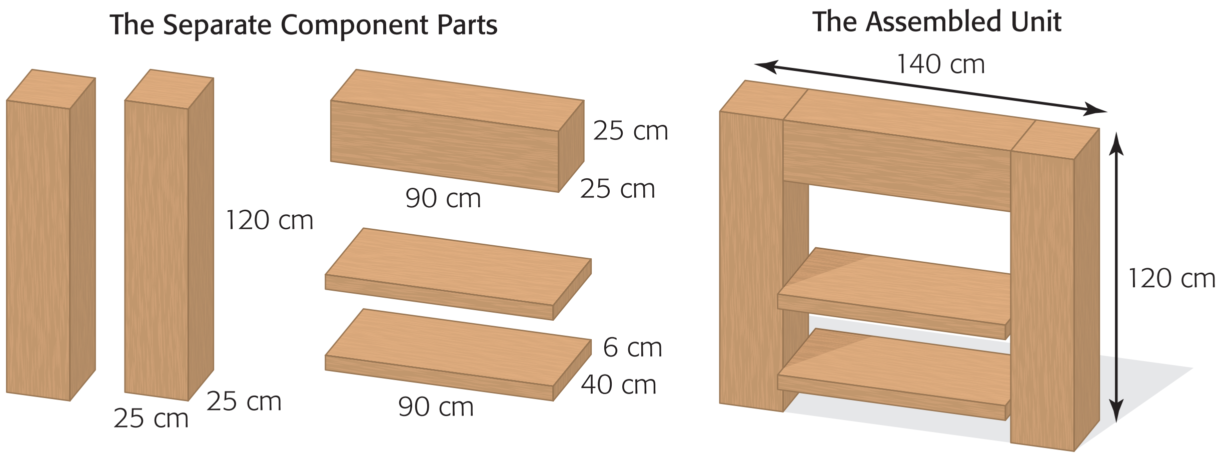
4.4 Calculating the Surface Area of Composite Objects

GOAL

Determine the surface area of composite 3-D objects.

LEARN ABOUT the Math

Derek must assemble an entertainment unit and then paint it.



What is the total area Derek needs to paint?

A. Calculate the total surface area of the components.

B. Calculate the area of overlap of the components when the unit is assembled.

C. What is the total area of the components that will not be painted? Explain your answer.

D. The ends of the stand that touch the floor will not be painted. Determine the total area to be painted.

Reflecting

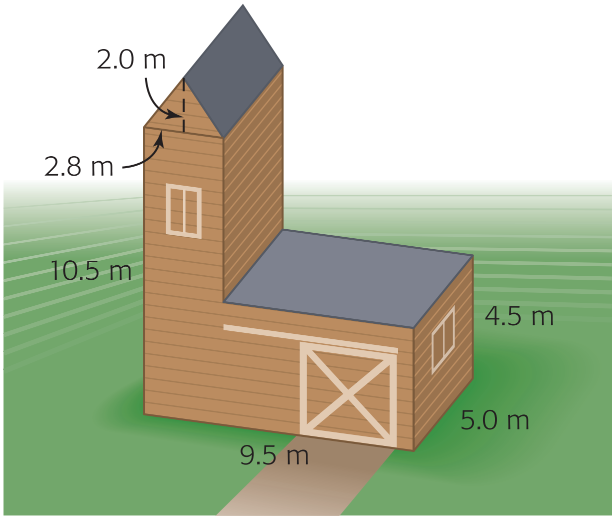
E. How can you use the area of overlap to determine the surface area of a composite shape?

F. Suppose the assembled unit had been cut into component parts in a different way. Would the total surface area change?

WORK WITH the Math

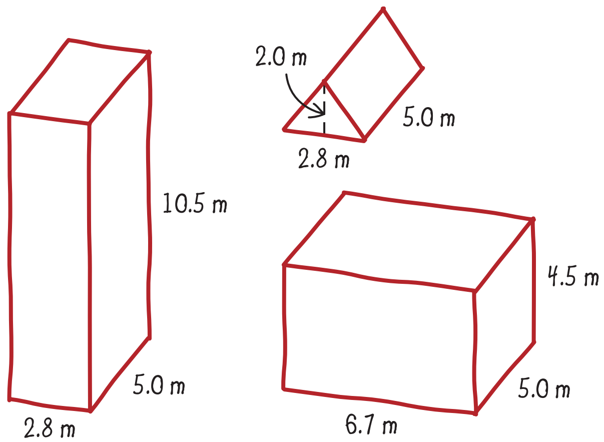
EXAMPLE 1 | **Calculating surface area of a composite object**

A farmer wants to paint a building on his farm, and needs to know the total area to be painted. Determine the total area of the building, including the shutters covering the windows.



