

# **PRECALCULUS**

# **MATHEMATICS 12**

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Review of basic functions;

Addition, subtraction, multiplication and division of functions; composition of functions.

- **48 PROBLEMS**

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- **118 PROBLEMS**

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Graphing and polynomial characteristics;

Solving equations and inequalities algebraically and graphically.

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- **51 PROBLEMS**

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- **165 PROBLEMS**
- **REVIEW PACKAGE**

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- **194 PROBLEMS**

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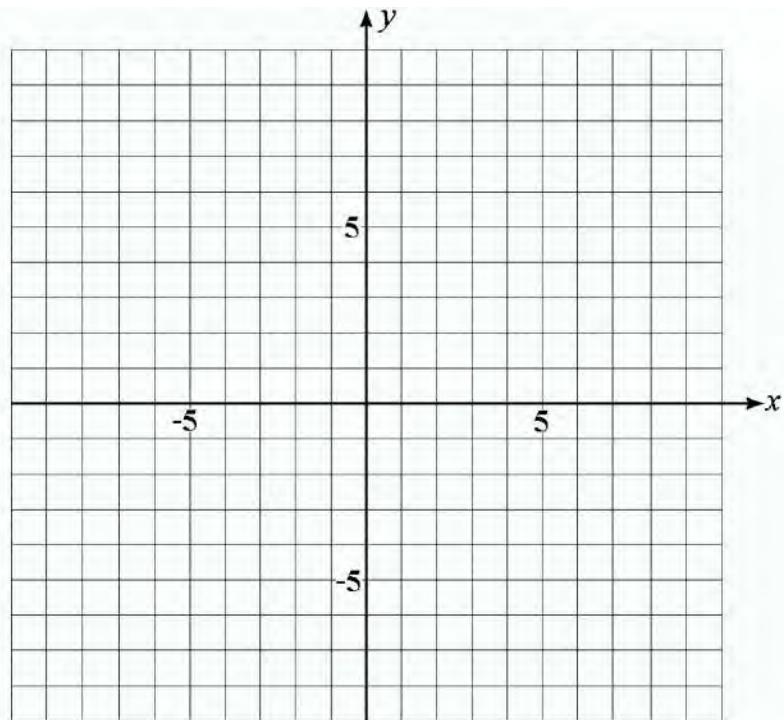
- **121 PROBLEMS**

# **COMBINING FUNCTIONS**

## REVIEW OF BASIC FUNCTIONS

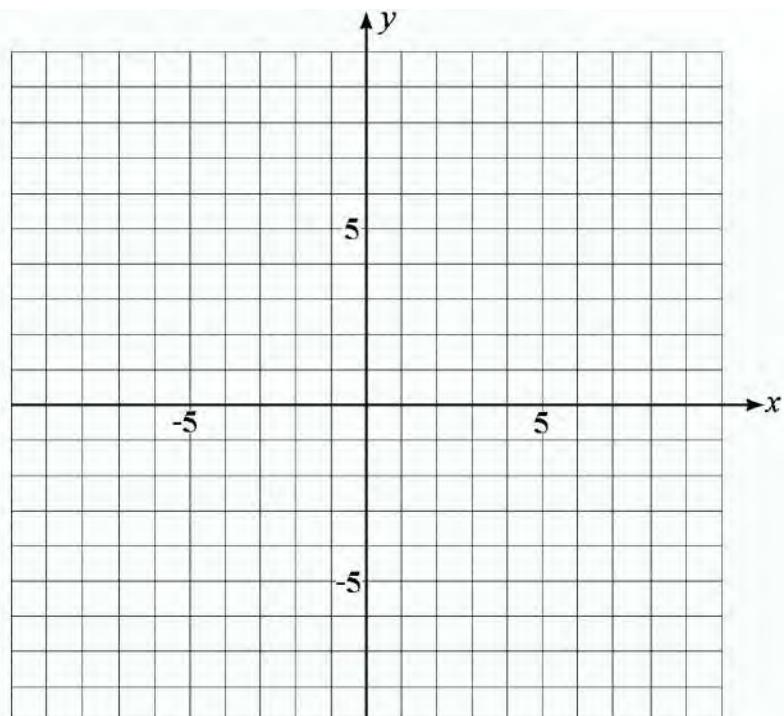
1. LINEAR FUNCTIONS: a)  $y = 3x - 4$       b)  $2x - 5y = 10$

	$a$	$b$
$x$ -intercept		
$y$ -intercept		
slope		



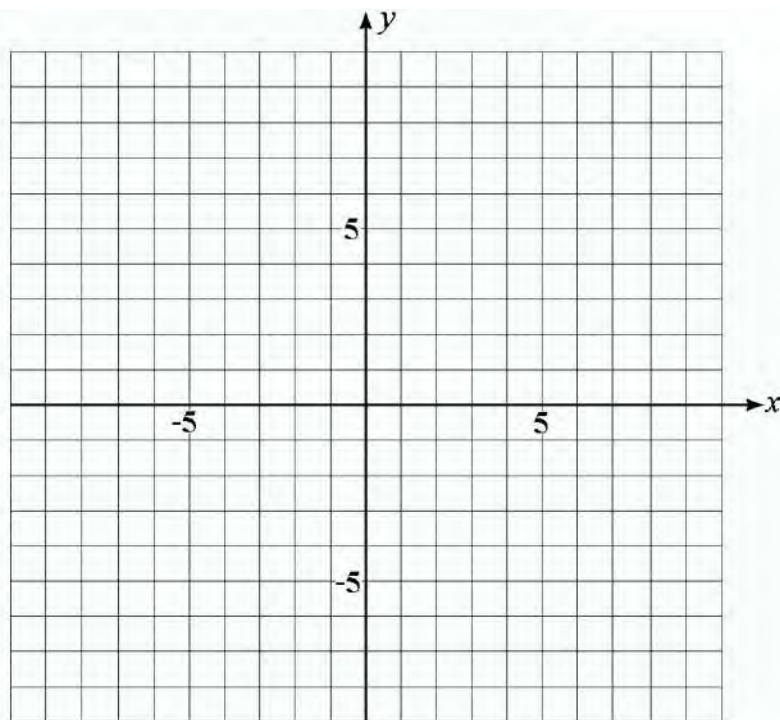
2. QUADRATIC FUNCTIONS: a)  $y = \frac{1}{4}(x - 3)^2 + 2$       b)  $y = 2x^2 - 8x + 3$

	$a$	$b$
$x$ -intercept		
$y$ -intercept		
Vertex		
Domain		
Range		



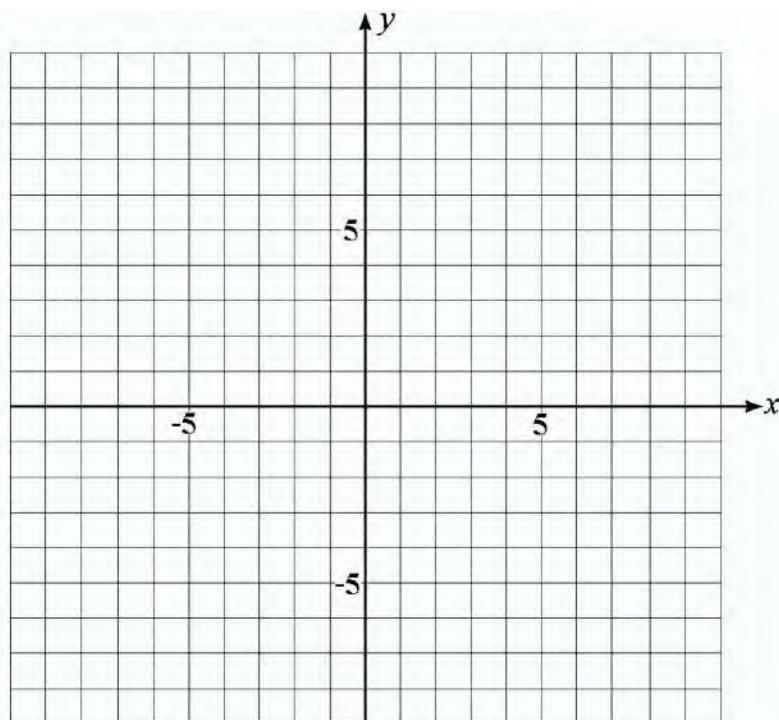
3. BASIC CUBIC FUNCTION:  $y = x^3$

$x$	$y$
-3	
-2	
-1	
0	
1	
2	
3	



4. RECIPROCAL FUNCTION:  $y = \frac{1}{x}$

$x$	$y$
-3	
-2	
-1	
-0.5	
0	
0.5	
1	
2	
3	



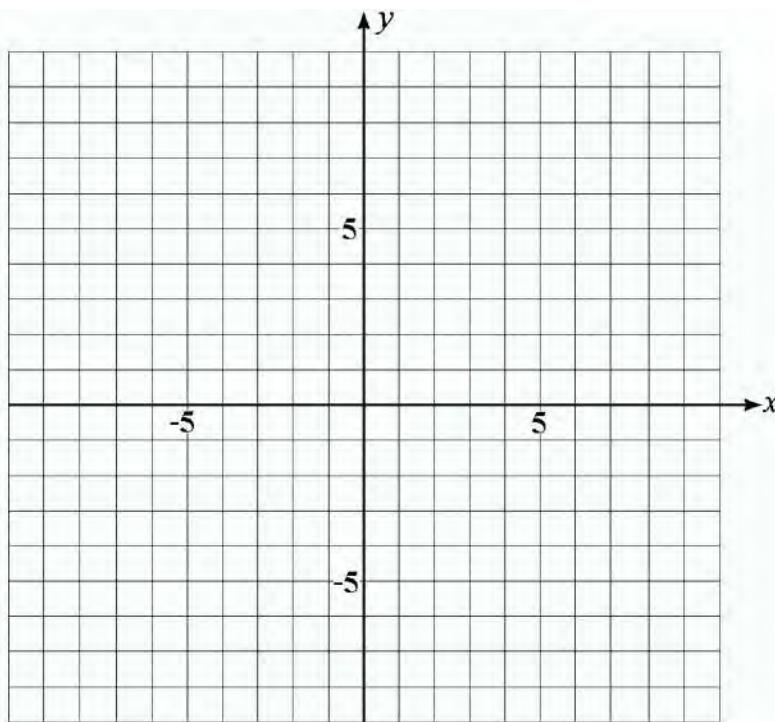
Domain:

