PRECALCULUS MATHEMATICS 12

TABLE OF CONTENTS

TRIGONOMETRY I

Examining angles in standard position in both radians and degrees;
Reference and coterminal angles; arc length;
Reciprocal trigonometric functions;
Solving first-degree equations;
Graphing and characteristics of primary trigonometric functions, including transformations.

- **165 PROBLEMS**
- **REVIEW PACKAGE**

TRIGONOMETRY II (EQUATIONS and IDENTITIES)

Trigonometric identities (Pythagorean, quotient, double angle, reciprocal, and sum and difference identities) and two-column proofs;
Solving second-degree equations (over restricted domains and all real numbers).

- **135 PROBLEMS**
- **TRIGONOMETRIC IDENTITIES PACKAGE**
- **REVIEW OF SUM and DIFFERENCE & DOUBLE ANGLE IDENTITIES WORKSHEET**

CONIC SECTIONS

The parabola, circle, ellipse and hyperbola: equations and graphs;
General to Standard Form;
Transformations and applications.

- **194 PROBLEMS**

GEOMETRIC SEQUENCES and SERIES

Common ratio, first term, general term;
Infinite geometric series and summation notation.

- **121 PROBLEMS**

TRIGONOMETRY II

JAN 1997

1. Which expression is equivalent to $\cot\theta + \tan\theta$?

A. $\dfrac{1}{\sin\theta\cos\theta}$ B. $\dfrac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$ C. 1 D. 2

2. Which expression is equivalent to $\dfrac{1-\cos 2\theta}{\sin 2\theta}$?

A. $\tan\theta$ B. $\cot\theta$ C. $-\tan\theta$ D. $-\cot\theta$

3. How many solutions does the equation $(2\sin\theta + 5)(3\cos\theta + 3)(\tan^2\theta - 2) = 0$ have over the interval $0 \le \theta < 2\pi$?

A. 4 B. 5 C. 6 D. 7

4. Solve: $3\cos^2 x - 5\cos x - 2 = 0,\ 0 \le x < 2\pi$

JUNE 1997

5. Simplify: $\dfrac{2\tan x}{\cos^2 x + \sin^2 x + \tan^2 x}$

A. $2\sin x$ B. $\sin 2x$ C. $\tan 2x$ D. $2\cot x$

6. Solve: $2\cos^2 x - 5\cos x + 2 = 0,\ 0 \le x < 2\pi$

A. 0 B. $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$ C. $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ D. $\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