Shelby's Solution: Using area of overlap

I'll divide the building into component parts.



I sketched each part. I knew that each part was 5.0 m wide.

The building is 9.5 m long and the tall rectangular prism is 2.8 m long, so the short rectangular prism is $9.5 - 2.8 = 6.7$ m long.

Surface area of tall rectangular prism

$= 2lw + 2lh + 2wh$

$= 2(2.8 \times 5.0) + 2(2.8 \times 10.5) + 2(5.0 \times 10.5)$

$= 191.8 \text{ m}^2$

I calculated the surface area of each rectangular prism by thinking about all of the faces. I knew that there were three sizes of faces and two of each of those sizes.

Surface area of short rectangular prism

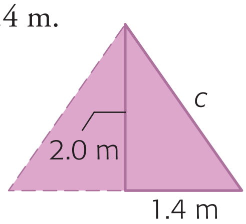
$= 2lw + 2lh + 2wh$

$= 2(6.7 \times 5.0) + 2(6.7 \times 4.5) + 2(5.0 \times 4.5)$

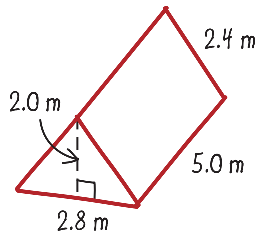
$= 172.3 \text{ m}^2$

Surface area of the triangular prism
Half of the base of the triangle is 1.4 m.
Let the hypotenuse be c.

First, I needed to calculate the dimensions of the triangular prism. I used the Pythagorean theorem.



$$c^2 = (1.4)^2 + (2.0)^2$$
$$c^2 = 5.96\ \text{m}^2$$
$$c \doteq 2.4\ \text{m}$$



Area of two triangles
$= 2(bh \div 2)$
$= bh$
$= 2.8 \times 2.0$
$= 5.6\ \text{m}^2$

Area of two roof faces
$= 2(2.4 \times 5.0)$
$= 24.0\ \text{m}^2$

Area of base
$= 2.8 \times 5.0$
$= 14.0\ \text{m}^2$

Total area of triangular prism
$= 5.6 + 24.0 + 14.0$
$= 43.6\ \text{m}^2$

Total surface area of the three prisms
$= 191.8 + 172.3 + 43.6$
$= 407.7\ \text{m}^2$

Areas of overlap

I needed to calculate the areas of overlap that wouldn't be painted. There are areas of overlap where the triangular prism and the tall rectangular prism meet, and where the tall and short rectangular prisms meet.

Tall rectangular prism and triangular prism
$= 2.8 \times 5.0$
$= 14.0\ \text{m}^2$

Tall and short rectangular prisms
$= 5.0 \times 4.5$
$= 22.5\ \text{m}^2$

Surface area of building
$= 407.7 - 2(14.0) - 2(22.5)$
$= 334.7\ \text{m}^2$

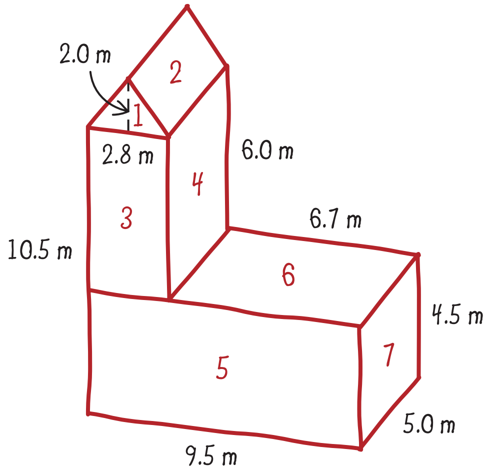
I had included the area of overlap between the triangular prism and the tall prism in both calculations of surface area, so I had to subtract the overlap area twice. The same was true of overlap of the tall and short rectangular prisms.