Range:

Asymptotes:

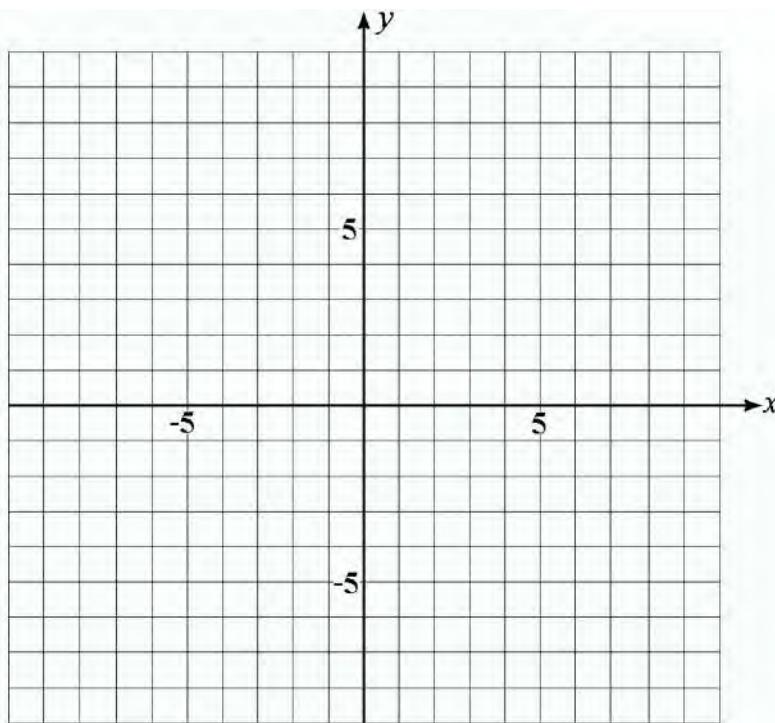
5. SQUARE ROOT FUNCTION:  $y = \sqrt{x}$

$x$	$y$
0	
1	
4	
9	



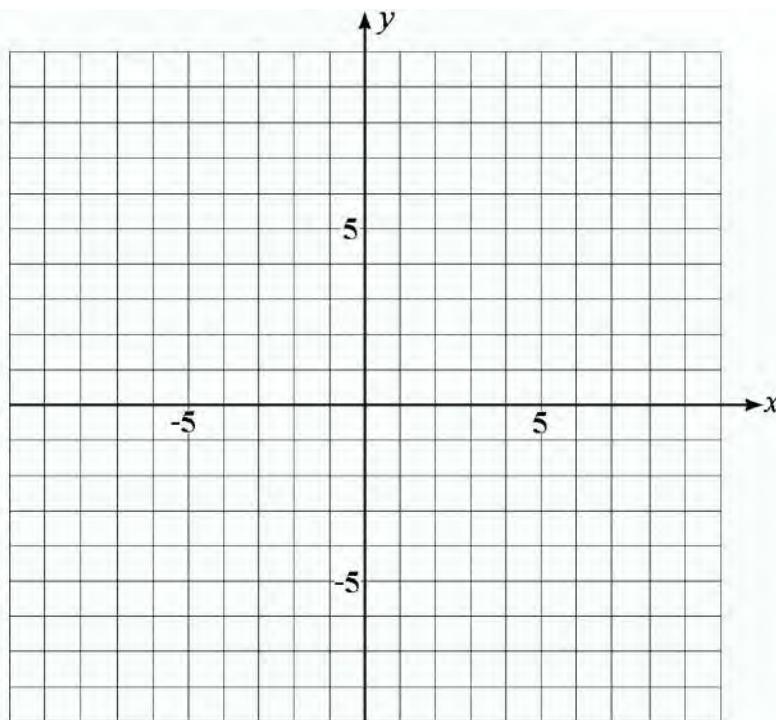
6. ABSOLUTE VALUE FUNCTION:  $y = |x|$

$x$	$y$
-3	
-2	
-1	
0	
1	
2	
3	



7. EXPONENTIAL FUNCTION:  $y = 2^x$

$x$	$y$
-3	
-2	
-1	
0	
1	
2	
3	



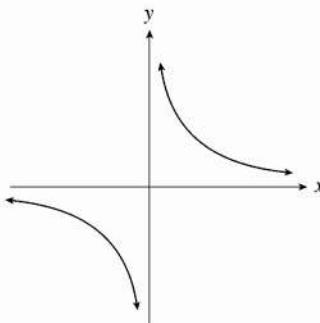
Domain:

Range:

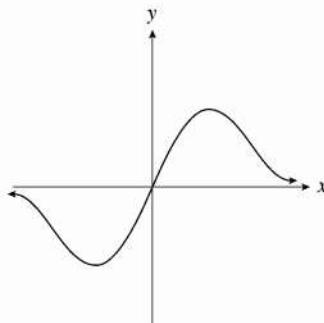
Asymptote:

8. Given  $f(x) = x$  and  $g(x) = \frac{1}{x}$ , which graph best represents  $y = f(x) + g(x)$  ?

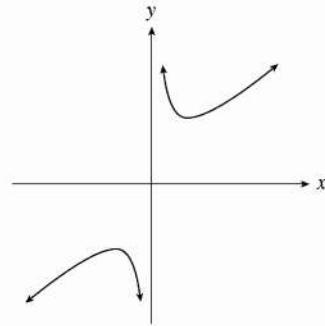
A.



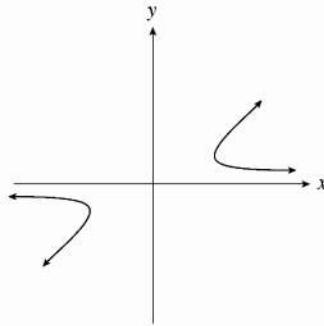
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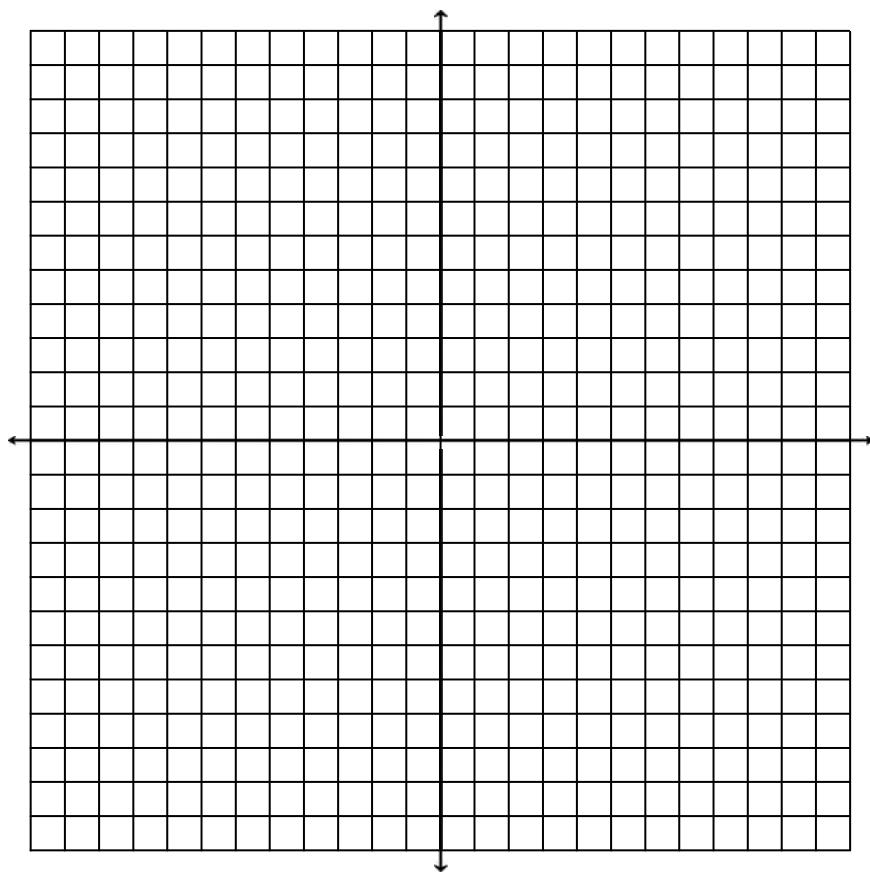
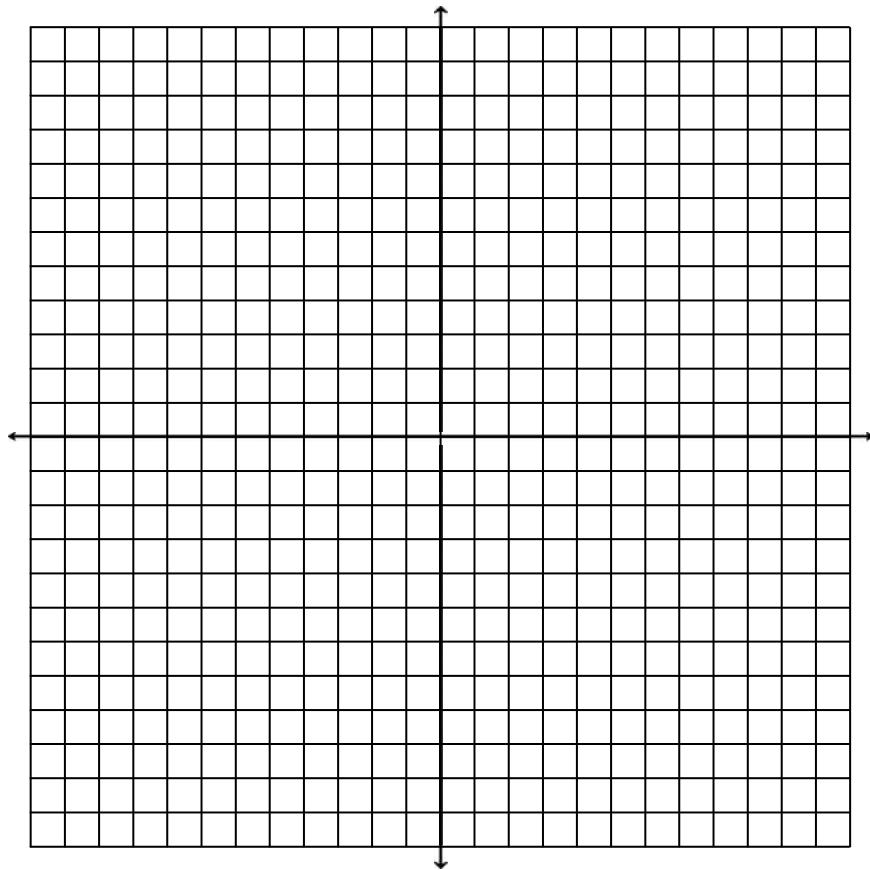
C.