7. How many solutions does $\cos 3x = -1$ have over the interval $0 \le x < 2\pi$?

A. 1 B. 2 C. 3 D. 6

8. Which expression is equivalent to $4\sin 6\theta \cos 6\theta$?

A. $\sin 6\theta$ B. $\sin 12\theta$ C. $2\sin 3\theta$ D. $2\sin 12\theta$

JAN 1998

9. Simplify: $2\cot x \ \sin^2 x$

A. $\dfrac{\sin 2x}{2}$ B. $\sin 2x$ C. $\dfrac{2\sin^3 x}{\cos x}$ D. $\cos 2x \sin x$

10. Solve: $2\sec^2 x + 5\sec x - 3 = 0, \ 0 \le x < 2\pi$

A. 1.23, 5.05 B. 1.91, 4.37

C. 3.48, 5.94 D. $\dfrac{\pi}{3}$, 1.91, 4.37, $\dfrac{5\pi}{3}$

11. Solve: $\cos \dfrac{\pi}{2}x = 1$, over all real numbers

A. $2n, \ n \in I$ B. $4n, \ n \in I$ C. $2 + 2n, \ n \in I$ D. $2 + 4n, \ n \in I$

12. Prove: $\dfrac{1 - \cos\theta}{\sin^2\theta} = \dfrac{1}{1 + \cos\theta}$

JUN 1998

13. Determine all restrictions for: $\dfrac{\cot\theta}{1 - \sin\theta}$

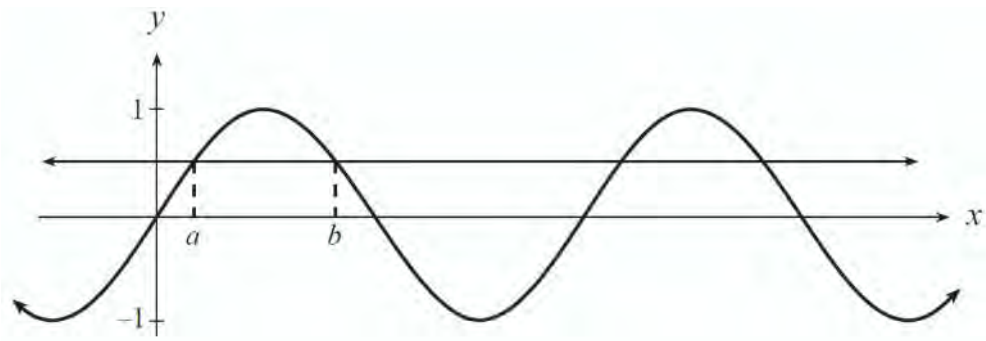
A. $\sin\theta \neq 1$ B. $\sin\theta \neq 0$

C. $\sin\theta \neq 1, \ \cos\theta \neq 0$ D. $\sin\theta \neq 0, \ \sin\theta \neq 1$

14. Given $\csc^2\theta + \sin^2\theta = 5.34$, find the value of: $\dfrac{1}{\csc^2\theta} + \dfrac{1}{\sin^2\theta}$

A. 0.19 B. 2.27 C. 5.14 D. 5.34

15. Solve: $\sin \pi x > \frac{1}{2}$, over the real numbers, using the graphs of $y = \sin \pi x$ and $y = \frac{1}{2}$ shown below. Express the answers in terms of a and b.



A. $a + n < x < b + n$ B. $a + 2n < x < b + 2n$

C. $a + \pi < x < b + \pi$ D. $a + 2\pi < x < b + 2\pi$

16. Prove: $\dfrac{\csc\theta}{\tan\theta + \cot\theta} = \cos\theta$

JAN 1999

17. Which expression is equivalent to $\dfrac{\sin\theta + \cos\theta\cot\theta}{\cot\theta}$?

A. $\csc\theta$ B. $\cos\theta$ C. $\sin\theta$ D. $\sec\theta$

18. Solve: $\sin 2\theta + 2\cos\theta = 0,\ 0 \le \theta < 2\pi$

A. $\frac{3\pi}{2}$ B. $\frac{\pi}{2}, \frac{3\pi}{2}$ C. $\frac{\pi}{2}, \pi$ D. $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$

19. Let A be an angle in standard position such that $0 < A < \frac{\pi}{2}$. If $\sin A = n$ and $\cos A = m$, determine an expression for: $\sin(\pi + A) + \cos(\pi + A)$

A. $-m - n$ B. $-m + n$ C. $m - n$ D. $m + n$

20. Prove: $\dfrac{\cot\theta}{\csc\theta - 1} = \dfrac{\csc\theta + 1}{\cot\theta}$

JUN 1999

21. How many solutions does $\tan^2 x + 5\cos x - 8 = 0$ have over the interval $0 \le x < 2\pi$?

A. 1 B. 2 C. 3 D. 4

22. Determine all restrictions for: $\dfrac{\sec x}{4\sin^2 x - 1}$

A. $\sin x \neq \pm\frac{1}{4}$ B. $\sin x \neq \pm\frac{1}{2}$

C. $\cos x \neq 0,\ \sin x \neq \pm\frac{1}{4}$ D. $\cos x \neq 0,\ \sin x \neq \pm\frac{1}{2}$

23. Solve: $\tan^2 x = \tan x,\ 0 \le x < 2\pi$

A. $0, \frac{\pi}{4}$ B. $\frac{\pi}{4}, \frac{5\pi}{4}$ C. $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$ D. $0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$

24. Prove: $\dfrac{\sin\theta + \tan\theta}{1 + \cos\theta} = \dfrac{\sin 2\theta}{2\cos^2\theta}$

JAN 2000

25. Determine all restriction(s) for: $\cot x$

A. $\sin x = 0$
B. $\cos x = 0$
C. $\cos x = 0,\ \sin x = 0$
D. no restriction(s)

26. Simplify: $\sin 5m \cos m + \cos 5m \sin m$

A. $\cos 4m$
B. $\cos 6m$
C. $\sin 4m$
D. $\sin 6m$

27. Solve: $2\sin x \cos x + \sin x = 0,\ 0 \le x < 2\pi$

A. $0,\ \pi$
B. $\frac{2\pi}{3},\ \frac{4\pi}{3}$
C. $0,\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3}$
D. $0,\ \frac{2\pi}{3},\ \pi,\ \frac{5\pi}{3}$