Surface to be painted
$= 334.7 - (2.8 \times 5.0) - (6.7 \times 5.0)$
$= 287.2\ \text{m}^2$

Since the farmer won't paint the bases of the two rectangular prisms, I needed to subtract those areas.

The farmer needs enough paint to cover $287.2\ \text{m}^2$.

Derek's Solution: Using exposed faces



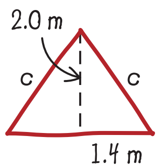
The sides and roof were made of rectangles and triangles. I decided to calculate the area of each shape that is visible, and then add. I numbered the visible shapes that make up the building.

height of sections 3 and 4 $= 10.5 - 4.5$
$= 6.0$ m

I calculated the missing dimensions and included them in my diagram.

width of section 2, c

$c^2 = (1.4)^2 + (2.0)^2$
$= 5.96 \text{ m}^2$
$c \doteq 2.4$ m



For section 2, I calculated the dimension I needed using the Pythagorean theorem $c^2 = a^2 + b^2$.

Area of section 1
$= \frac{1}{2}(2.8 \times 2.0)$
$= 2.8 \text{ m}^2$

Section 1 is a triangle. I used $A = \frac{1}{2}bh$.

Area of section 2
$= 5.0 \times 2.4$
$= 12.0 \text{ m}^2$

Area of section 3
$= 6.0 \times 2.8$
$= 16.8 \text{ m}^2$

All of the other sections are rectangles. I used $A = lw$.

Area of section 4
$= 6.0 \times 5.0$
$= 30.0 \text{ m}^2$

Area of section 5
$= 9.5 \times 4.5$
$= 42.75 \text{ m}^2$

I kept the hundredths in the answer for section 5 because I knew that I was going to have to do another calculation with the number.

Area of section 6
$= 6.7 \times 5.0$
$= 33.5 \text{ m}^2$

Area of section 7
$= 5.0 \times 4.5$
$= 22.5 \text{ m}^2$

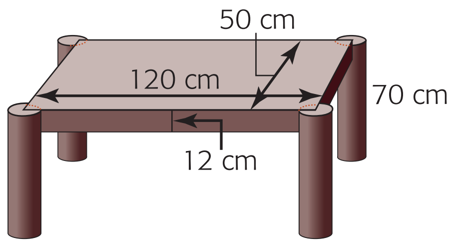
Area to be painted
$= 2(\text{section } 1) + 2(\text{section } 2) + 2(\text{section } 3)$
$+ 2(\text{section } 4) + 2(\text{section } 5) + \text{section } 6 + 2(\text{section } 7)$
$= 2(2.8) + 2(12.0) + 2(16.8) + 2(30.0)$
$+ 2(42.75) + 33.5 + 2(22.5)$
$= 287.2 \text{ m}^2$

I needed to include two of all the sections except section 6.

The farmer needs enough paint to cover 287.2 m^2.

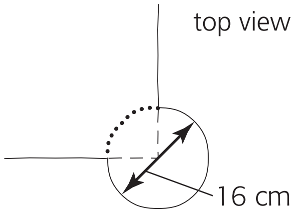
EXAMPLE 2 | Calculating surface area of objects with cylinders

Austin's sister builds tables to sell at craft fairs. Austin helps her by painting them. This design has legs of diameter 16 cm. A 1 L can of paint covers 7 m^2. How many cans of paint will Austin need to paint 12 tables?



Austin's Solution

The tabletop is a rectangle plus four $\frac{3}{4}$ circles.



I imagined the tabletop as a rectangle with its corners at the centre of each leg. I drew a diagram of the top.
I decided to paint only the visible surfaces.

Area of rectangle
$= lw$
$= 120 \times 50$
$= 6000 \text{ cm}^2$

Area of $\frac{3}{4}$ of circle
$= \left(\frac{3}{4}\right)\pi r^2$
$= \left(\frac{3}{4}\right)\pi(8)^2$
$\doteq 151 \text{ cm}^2$

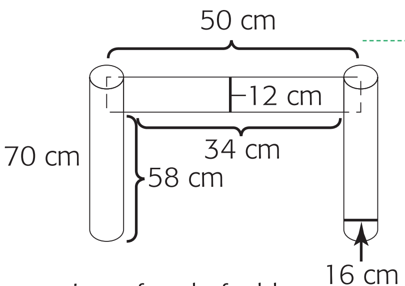
Each corner overlaps $\frac{1}{4}$ of the circular legs. I calculated the area of the rectangle and the area of $\frac{3}{4}$ of one circle.