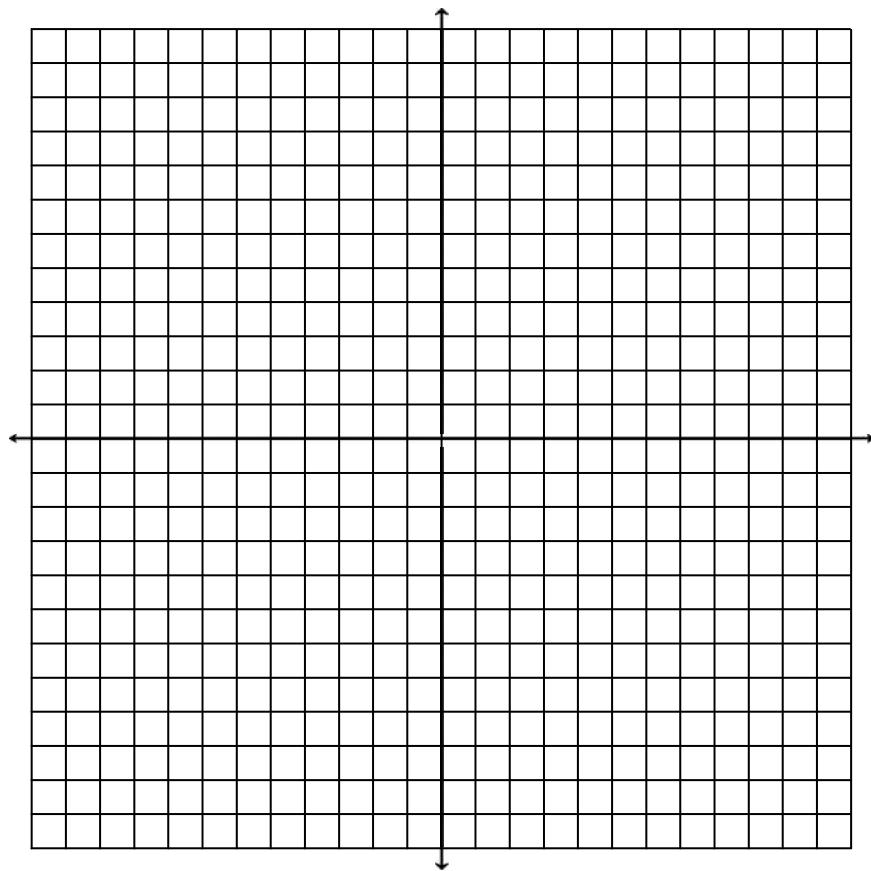
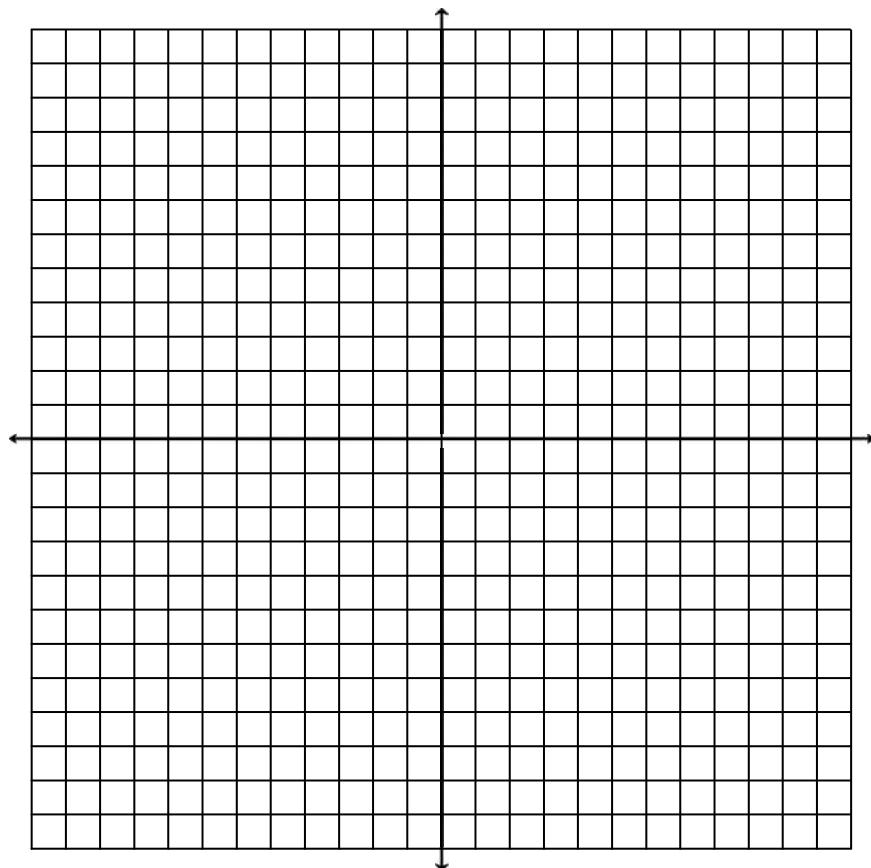
D.



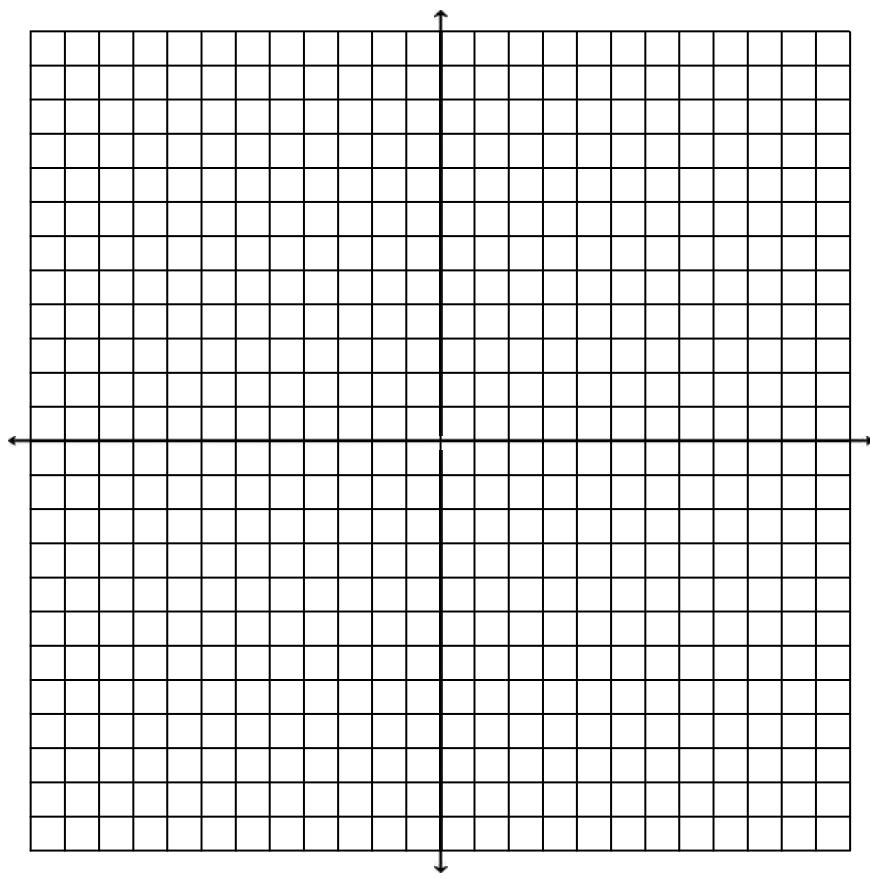
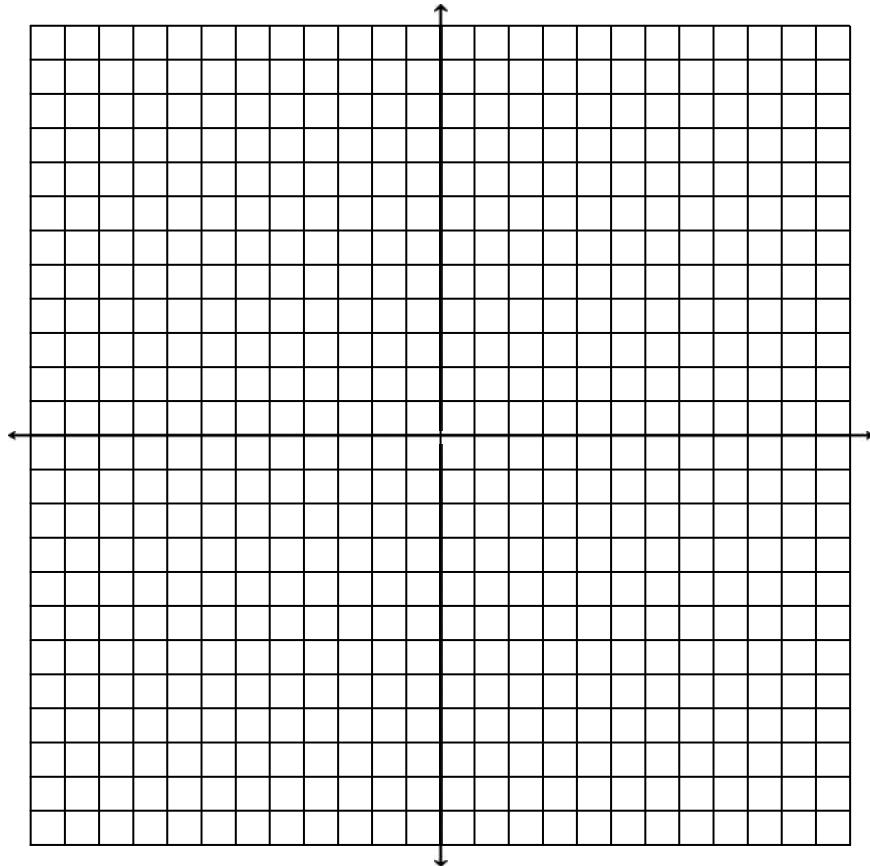
9. Given the functions  $f(x) = |x|$  and  $g(x) = \frac{1}{2}x - 4$ , graphically combine these to obtain graphs of:
- $f(x) + g(x)$
  - $y = f(x)g(x)$
  - $y = \frac{f(x)}{g(x)}$