28. Simplify: $\dfrac{\sin 2x}{1 - \cos 2x}$

A. $\cot x$
B. $\tan x$
C. $2\cot x$
D. $2\tan x$

29. Solve: $2 - x = \sin^2 x$

A. 1.06
B. 1.16
C. 2.43
D. 1.08, 1.68, 2.42

30. Prove: $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

JUN 2000

31. Solve: $\tan\theta - \cos^2\theta = \frac{1}{2},\ 0 \le \theta < 2\pi$

A. 0.36, 3.50
B. 0.79, 3.93
C. 0.86, 2.74
D. 0.88, 2.94, 3.26, 3.74

32. Simplify: $\sin A \cos B + \cos A \sin B$ if $A = \left(\frac{\pi}{2} - B\right)$

A. -1 B. 0 C. 1 D. $\frac{\pi}{2}$

33. Determine the number of solutions for: $(\csc\theta)(2\sec\theta + 1) = 0,\ 0 \le \theta < 2\pi$

A. 0 B. 2 C. 3 D. 4

34. Prove: $\dfrac{1}{\sec\theta + \tan\theta} = \dfrac{1 - \sin\theta}{\cos\theta}$

JAN 2001

35. Which expression is equivalent to $\dfrac{\cot\theta \sin\theta}{\sec\theta}$?

A. $\sin^2\theta$ B. $\cos^2\theta$ C. $\sec^2\theta$ D. $\csc^2\theta$

36. Solve: $3\tan\left(\frac{1}{2}x - 2\right) = 4\sin 2x,\ 0 \le x < 2\pi$

A. 1.48, 5.28, 5.94 B. 1.64, 3.56, 3.84 C. 2.20, 2.90, 4.60 D. 3.28, 4.90, 5.74

37. Which of the following is equivalent to $\cos(2\theta + \pi)$?

A. $2\sin\theta\cos\theta$ B. $-2\sin\theta\cos\theta$ C. $1 - 2\sin^2\theta$ D. $2\sin^2\theta - 1$

38. Determine the amplitude of the function $y = k\sin\theta\cos\theta$, where k is a positive constant.

A. $\frac{k}{2}$ B. k C. $2k$ D. $4k$

39. Prove: $\dfrac{\cot\theta - 1}{1 - \tan\theta} = \dfrac{\csc\theta}{\sec\theta}$

JUN 2001

40. Simplify: $\dfrac{2\cos\theta}{\sin 2\theta}$

A. $\sin\theta$ B. $\cot\theta$ C. $\sec\theta$ D. $\csc\theta$

41. Solve: $\sec\theta + \cot\theta = 2,\ 0 \le \theta < 2\pi$

A. 0.64 B. 0.93 C. 3.46, 5.13 D. 4.29, 5.97

42. Determine all restrictions for: $\dfrac{\csc\theta - 1}{\csc\theta + 1}$

A. $\sin\theta \neq 0$ B. $\sin\theta \neq -1$

C. $\sin\theta \neq 0$, $\sin\theta \neq -1$ D. $\sin\theta \neq 0$, $\sin\theta \neq \pm 1$

43. Prove: $\dfrac{\sin\theta\cos\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\tan\theta}$

SPECS/SAMPLE 2001

44. Prove: $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$

45. Solve: $\tan x + \sin x = 1,\ 0 \le x < 2\pi$

A. 0.49, 4.22
B. 2.06, 5.80
C. 0.49, 1.57, 4.22, 4.71
D. 1.57, 2.06, 4.71, 5.80

46. Which expression is equivalent to $\dfrac{\cos 8x + 1}{2}$?

A. $\cos^2 4x$
B. $\sin^2 4x$
C. $\cos^2 16x$
D. $\sin^2 16x$

47. Determine the general solution for: $3\sin 5x = 1$

A. $x = 0.07 + \dfrac{2n\pi}{5},\ x = 0.56 + \dfrac{2n\pi}{5},\ n \in I$
B. $x = 0.07 + \dfrac{2n\pi}{5},\ x = 5.94 + \dfrac{2n\pi}{5},\ n \in I$
C. $x = 0.07 + 2n\pi,\ x = 0.56 + 2n\pi,\ n \in I$
D. $x = 0.07 + 2n\pi,\ x = 0.56 + 2n\pi,\ n \in I$