Area of top of table
$\doteq 6000 + 4(151)$
$\doteq 6604 \text{ cm}^2$

I added all the areas. I multiplied the area of the $\frac{3}{4}$ circle by 4 since there are four legs.

Width of end rectangle
$= 50 - 2 \times 8$
$= 34 \text{ cm}$

Length of side rectangle
$= 120 - 2 \times 8$
$= 104 \text{ cm}$



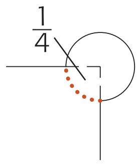
view of end of table

I determined the areas of the rectangular sides of the table. The legs overlapped the edges of the table, reducing the width of the side rectangles by $2r$.

Total area of sides
$= 2(34 \times 12 + 104 \times 12)$
$= 3312 \text{ cm}^2$

I multiplied the 34 and the 104 both by 12 because the height of the sides is 12 cm.

I'll calculate the surface area of one leg and subtract the $\frac{1}{4}$ cylinder in the tabletop.



Surface area of one leg

$= (2\pi rh) - \text{overlap}$

$= (2\pi(8) \times 70) - \frac{1}{4}(2\pi(8) \times 12)$

$= 1120\pi - 48\pi$

$\doteq 3368 \text{ cm}^2$

Since $\frac{1}{4}$ of the top 12 cm of each leg was not visible, I reduced the area of each leg by that amount. The area of the side of a cylinder is $2\pi rh$. I had included the area for the top of each leg when I calculated the area of the tabletop, and I wouldn't paint the bottom of the legs.

Surface area of one table

$= \text{top} + \text{sides} + 4 \text{ legs}$

$\doteq (6604 + 3312 + 4(3368))$

$\doteq 23\ 388 \text{ cm}^2$

The underside of the table is not visible, so I decided not to paint it.

Surface area of 12 tables

$\doteq 12 \times 23\ 388$

$\doteq 280\ 656 \text{ cm}^2$

To calculate the paint needed for 12 tables, I multiplied by 12.

Surface area in m^2

$\doteq 280\ 656 \div (100)^2$

$\doteq 28 \text{ m}^2$

I wrote this value in square metres since the paint coverage was given in square metres. I knew 100 cm = 1 m, so $100^2 \text{ cm}^2 = 1 \text{ m}^2$.

Number of cans needed $= \frac{28}{7} = 4$

I need 4 cans of paint.

I divided the total area by the area that one can of paint covers.

In Summary

Key Ideas

- One way to determine a composite shape's surface area is to calculate the surface area of each component, and then subtract twice the area of overlap of each component.
- Another way is to determine the area of each exposed surface and add.

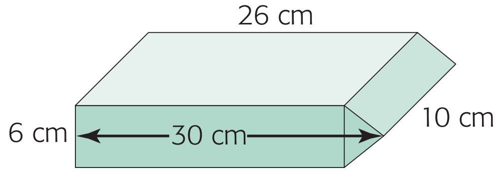
Need to Know

- When you are determining surface area, keep the context in mind. For example, to determine how much paint is needed to paint a flat-bottom dresser, omit the area of the bottom because it would not be painted.
- The area of overlap of component parts cannot be seen on the outside of the composite object.
- No matter how you decompose an object, its surface area will be the same.

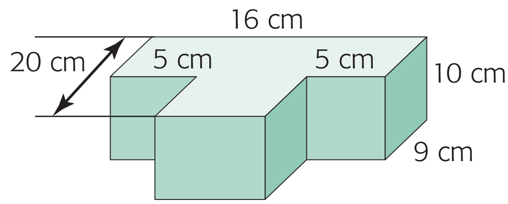
Checking

1. Calculate the surface area of each shape.

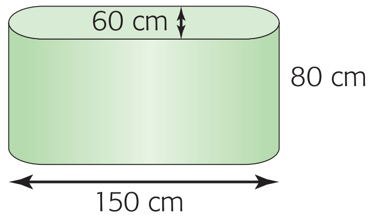
a)



b)