10. Given the functions  $f(x) = x - 3$  and  $g(x) = x^3$ , graphically combine these to obtain graphs of:  
a)  $g(x) - f(x)$       and      b)  $f(x)g(x)$

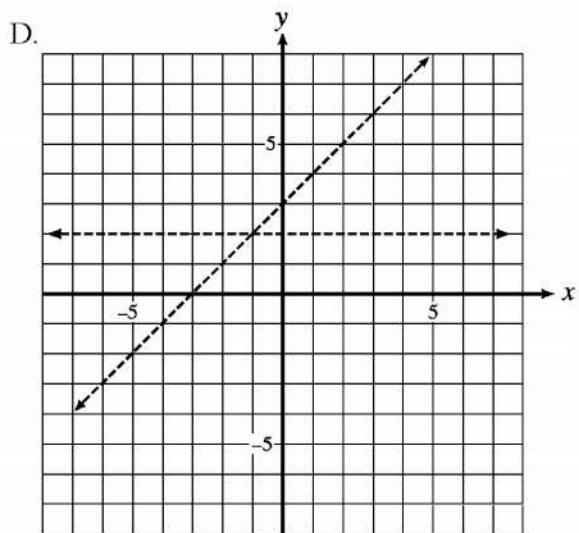
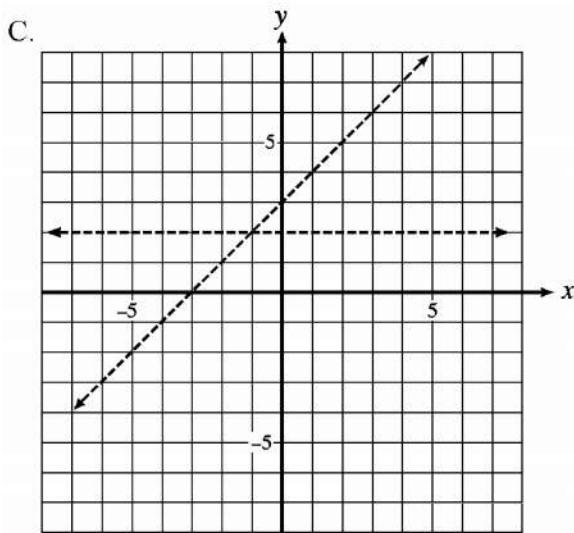
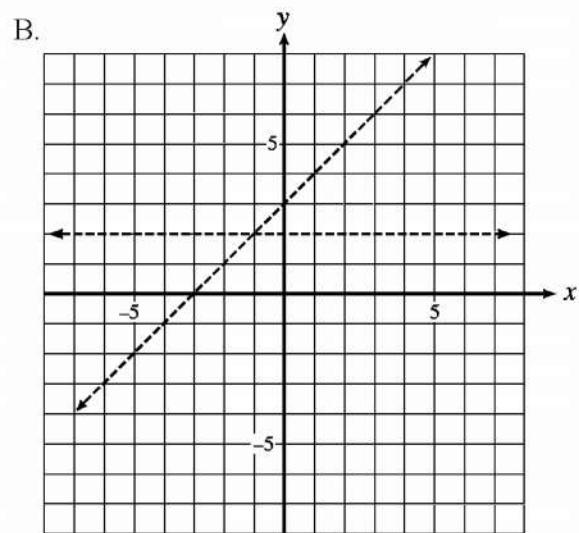
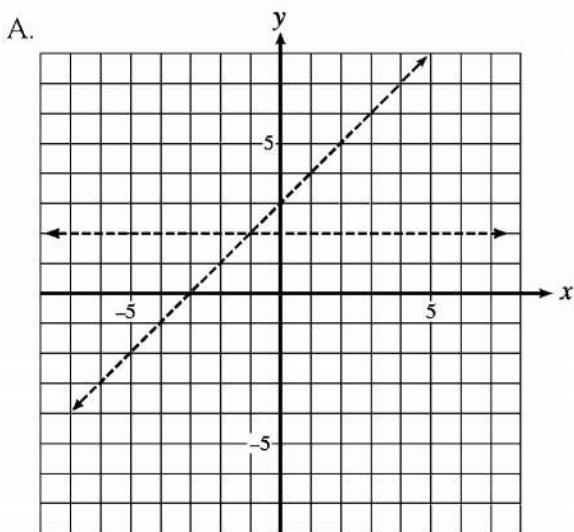
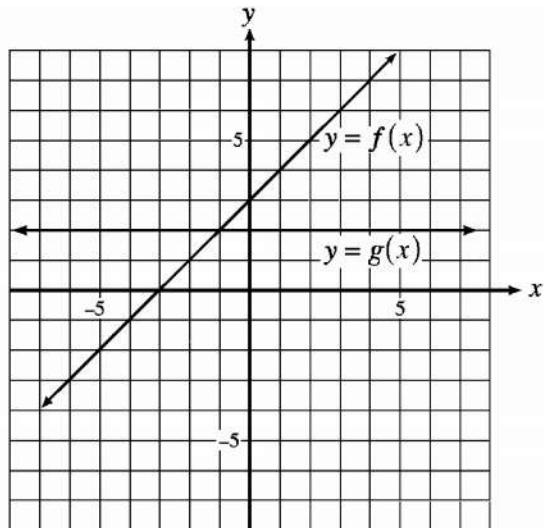


11. Given the functions  $f(x) = 2^x$  and  $g(x) = x^2$ , graphically combine these to obtain graphs of:  
a)  $f(x) - g(x)$       and      b)  $\frac{f(x)}{g(x)}$

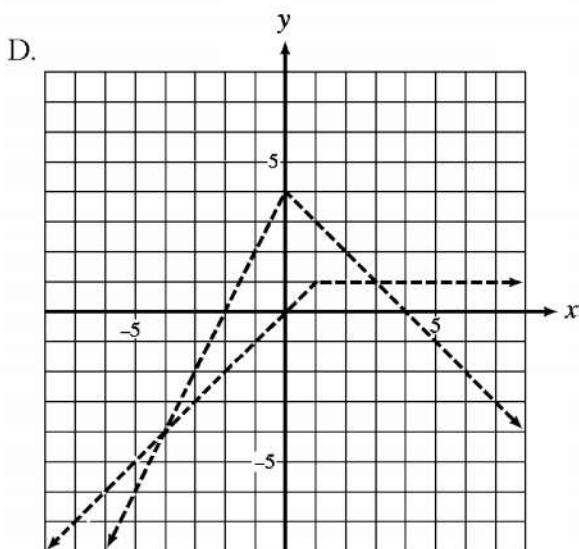
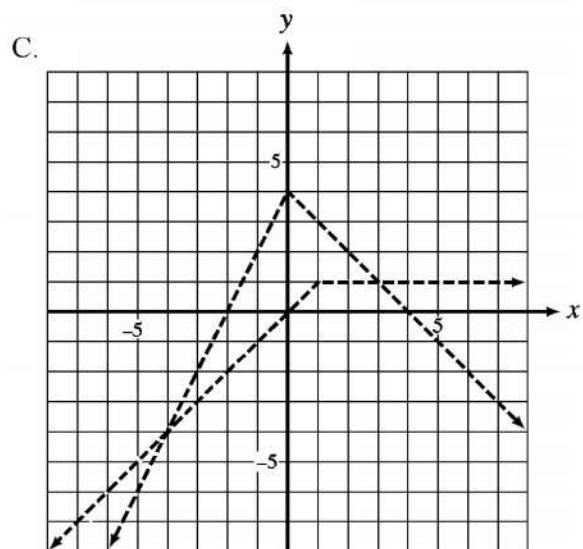
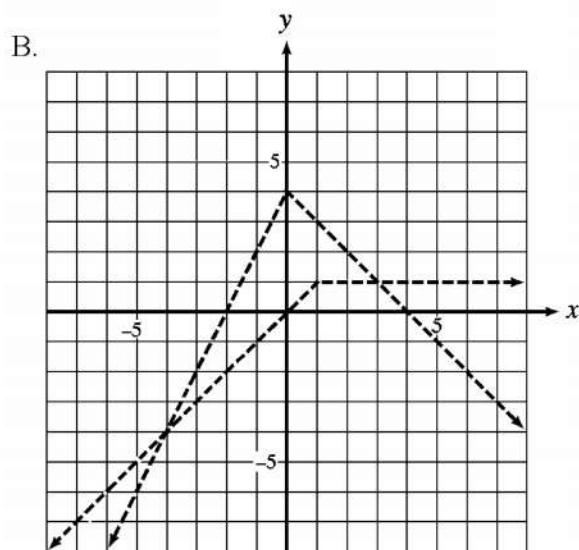
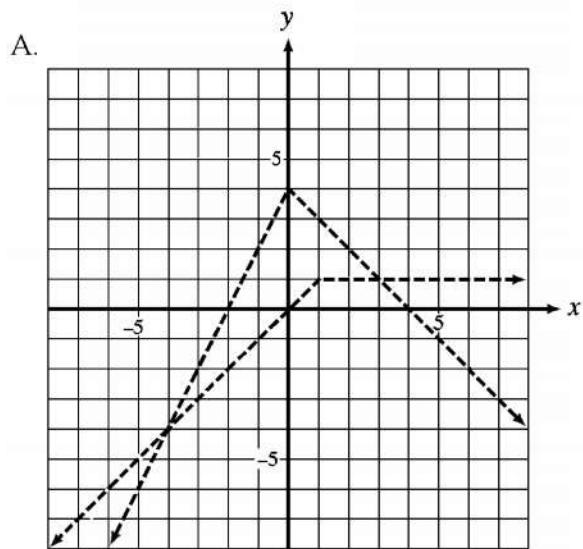
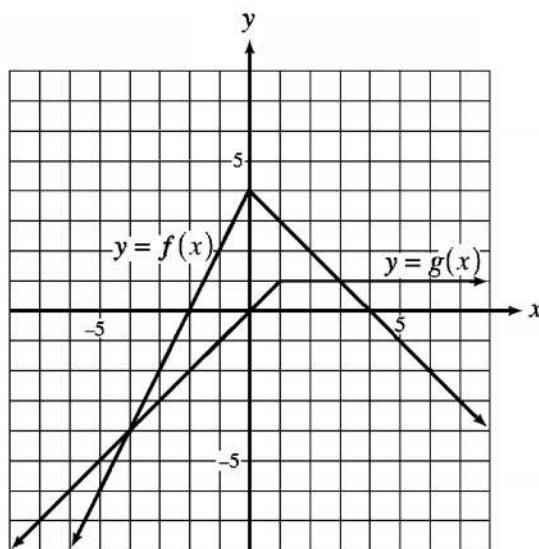