48. Prove: $\dfrac{\cot\theta}{\sin\theta - \csc\theta} = -\sec\theta$

JAN 2002

49. Simplify: $\dfrac{\sin 2\theta}{\sin\theta}$

A. 2
B. $\sin\theta$
C. $\cos\theta$
D. $2\cos\theta$

50. Which expression is equivalent to $\dfrac{\cos x + \cot x}{\sin x + 1}$?

A. $\sec x$ B. $\csc x$ C. $\cot x$ D. $\tan x$

51. Solve: $\sin 3x + \tan x = 3,\ 0 \le x < 2\pi$

A. 1.31, 4.34
B. 2.44, 3.85
C. 1.31, 1.57, 4.34, 4.71
D. 0, 2.44, 3.14, 3.85

52. Which expression is equivalent to $\sin\left(x + \dfrac{\pi}{3}\right) + \sin\left(x - \dfrac{\pi}{3}\right)$?

A. $\dfrac{\sqrt{3}}{4}\sin x$ B. $\sin x$ C. $\sqrt{3}\sin x$ D. $2\sin x$

53. Solve: $2\cos^2 x + \cos x - 1 = 0$, over the set of real numbers

APR 2002

54. Solve: $\cos x = 2x,\ 0 \le x < 2\pi$

A. 0.45 B. 0.58 C. 0.90 D. no solution

55. The expression $\cos 3x \cos 2x - \sin 3x \sin 2x$ is equal to

A. $\sin x$ B. $\sin 5x$ C. $\cos x$ D. $\cos 5x$

56. Solve: $2\cos^2 x - 1 = 0,\ 0 \le x < 2\pi$

A. $\dfrac{\pi}{4}, \dfrac{7\pi}{4}$ B. $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$ C. $\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ D. $\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

57. Simplify: $\dfrac{\cos\theta}{\cot\theta} + \dfrac{1}{\csc\theta}$

A. $\csc\theta$ B. $2\sin\theta$ C. $2\cot\theta$ D. $\sin\theta + \cos\theta$

58. Prove: $\dfrac{\sin 2x}{1+\cos 2x} = \dfrac{\sec^2 x - 1}{\tan x}$

JUN 2002

59. Determine an expression equivalent to: $\sec\theta\cot\theta\sin\theta$

A. 1 B. $\cot\theta$ C. $\csc\theta$ D. $\tan\theta$

60. Solve: $\sin 2x + \cos 3x = 1.5,\ 0 \le x < 2\pi$

A. 3.84, 4.37 B. 4.97, 5.12
C. 5.07, 5.58 D. 1.20, 1.90, 3.76, 5.64

61. Simplify: $\sin(2x+\pi)$

A. $\sin 2x$ B. $\cos 2x$ C. $-\sin 2x$ D. $-\cos 2x$

62. If the two smallest positive solutions of $\sin 3x = 0.4$ are $x = 0.14$ and $x = 0.91$, determine the general solution.

A. $x = 0.14 + 2n\pi,\ x = 0.91 + 2n\pi,\ n \in I$ B. $x = 0.14 + 6n\pi,\ x = 0.91 + 6n\pi,\ n \in I$
C. $x = 0.14 + \dfrac{n\pi}{3},\ x = 0.91 + \dfrac{n\pi}{3},\ n \in I$ D. $x = 0.14 + \dfrac{2n\pi}{3},\ x = 0.91 + \dfrac{2n\pi}{3},\ n \in I$