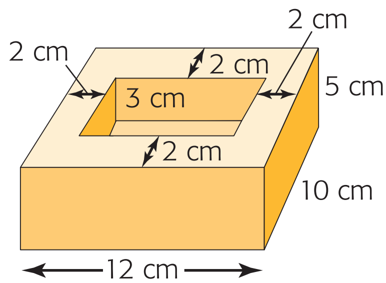


2. Determine the surface area of the rounded aquarium stand shown at right, not including the bottom.

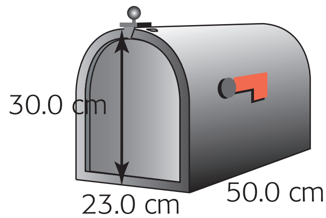


Practising

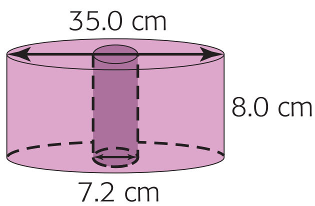
3. Determine the surface area of the paperclip holder shown at right.



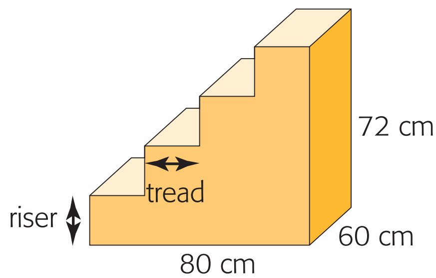
4. Jordy is making a mailbox, as shown. Calculate the amount of metal required, not including the flag and the top catch.



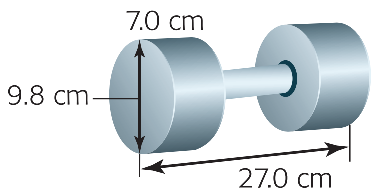
5. A Bundt cake has a cylindrical hole in the centre as shown. Calculate the amount of icing required to ice the top and exposed sides of the cake.



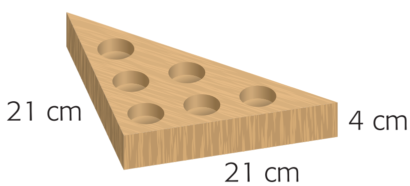
6. Determine the amount of carpeting needed to cover the entire surface of these pet stairs.

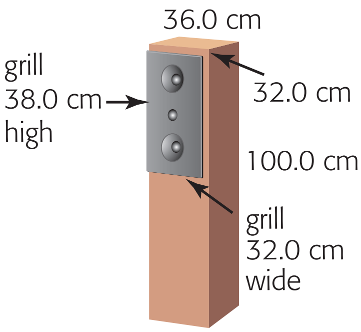


7. The hand grip on this dumbbell has a circumference of 10 cm. Calculate the total surface area of the dumbbell.



8. A wooden tea light candle holder is in the shape of an isosceles right triangle. Each insert for a tea light is 3.5 cm in diameter and 1.5 cm deep. The entire holder, including the inserts, is to be stained. Calculate the total area that will be stained.



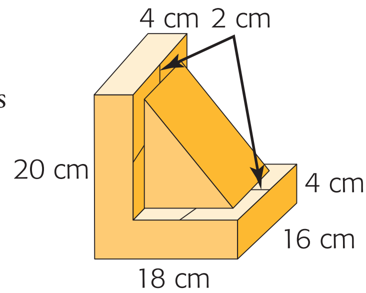


9. This stereo speaker has the measurements shown. Determine the surface area of the speaker, not including the grill.

10. Kelly makes and sells bookends as shown. The triangular prism support is inset by 2 cm on all sides. She paints the bookends on all surfaces.

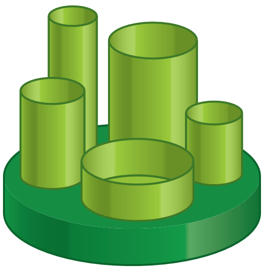
a) Determine the total surface area for a pair of bookends.

b) One litre of the paint Kelly uses covers 5 m^2. Determine how many litres of paint she will need to paint 100 pairs of bookends.



11. Multiple choice. The metal frame of a greenhouse is covered with polycarbonate panels. It is 3.6 m long and 2.4 m wide, with walls 1.5 m high and a total height of 2.3 m. Which choice is the best estimate for total area of polycarbonate needed?