For the next 3 questions, graphs of  $y = f(x)$  and  $y = g(x)$  are given. Graphically combine these functions to obtain the graphs of: A.  $y = f(x) + g(x)$  B.  $y = f(x) - g(x)$  C.  $y = f(x)g(x)$  D.  $y = \frac{f(x)}{g(x)}$

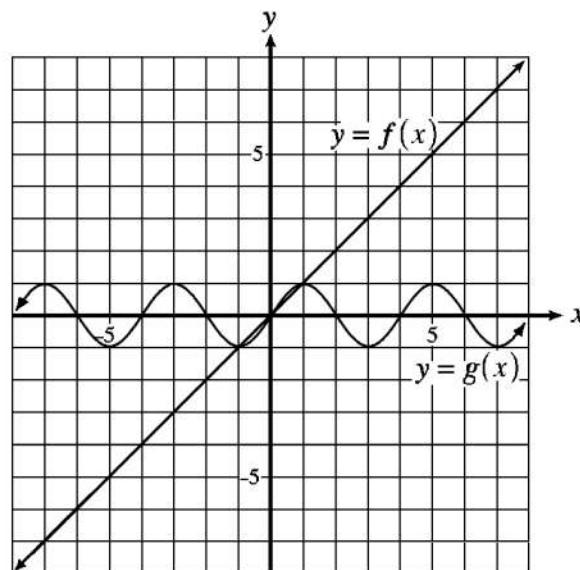
12.



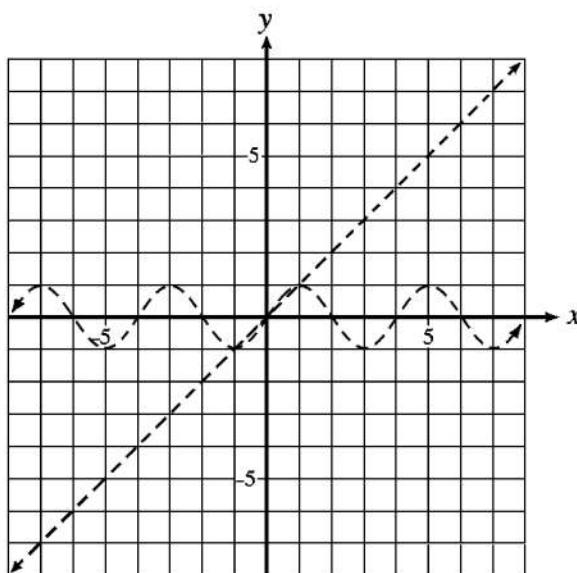
13.



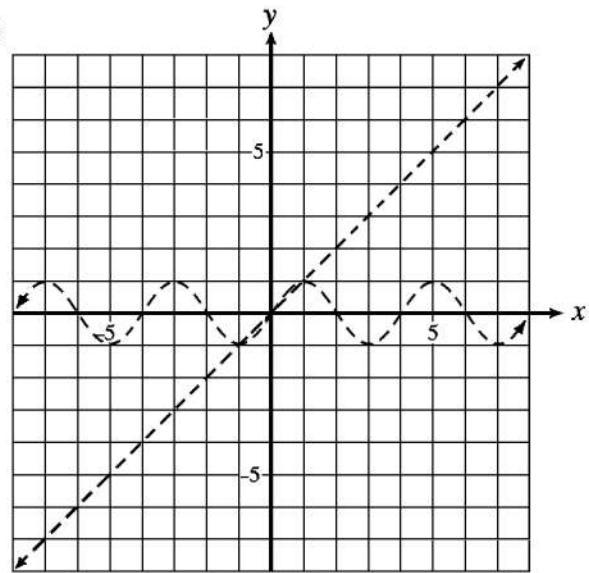
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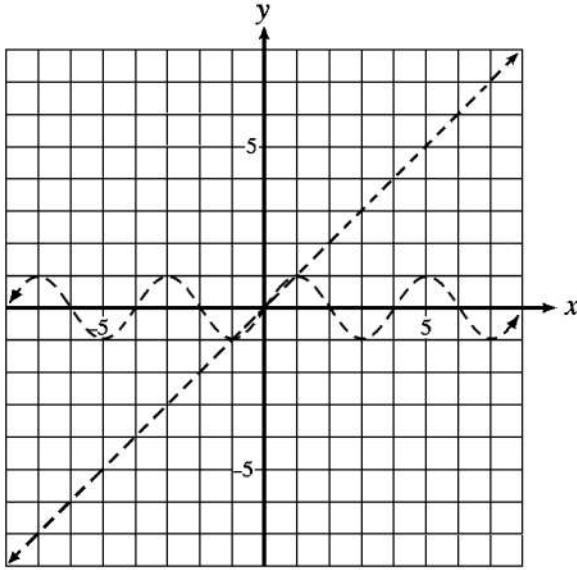
A



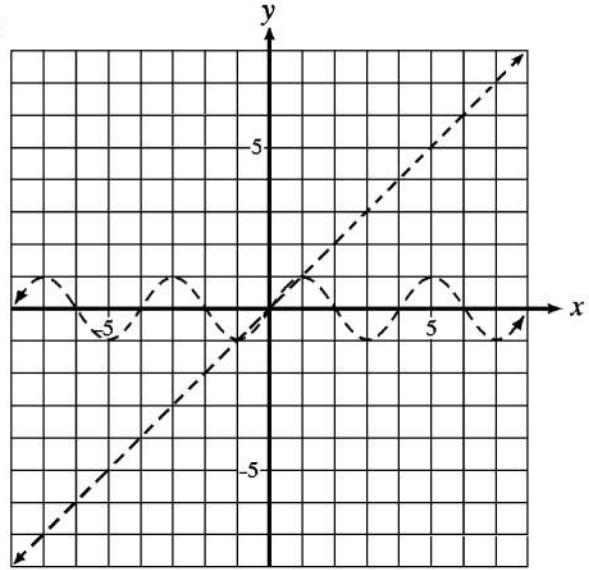
B



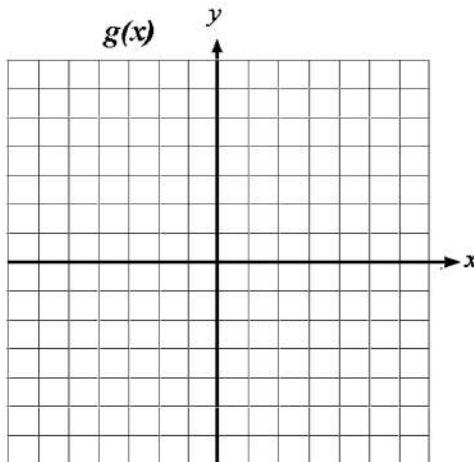
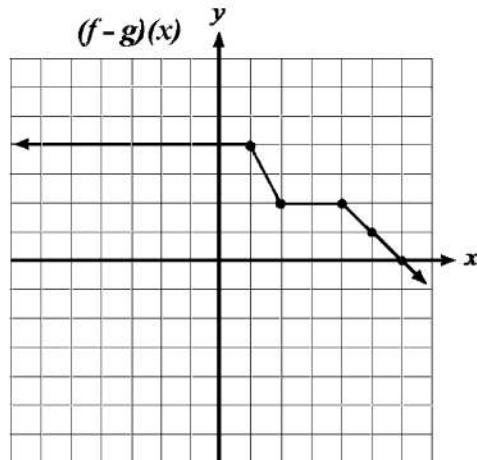
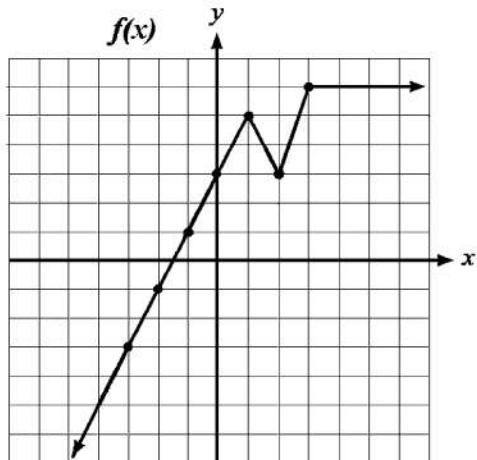
C



D



15. Given the graphs of  $f(x)$  and  $(f - g)(x)$ , sketch the graph of  $g(x)$ .



16. If  $f(x) = x^2 - 3x + 4$  and  $g(x) = 5x + 2$ , determine the equation for  $f(x) - g(x)$ .

- A.  $x^2 - 8x + 2$       B.  $x^2 + 2x + 6$   
 C.  $x^2 - 8x + 6$       D.  $-5x^3 + 13x^2 - 14x - 8$

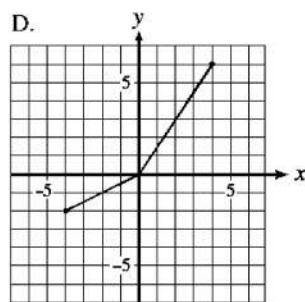
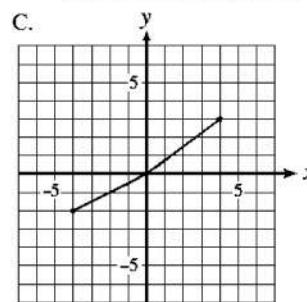
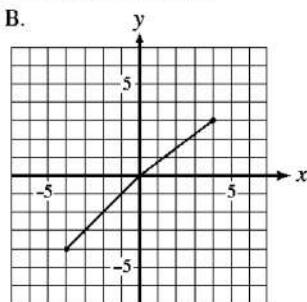
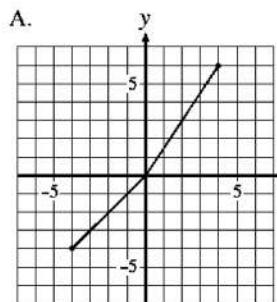
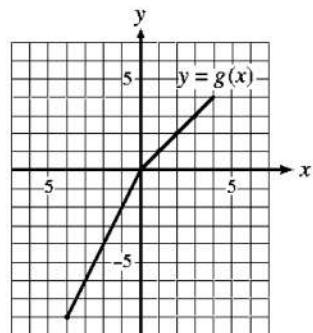
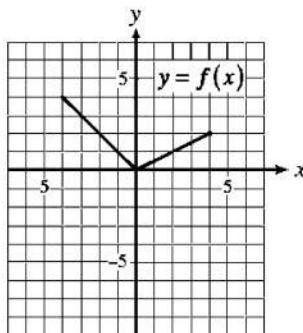
17. Given the functions  $f(x) = x^2 - 3$  and  $g(x) = -9 - x$ , determine the equation for the combined function  $y = f(x) + g(x)$ .