63. Prove: $\sin 2x(\tan x + \cot x) = 2$

64. Simplify: $\dfrac{\csc^2 x - 1}{\csc^2 x}$

A. $\cos^2 x$ B. $\sin^2 x$ C. $-\cos^2 x$ D. $-\sin^2 x$

65. Solve: $3\cos 2x = -x,\ 0 \le x < 2\pi$

A. 0.67 B. 0.52, 1.57 C. 0.67, 3.07 D. 0.95, 1.99

66. Determine the number of solutions for: $(a\sin x + a)(b\cos x - c) = 0$, where $0 \le x < 2\pi$, and $1 < a < b < c$

A. 1 B. 2 C. 3 D. 4

JAN 2003

67. Solve: $\sin 2x - \cos x = 1,\ 0 \le x < 2\pi$

A. 0, 5.07 B. 3.14, 4.32
C. 3.14, 4.36 D. 0.42, 1.89, 2.95, 4.21

68. Simplify: $\cos(\pi - 2x)$

A. $-\cos 2x$ B. $-\sin 2x$ C. $\cos 2x$ D. $\sin 2x$

69. Determine a cosine equation that has the following general solution: $\frac{\pi}{2}+n\pi, \frac{\pi}{6}+2n\pi, \frac{11\pi}{6}+2n\pi$, where n is an integer

A. $\cos x(2\cos x+\sqrt{2})=0$

B. $\cos x(2\cos x+\sqrt{3})=0$

C. $\cos x(2\cos x-\sqrt{2})=0$

D. $\cos x(2\cos x-\sqrt{3})=0$

70. Solve: $3\cos^2 x+\cos x-2 = 0,\ 0 \le x < 2\pi$

71. Prove: $(\csc\theta-\sin\theta)\tan\theta = \frac{\sin 2\theta}{2\sin\theta}$

JUN 2003

72. Solve: $\tan x - \cos x = -2,\ 0 \le x < 2\pi$

A. 1.17, 4.10

B. 1.97, 5.32

C. 1.17, 1.57, 4.10, 4.71

D. 1.57, 1.97, 4.71, 5.32

73. Solve: $4\cos^2 x = 3,\ 0 \le x < 2\pi$

A. $\frac{\pi}{6}, \frac{11\pi}{6}$

B. $-\frac{\pi}{3}, \frac{5\pi}{3}$

C. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

D. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

74. Determine an expression equivalent to: $\tan\theta + \cot\theta$

A. 1 B. $\sin\theta\cos\theta$ C. $\sec\theta\csc\theta$ D. $\sin\theta + \cos\theta$

75. Which expression is equivalent to $6\sin 8x\cos 8x$

A. $\sin 8x$ B. $\sin 16x$ C. $3\sin 4x$ D. $3\sin 16x$

76. Solve: $2\sin^2 x - \sin x = 0$

a) where $0 \le x \le 2\pi$

b) over the set of real numbers

JAN 2004

77. Solve: $2\sin x = \cos 3x,\ 0 \le x < 2\pi$

A. 0.31, 3.45 B. 2.83, 5.98
C. 0.39, 2.75, 4.03, 5.30 D. 0.98, 2.16, 3.55, 5.89

78. Simplify: $4\cos^2 6x - 2$

A. $2\cos 3x$ B. $4\cos 3x$ C. $2\cos 12x$ D. $4\cos 12x$

79. If the smallest positive solution of $\tan bx = 2$ is $x = 0.3$, determine the general solution.

A. $0.3 + 2n\pi,\ n \in I$ B. $0.3 + 2bn\pi,\ n \in I$ C. $0.3 + \frac{n\pi}{b},\ n \in I$ D. $0.3 + \frac{2n\pi}{b},\ n \in I$