A. 30 m^2 **B.** 40 m^2 **C.** 50 m^2 **D.** 60 m^2

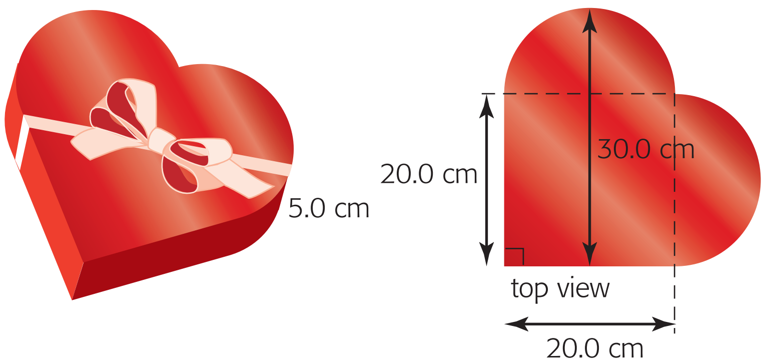


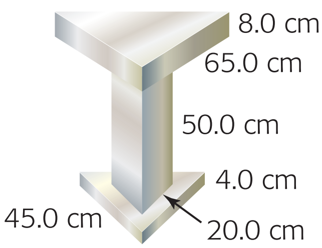
12. Multiple choice. A desk organizer has a circular base with diameter of 16 cm, and height of 2.5 cm. The organizing cylinders have diameters of 3 cm, 3.5 cm, 4 cm, 5.5 cm, and 7 cm.

Which choice is the best estimate for the exposed dark green surface area of the base of the organizer, including the bottom?

A. 690 cm^2 **B.** 420 cm^2 **C.** 325 cm^2 **D.** 235 cm^2

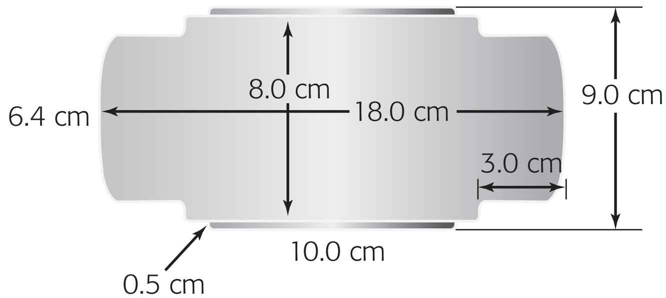
13. Determine the total surface area of the heart-shaped box.



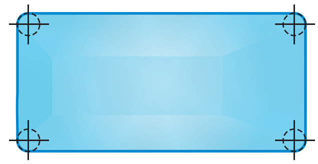


14. Each section of the acrylic table shown is an isosceles right triangular prism. Determine the total surface area, including the undersides of the top and the base.

15. The carrying case of a portable video game is 18.0 cm long by 8.0 cm wide by 4.0 cm high. The hinge on one side and the clasp on the other side are both 10.0 cm long by 0.5 cm wide by 1.5 cm high. Calculate the total surface area of the case.



16. A swimming pool is 12.7 m long, 6.0 m wide, and 1.5 m deep. Its circular corners have a radius of 0.7 m. Explain how you would determine the surface area of the pool's vinyl liner.

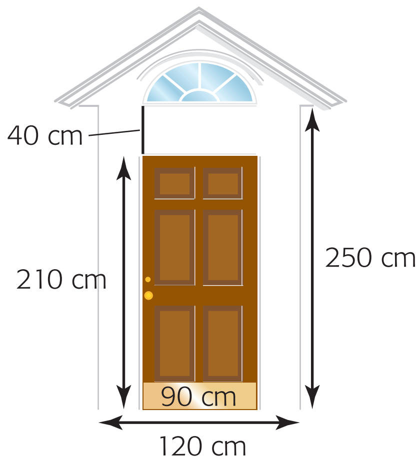


Closing

17. When you are determining the surface area of a composite object, do you always need to know the area of the overlapping faces? Explain with an example.

Extending

18. This door frame is 315 cm high, 140 cm wide, and 6 cm deep. Determine the surface area of the frame that will be painted, not including the pieces in the window.



19. This inground pool is 1.4 m deep. Calculate the area of the vinyl liner for the pool.