- A.  $y = x^2 - x + 6$       B.  $y = x^2 + 27x + 6$       C.  $y = x^2 - 27x - 12$       D.  $y = x^2 - x - 12$

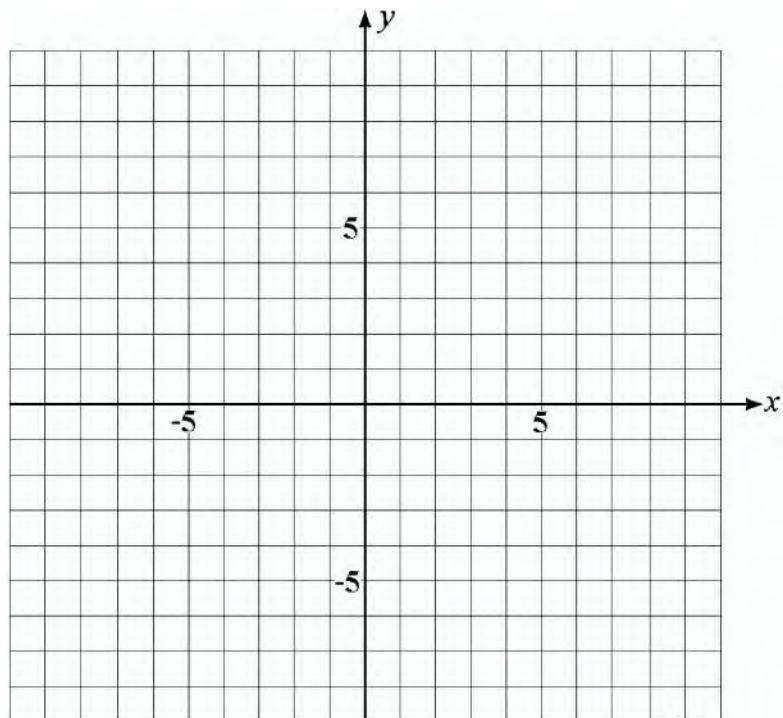
18. Given the functions  $f(x) = x + 3$  and  $g(x) = x^2 - 4$ , determine the value of  $(f + g)(-2)$ .

- A. 0      B. 1      C. 3      D. 5

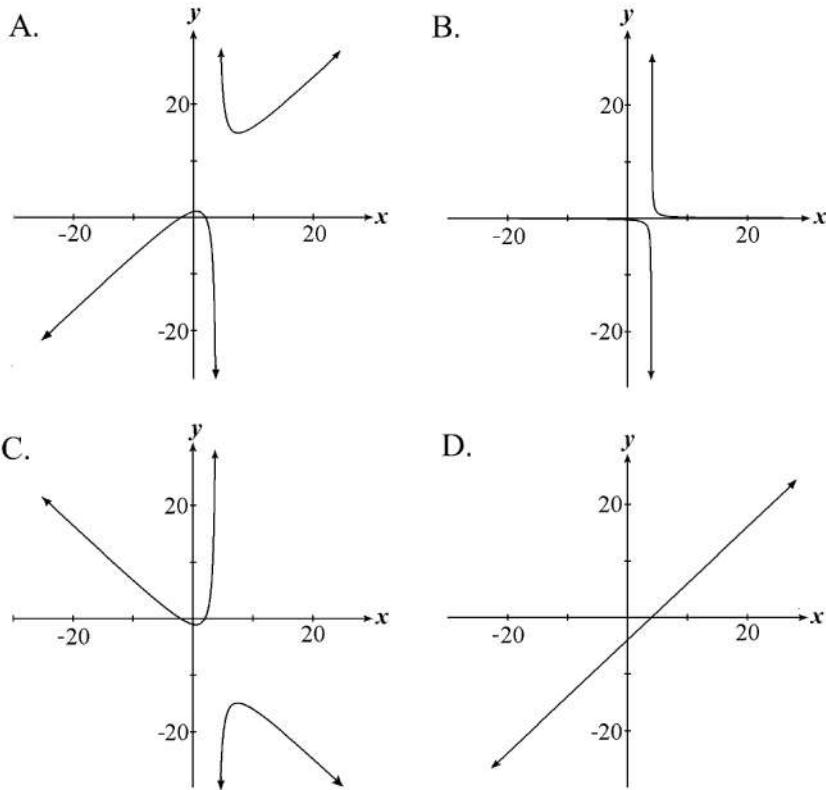
19. The graphs of  $y = f(x)$  and  $y = g(x)$  are graphed below. Which graph represents the graph of  $y = f(x) + g(x)$ ?



20. Given the functions  $f(x) = x^2 - 3$  and  $g(x) = -2x + 3$ , sketch the graph of the combined function  $y = (f + g)(x)$  and state its domain and range.



21. Given the functions  $f(x) = x^2 - 4$  and  $g(x) = x - 4$ , a graph of the combined function  $h(x) = \frac{f(x)}{g(x)}$  most likely resembles



22. Given the functions  $f(x) = x^3 - 81x$  and  $g(x) = x + 9$ , determine the simplified equation for the combined function  $\frac{f(x)}{g(x)}$ .

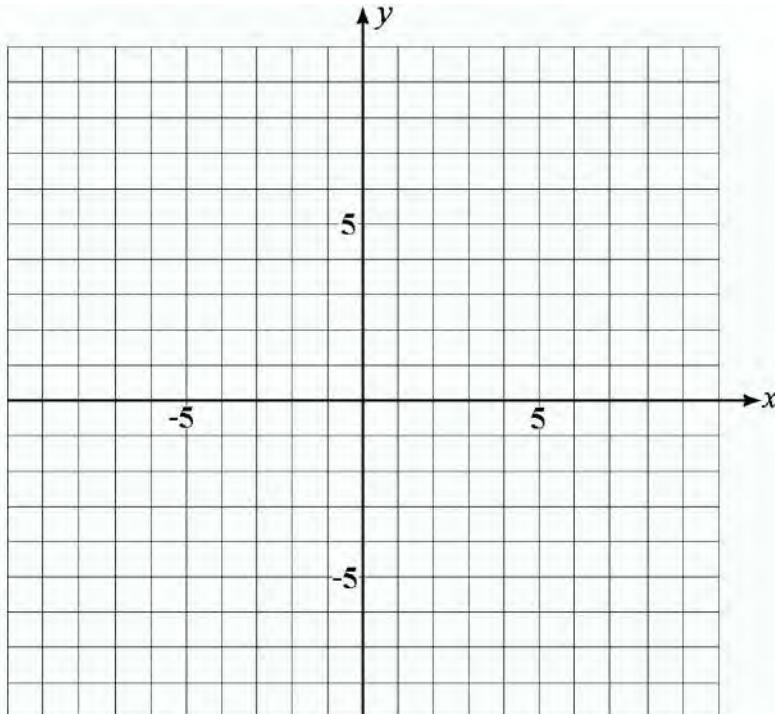
- A.  $x(x - 9)$ ,  $x \neq -9$       B.  $\frac{x}{x + 9}$ ,  $x \neq -9$       C.  $\frac{x}{x - 9}$ ,  $x \neq -9, 9$       D.  $x(x + 9)$ ,  $x \neq -9$

23. Given  $f(x) = x^2 - 5$ ,  $g(x) = x - 2$ , and  $h(x) = \frac{2x^2 - 6x + 5}{x - 2}$ , determine  $j(x) = \frac{f(x)}{g(x)} + h(x)$ , and state the domain and range of  $j(x)$ .

24. If  $f(x) = x^3$  and  $g(x) = 2x - 3$ , determine the value of  $\left(\frac{f}{g}\right)(-1)$ .

25. If  $f(x) = \frac{1}{x-2}$  and  $g(x) = x - 2$ , determine the domain of  $f(x) \cdot g(x)$ .

26. Given the functions  $f(x) = x^2 - 1$  and  $g(x) = x + 1$ , sketch the graph of  $y = \frac{g(x)}{f(x)}$ .



27. Given that  $h(x) = 2x^2 + 5x - 3$  and that  $h(x) = f(x)g(x)$ , determine possible equations for  $f(x)$  and  $g(x)$ .

28. Consider the functions  $y = f(x)$  and  $y = g(x)$ , as shown. Determine the values of  $h(-2)$ , for

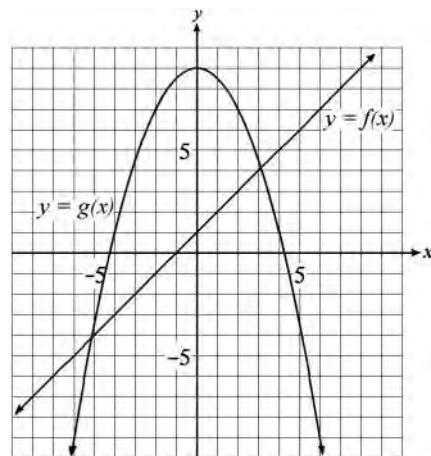
a)  $h(x) = (f + g)(x)$

b)  $h(x) = (f - g)(x)$

c)  $h(x) = (fg)(x)$

d)  $h(x) = \left(\frac{f}{g}\right)(x)$

e)  $h(x) = f(g(x))$



29. A polynomial function  $f$  has zeros at  $1$ ,  $-1$ , and  $2$ . Given the function  $g(x) = \frac{x+1}{x-2}$ , determine the domain of the function  $h(x) = \frac{f(x)}{g(x)}$ .

