether the answer will be definitely **positive**, **negative** or **dependent** (on the size of the numbers). Give 2 examples to support your answer for each question.

19. A positive minus a positive is _________:

20. A positive minus a negative is ________:

21. A negative minus a positive is ________:

22. A negative minus a negative is ________:

23. Explain what you are thinking to do a question such as $-8-3$:

24. Explain what you are thinking to do a question such as $-8-(-3)$:

1-8 Integers: Multiplication and Division

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1. $7(8) =$ _____

2. $-9(6) =$ _____

3. $(-5)(12) =$ _____

4. $(-4)(-8) =$ _____

5. $-15 \times -3 =$ _____

6. $4(-17) =$ _____

7. $8(13) =$ _____

8. $-12(7) =$ _____

9. $20(-15) =$ _____

10. $(-4)(-15) =$ _____

11. $63 \div (-9) =$ _____

12. $51 \div (-17) =$ _____

13. $-48 \div (-6) =$ _____

14. $-32 \div 2 =$ _____

15. $-64 \div (8) =$ _____

16. $\frac{-75}{-5} =$ _____

17. $\frac{-400}{25} =$ _____

18. $-6 \div \_\_\_\_ = 1$

19. $-36 \div \_\_\_\_ = 9$

20. $6 \times \_\_\_\_ = -30$

21. $-12 \times \_\_\_\_ = -108$

22. $-48 \div \_\_\_\_ = -4$

23. $\_\_\_\_ \div (-6) = 7$

24. $\_\_\_\_ \times (-8) = 72$

25. Give a general rule about multiplying 2, 3, 4, or many negative numbers together.

Order of Operations

B: Simplify expressions inside brackets first.

E: Simplify expressions with exponents next (coming in unit 5).

D: Do division and multiplication next, in the order they appear going
M: from left to right.

A: Do addition and subtraction last, in the order they appear going
S: from left to right.

1. $-4 + (-8) + 15 =$ ______

2. $(-5) + (-12) + 23 =$ ______

3. $8 + (-6) + 1 + (-7) =$ ______

4. $8 - 6 - 1 - 7 =$ ______

5. $-7 - 24 - 5 =$ ______

6. $-4 - (-8) - 15 =$ ______

7. $8 - (2 - 7) =$ ______

8. $-5 - (-12) - 23 =$ ______

9. $-6 - (3 - 7) =$ ______

10. $-(11 - 4) =$ ______

11. $(-6)(4)(3) =$ _____

12. $(-2)(4)(-8) =$ _____

13. $(-4)(-2)(-1)(-5) =$ _____

14. $\dfrac{(-4)(-6)(10)}{5(-3)} =$ _____

15. $\dfrac{-8(5-8)}{2-8} =$ _____

16. $4+(-3-4)(-2) =$ ______

17. $8-4(-2)\div 2 =$ ______

18. $-3\times 12\div(-4) =$ _____

19. $-3(2-7)+4(-2+5) =$ ______

20. $15+12\div(-3)+1-2(-4) =$ ______

21. $-3\left(5\times 2-7\right)+4\left(-3+6\div 3\ \right) =$ ______

22. $-3\left(5\times\left(2-7\right)\right)+4\left(\left(-3+6\right)\div 3\ \right) =$ _______

1-9 Zero and One

1-9 Zero and One

Properties of Zero:

I) $n+0=n$ II) $n-0=n$ III) $n-n=0$

IV) $n\times 0=0$ V) $0\div n=0$ $(n\neq 0)$ VI) $n\div 0$ is undefined.

(where n can be any real number)

NOTE: It is mathematically impossible to divide by zero!

Properties of One:

I) $n\times 1=n$ II) $n\div 1=n$ III) $n\div n=1$ $(n\neq 0)$

IV) $n^1=n$ V) $1^n=1$

(where n can be any real number)

Evaluate:

1. $a\times 0=$____ 2. $5\times 0=$____ 3. $0+0=$____ 4. $0\times 5=$____

5. $16\div 16=$____ 6. $x-x=$____ 7. $0+1=$____ 8. $0\times 1=$____

9. $6+0=$____ 10. $6\div 6=$____ 11. $6\times 1=$____ 12. $6\div 1=$____

13. $\frac{49}{49}=$____ 14. $49^1=$____ 15. $49\div 1=$____ 16. $49\div 0=$____

17. $r\times 1=$____ 18. $\frac{r}{r}=$____ $(r\neq 0)$ 19. $0\times r=$____ 20. $r^1=$____

21. $0+b=$____ 22. $y\div 1=$____ 23. $1\times c=$____ 24. $f-0=$____

25. $n - 0 =$____ 26. $p - p =$____ 27. $0 \div 8 =$____ 28. $d \div 0 =$____

29. $66 - 66 =$____ 30. $0 \div g =$ ____ $(g \neq 0)$ 31. $24 \div 24 =$____

32. $0 + 36 =$____ 33. $1 \times 0 =$____ 34. $(-4)(1) =$____

35. $(-4)^1 =$____ 36. $-4 \times 0 =$____ 37. $0 \div (-4) =$____

38. $(-4) \div 0 =$____ 39. $0 + (-4) =$___ 40. $12^1 \div 1^{12} =$____

41. Explain why you can't divide by zero.

42. Is zero positive or negative?