Solids of Revolution

Tuesday, October 11, 2016 1:12 PM

Solution to todays challenge:

$$V = \pi \int_0^{\pi} \sin^2(x)\, dx$$

Let $u = \sin(x)$

then $\frac{du}{dx} = \cos(x)$

so $dx = \frac{du}{\cos(x)}$ still an x!

This substitution does not work.

Consider $\cos(2x) = 1 - 2\sin^2(x)$

so $2\sin^2(x) = 1 - \cos(2x)$

$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$$\therefore \pi \int_0^{\pi} \sin^2(x)\, dx = \pi \int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos(2x)\, dx$$

$$= \frac{\pi}{2} \int_0^{\pi} 1 - \cos(2x)\, dx$$ * see below

$$= \frac{\pi}{2} \left[x - \frac{\sin(2x)}{2} \right]_0^{\pi}$$

$$= \frac{\pi}{2} \left[\pi - \frac{\sin(2\pi)}{2} - \left(0 - \frac{\sin(2 \times 0)}{2} \right) \right]$$

$$= \frac{\pi}{2}\left(\pi\right)$$

$$= \frac{\pi^2}{2}$$

* To integrate $\cos(2x)$ you could substitute:

$$\int \cos(2x)\,dx$$

Let $u(x) = 2x$

Then $\frac{du}{dx} = 2$

So $dx = \frac{1}{2}\,du$

$$\therefore \int \cos(2x)\,dx = \int \cos(u)\,\frac{1}{2}\,du$$

$$= \frac{1}{2}\int \cos(u)\,du$$

Remember you're working with a definite integral and your limits of integration would change also.