A. all real numbers

B. all real numbers,  $x \neq 2$

C. all real numbers,  $x \neq -1$

D. all real numbers,  $x \neq -1$ ,  $x \neq 2$

30. Given  $f(x) = x^2 - 2x - 3$  and  $g(x) = x + 1$ , write the equations of:

a)  $y = f(g(x))$

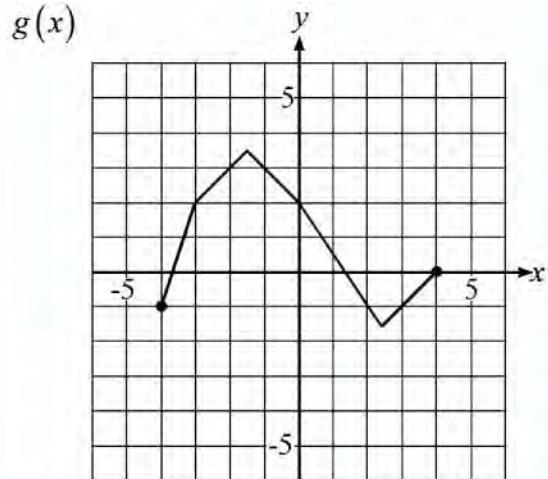
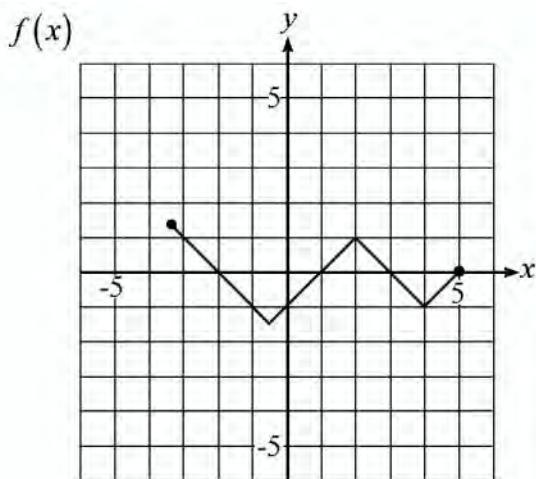
b)  $y = g(f(x))$

c)  $y = f(f(x))$

d)  $y = g(g(x))$

31. Given  $f(x) = \sqrt{x - 2}$  and  $g(x) = 3x$ , write the equation for  $h(x) = f(g(x))$  and state its domain.
32. Given  $f(x) = x - 1$  and  $g(x) = x^2$ , write the equation of  $y = f(g(x))$ .
33. Given that  $f(x) = (1, 3), (2, 5), (3, 4), (4, 2)$ , determine  $f(f(3))$ .
34. Given the following two functions,  $f(x) = \sqrt{x - 1}$  and  $g(x) = x^2 + 1$ , evaluate  $g(f(3))$ .
35. Given  $f(x) = x^2 - 1$  and  $g(x) = \sqrt{x + 1}$ , determine a simplified equation for  $y = f(g(x))$  and state its domain.
36. Given  $f(x) = x^2 - 2x - 3$  and  $g(x) = x + 1$ ,
- Write the equation of  $y = f(g(x))$ .
  - Write the equation of  $y = g(f(x))$ .

37. Given the following graphs:



- a) Determine the value of  $[f \cdot g](0)$
- b) Determine the value of  $g(f(4))$
- c) Determine a value for  $k$  where  $f(k) = 1$
38. Given  $f(x) = x + 2$  and  $g(x) = x^2 + 3x - 1$ , determine the value of  $f(g(3))$ .
- A. 16      B. 17      C. 19      D. 39
39. Given  $f(x) = 7 \log_2 x$  and  $g(x) = 5x + 6$ , determine the value of  $f(g(2))$ .
40. Using the function  $f(x) = \frac{x^2 + 4}{x - 2}$ , determine the value of  $f(f(4))$ .
41. Given  $f(x) = \sqrt{x - 1}$ ,  $g(x) = x^2 + 3$ , and  $h(x) = 2x - 5$ , determine  $k(x) = h(f(g(x)))$ , including restrictions on the domain.

42. Given the functions  $f(x) = 7 - x$  and  $g(x) = 2x + 1$ , determine the equation  $y = h(x)$  for each question below and state the domain and range.

a)  $h(x) = f(x) - g(x)$

b)  $h(x) = f(x)g(x)$

c)  $h(x) = \frac{g}{f}(x)$

d)  $h(x) = g(f(x))$

43. If  $h(g(x)) = \sqrt{x^3 - 5}$ , which of the following gives possible equations for  $g(x)$  and  $h(x)$ ?

A.  $g(x) = x^3$ ,  $h(x) = x$

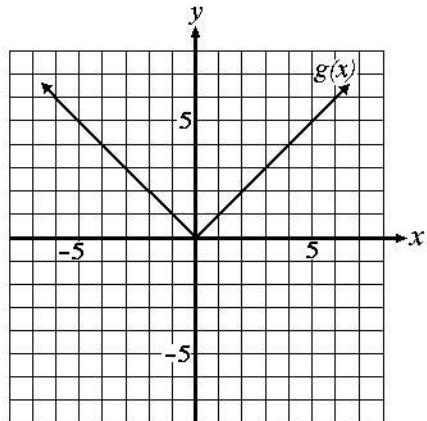
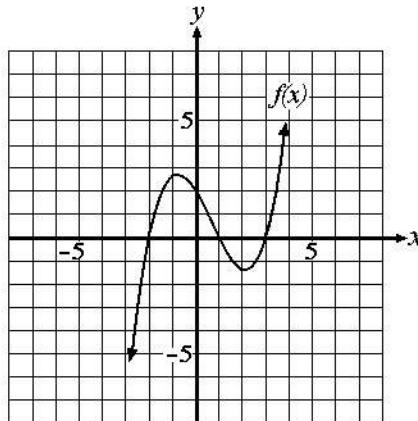
B.  $g(x) = x$ ,  $h(x) = x^3$

C.  $g(x) = \sqrt{x}$ ,  $h(x) = x^3 - 5$

D.  $g(x) = x^3 - 5$ ,  $h(x) = \sqrt{x}$

44. If  $f(x) = 3x - 5$  and  $g(x) = (x + 2)^2 - 3$ , determine  $g(f(4))$ .

45. The graphs of  $f(x)$  and  $g(x)$  are given below. Determine  $f(g(-3))$ .



A. -6

B. -2

C. 0

D. 3

46. Given the functions  $f(x) = x^2 - 8$  and  $g(x) = -5 - x$ , determine an equation for the combined function  $h(x) = f(g(x))$ .
- A.  $h(x) = x^2 + 10x + 17$       B.  $h(x) = x^2 - 10x + 3$   
C.  $h(x) = -x^2 - 13$       D.  $h(x) = -x^2 + 3$
47. Given the function  $f(x) = x^2 - 5$ , determine the value of  $f(f(-1))$ .
- A.  $-4$       B.  $11$       C.  $16$       D.  $-8$
48. Given the functions  $f(x) = \sqrt{4+x}$  and  $g(x) = |3x - 6|$ ,
- a) Evaluate  $f(g(-5))$ .  
  
b) Is it possible to evaluate  $g(f(-5))$ ?

## COMBINING FUNCTIONS ANSWER KEY

1a) x-intercept:  $\frac{4}{3}$       b) x-intercept: 5  
 $\frac{3}{3}$

y-intercept: -4      y-intercept: -2

slope: 3      slope:  $\frac{2}{5}$

2a) x-intercept(s): none      b) x-intercept(s):  $2 \pm \sqrt{\frac{5}{2}}$

y-intercept:  $\frac{17}{4}$       y-intercept: 3

Vertex: (3, 2)

Domain:  $x \in \mathbb{R}$

Range:  $y \geq 2$

Vertex: (2, -5)