80. Solve: $2\tan x\sin x - \tan x = 0,\ 0 \le x < 2\pi$

JUN 2004

81. Determine an expression equivalent to: $\tan^2\theta\csc\theta + \dfrac{1}{\sin\theta}$

A. $\sec^3\theta$ B. $\csc^3\theta$ C. $\csc^2\theta\sec\theta$ D. $\sec^2\theta\csc\theta$

82. Solve: $2\sin^2 x - 5\sin x - 3 = 0$, over the set of real numbers

A. $\dfrac{\pi}{6} + n\pi,\ \dfrac{5\pi}{6} + n\pi,\ n \in I$

B. $\dfrac{7\pi}{6} + n\pi,\ \dfrac{11\pi}{6} + n\pi,\ n \in I$

C. $\dfrac{\pi}{2} + 2n\pi,\ \dfrac{5\pi}{6} + 2n\pi,\ n \in I$

D. $\dfrac{7\pi}{6} + 2n\pi,\ \dfrac{11\pi}{6} + 2n\pi,\ n \in I$

83. Prove: $\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$

AUG 2005

84. Solve: $\sin^2 x = 3 - x$

A. 2.18 B. 2.97 C. 3.02 D. 3.09

85. Solve: $2\cos^2 x + 3\cos x + 1 = 0$

a) where $0 \le x \le 2\pi$

b) over the set of real numbers

86. Prove: $\cos 2x = \dfrac{\cot x - \sin 2x}{\cot x}$

AUG 2006

87. Determine an equivalent expression to: $\sin(2x - \pi)$

A. $2\sin x\cos x$ B. $-2\sin x\cos x$ C. $\cos^2 x - \sin^2 x$ D. $\sin^2 x - \cos^2 x$

88. Determine the number of solutions for: $(a\sin x - b)(a\cos x - a)(b\sin x + a) = 0$, where $0 \le x < 2\pi$ and $0 < a < b$

A. 3 B. 4 C. 5 D. 6

89. Solve: $3\cos^2 x - 8\cos x + 4 = 0$, over all real numbers

90. Prove: $\dfrac{\tan x(\cos x + \cot x)}{\sec x + \tan x} = \dfrac{\sin x \sin 2x}{2 - 2\cos^2 x}$

SAMPLE 2008

91. Which expression is equivalent to $\sin(\pi + 2x)$?

A. $2\cos^2 x - 1$ B. $1 - 2\cos^2 x$ C. $2\sin x\cos x$ D. $-2\sin x\cos x$

92. Solve: $\sqrt{3}\cos x\tan x + \cos x = 0,\ \ 0 \le x < 2\pi$

A. $\frac{\pi}{6}, \frac{7\pi}{6}$ B. $\frac{5\pi}{6}, \frac{11\pi}{6}$ C. $\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$ D. $\frac{5\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

93. Solve: $\cos 2x - 3\sin x = 2\ ,\ \ -\pi \le x \le \pi$

A. $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$ B. $\frac{4\pi}{3}, \frac{5\pi}{3}, \frac{3\pi}{2}$ C. $-\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2}$ D. $-\frac{\pi}{3}, -\frac{2\pi}{3}, -\frac{\pi}{2}$

94. Solve: $2\cos x = 2^x\ ,\ \ -\pi \le x \le \pi$

A. $-1.45,\ 0.57$ B. $-1.38,\ 0.66$

C. $-1.38,\ 0,\ 0.66$ D. $-1.11,\ 1.72,\ 2.93$

95. Determine all restrictions for: $\dfrac{3 + 2\csc\theta}{2\sec\theta - 3}$

A. $\sin\theta \neq 0$

B. $\cos\theta \neq \frac{2}{3},\ \cos\theta \neq 0$

C. $\cos\theta \neq \frac{2}{3},\ \sin\theta \neq 0,\ \cos\theta \neq 0$

D. $\sin\theta \neq -\frac{2}{3},\ \cos\theta \neq \frac{2}{3},\ \sin\theta \neq 0,\ \cos\theta \neq 0$

96. If the two smallest positive solutions for $\cos 4x = 0.6$ are $x = 0.232$ and $x = 1.339$, determine the general solution.

97. Prove: $\dfrac{\tan x}{\sec x + 1} = \dfrac{2\cos x - 2\cos^2 x}{\sin 2x}$

JAN 2008

98. Determine an equivalent expression for: $\sin 3x \cos x + \cos 3x \sin x$

A. $4\sin x$ B. $2\sin x \cos x$ C. $4\sin x \cos x$ D. $2\sin 2x \cos 2x$

99. Solve: $4\cos^2 x = 3,\ 0 \le x < 2\pi$

A. $\dfrac{\pi}{6}, \dfrac{11\pi}{6}$ B. $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$ C. $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$ D. $\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