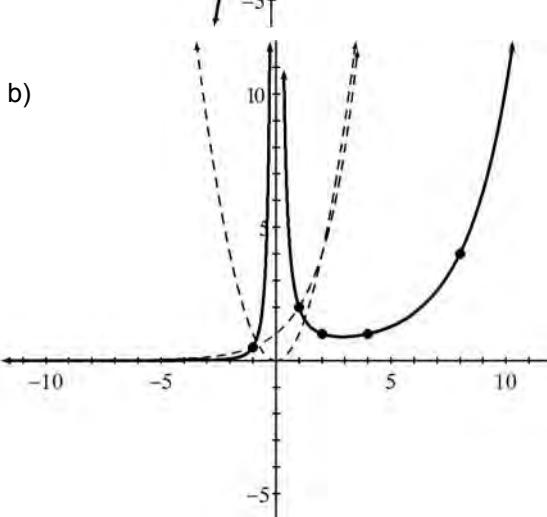
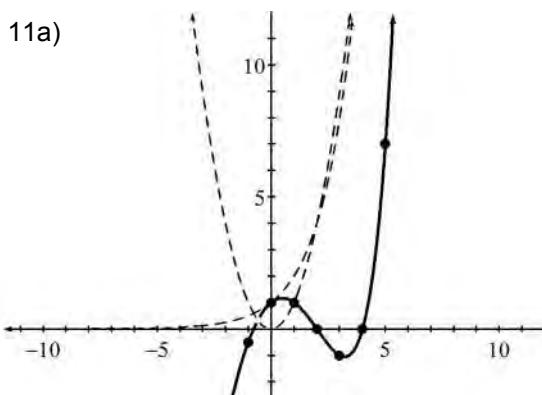
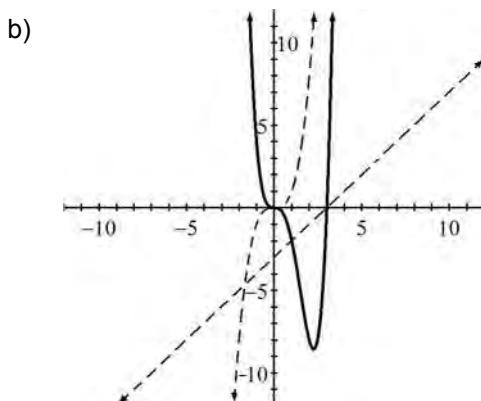
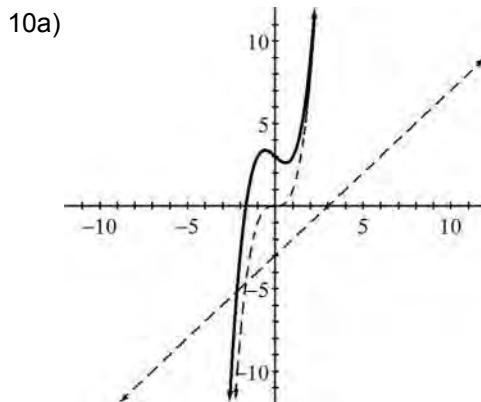
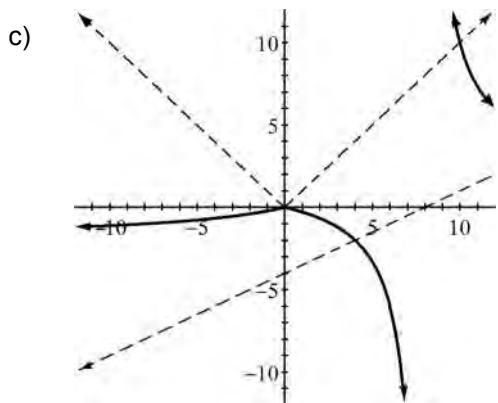
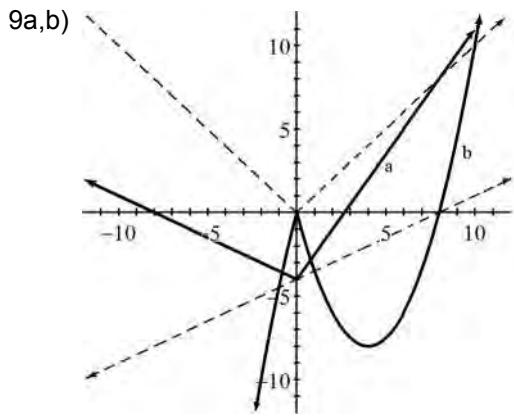
Domain:  $x \in \mathbb{R}$

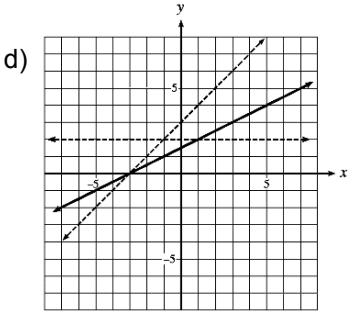
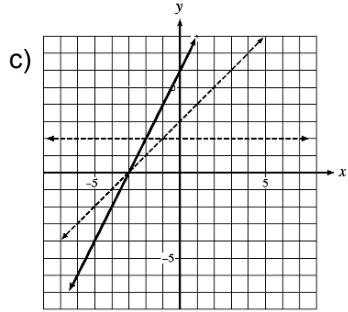
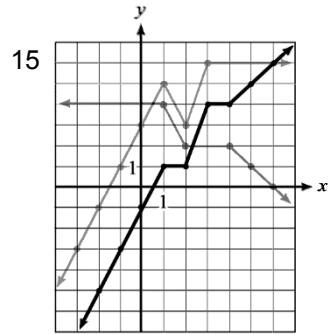
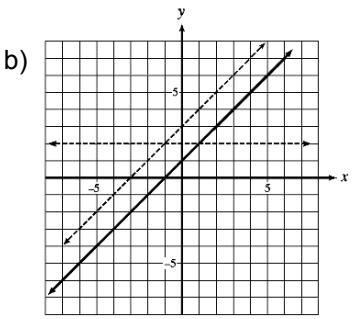
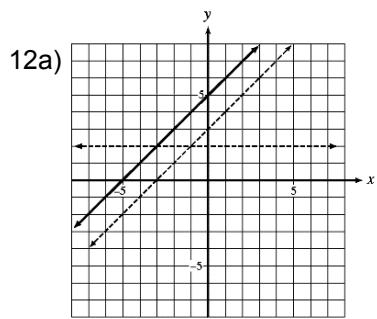
Range:  $y \geq -5$

4 Domain:  $x \in \mathbb{R}, x \neq 0$ ; Range:  $y \in \mathbb{R}, y \neq 0$   
 Asymptotes:  $x = 0, y = 0$

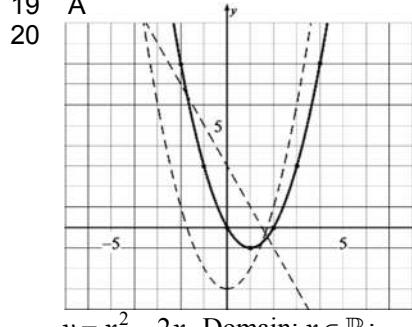
7 Domain:  $x \in \mathbb{R}$ ; Range:  $y > 0$   
 Asymptote:  $y = 0$

8 C

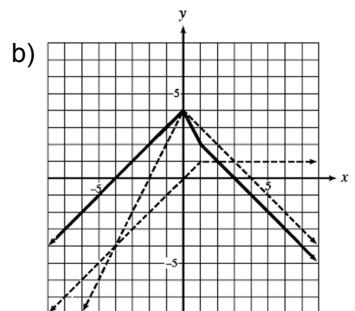
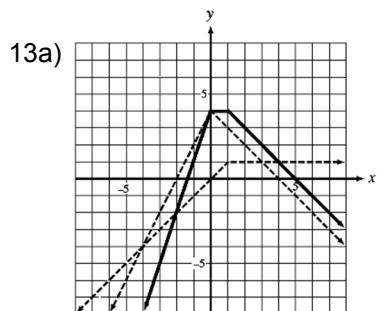




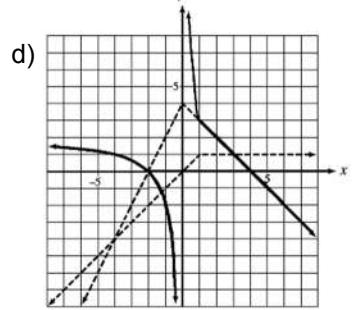
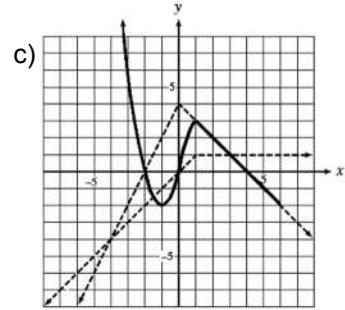
- 16 A  
17 D  
18 B  
19 A



$$y = x^2 - 2x, \text{ Domain: } x \in \mathbb{R}; \\ \text{Range: } y \geq -1$$



- 21 A  
22 A  
23  $j(x) = 3x$ , Domain:  $x \neq 2$ ;  
Range:  $y \neq 6$



- 24  $\frac{1}{5}$   
25  $x \neq 2$   
26  $x \in \mathbb{R}, x \neq \pm 1$   
27  $f(x) = 2x - 1; g(x) = x + 3$   
Other answers possible  
28 a) 6      b) -8      c) -7  
d)  $-\frac{1}{7}$       e) 8

- 29 D  
30a)  $f(g(x)) = x^2 - 4$   
b)  $g(f(x)) = x^2 - 2x - 2$   
c)  $f(f(x)) = x^4 - 4x^3 - 4x^2 + 16x + 12$   
d)  $g(g(x)) = x + 2$

31  $h(x) = \sqrt{3x - 2}, \text{ Domain: } x \geq \frac{2}{3}$

32  $f(g(x)) = x^2 - 1$

33 2

34 3

35  $f(g(x)) = x; \text{ Domain: } x \geq -1$

36a)  $x^2 - 4$

b)  $x^2 - 2x - 2$

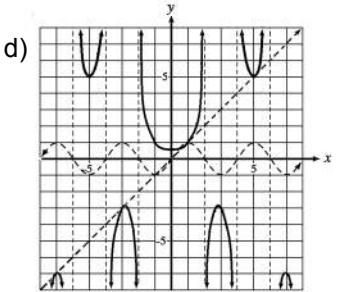
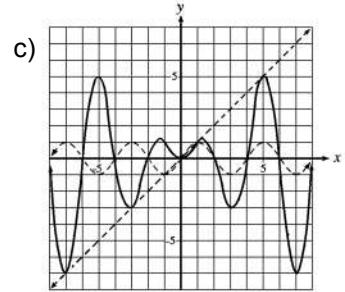
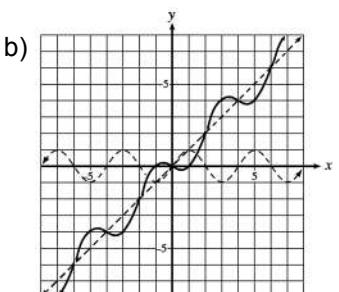
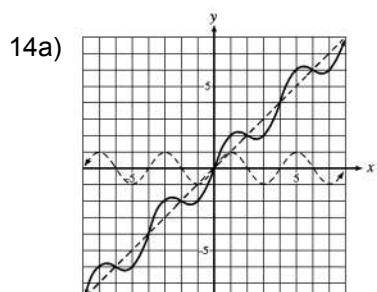
37a) -2    b) 3    c)  $k = 2 \text{ or } -3$

38 C

39 28

40 13

41  $h(f(g(x))) = 2\sqrt{x^2 + 2} - 5;$   
Domain:  $x \in \mathbb{R}$



42a)  $h(x) = -3x + 6$ ; Domain:  $x \in \mathbb{R}$

Range:  $y \in \mathbb{R}$

b)  $h(x) = -2x^2 + 13x + 7$ ;

Domain:  $x \in \mathbb{R}$ ; Range:  $y \leq 28.125$

c)  $h(x) = \frac{2x+1}{7-x}$ , Domain:  $x \neq 7$ ;

Range:  $y \in \mathbb{R}$

d)  $h(x) = -2x + 15$ , Domain:  $x \in \mathbb{R}$ ;

Range:  $y \in \mathbb{R}$

43 D

44 78

45 C

46 A

47 B

48 a) 5 b) not possible