100. Solve: $5\sin^2 x = \cos x$

A. 0.43, 1.78 B. 0.44, 5.84 C. 0.82, 1.73 D. 2.87, 3.58

101. Determine all restrictions for: $\dfrac{\sec x}{2\sin x + 1}$

A. $\sin x \neq -\dfrac{1}{2}$ B. $\sin x \neq 0,\ \sin x \neq -\dfrac{1}{2}$

C. $\cos x \neq 0,\ \sin x \neq -\dfrac{1}{2}$ D. $\cos x \neq 0,\ \sin x \neq 0,\ \sin x \neq -\dfrac{1}{2}$

102. Solve: $2\sin^2 x + \sqrt{3}\sin x = 0,\ 0 \le x < 2\pi$

103. If the smallest positive solution for $\tan 3x = 0.6$ is $x = 0.180$, determine the general solution.

104. Prove: $\dfrac{\sin\theta}{1-\sin\theta} + \dfrac{\sin\theta}{1+\sin\theta} = \sin 2\theta \sec^3\theta$

ADDITIONAL QUESTIONS

105. Express $\sin x \cos x$ in terms of a single trigonometric function.

A. $\dfrac{\sin 2x}{2}$ B. $\dfrac{\cos^2 x}{2}$ C. $2\sin 2x$ D. $2\cos^2 x$

106. What is the period of the graph of $y = 2\cos^2 5x - 1$?

A. $\dfrac{\pi}{5}$ B. $\dfrac{2\pi}{5}$ C. π D. 2π

107. Which expression is equivalent to $\left(\sin^2\theta - \cos^2\theta\right)^2 - \sin^2 2\theta$?

A. $-2\sin^2 2\theta$ B. $2\sin^2 2\theta$ C. $-\cos 4\theta$ D. $\cos 4\theta$

108. Determine the amplitude of the function: $y = 6\sin x \cos x$

A. 2 B. 3 C. 6 D. 12

109. Simplify: $\cos^4\theta - \sin^4\theta$

A. -1 B. $-2\cos 2\theta$ C. $\cos 2\theta$ D. $\cos 4\theta$

110. Determine a single geometric mean between $\sec x - 1$ and $\sec x + 1$.

A. -1 B. 1 C. $\cos x$ D. $\tan x$

2009 SAMPLE

111. Solve: $2\cos^2 x - \cos x - 1 = 0,\ 0 \le x < 2\pi$

A. $0,\ \frac{5\pi}{6},\ \frac{7\pi}{6}$ B. $0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}$ C. $\frac{\pi}{6},\ \pi,\ \frac{11\pi}{6}$ D. $\frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}$

112. Solve: $2\sin x = 3\cos x,\ 0 \le x < 2\pi$

A. 0.31, 3.45 B. 2.83, 5.98
C. 0.39, 2.75, 4.03, 5.30 D. 0.98, 2.16, 3.55, 5.89

113. Determine the number of solutions in the interval $0 \le x < 2\pi$ for: $\sin ax = \frac{1}{3}$
(a is an integer, $a \ge 1$)

A. 2 B. $\frac{a}{2}$ C. a D. $2a$

114. Solve: $\sin 2x = \frac{1}{\sqrt{2}},\ 0 \le x < 2\pi$

A. $\frac{\pi}{8},\ \frac{3\pi}{8}$ B. $\frac{\pi}{8},\ \frac{3\pi}{8},\ \frac{9\pi}{8},\ \frac{11\pi}{8}$
C. $\frac{\pi}{4},\ \frac{3\pi}{4}$ D. $\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$

115. Solve: $\sin x = \cos 2x,\ 0 \le x < 2\pi$

116. Solve: $2\tan x\cos x - \sqrt{3}\tan x = 0,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$

117. Solve: $\sin\frac{1}{3}x = \frac{\sqrt{3}}{2}$

a) where $0 \le x < 2\pi$

b) over the set of real numbers

118. Determine the general solution: $\sin 2x = -\frac{1}{2}$

A. $\frac{7\pi}{12} + 2n\pi,\ \frac{11\pi}{12} + 2n\pi,\ n \in I$

B. $\frac{7\pi}{12} + n\pi,\ \frac{11\pi}{12} + n\pi,\ n \in I$

C. $\frac{13\pi}{12} + 2n\pi,\ \frac{21\pi}{12} + 2n\pi,\ n \in I$

D. $\frac{13\pi}{12} + n\pi,\ \frac{21\pi}{12} + n\pi,\ n \in I$

119. Solve: $\cos^2 x = \cos x$ over the set of real numbers

120. The two smallest positive solutions of $\sin 3x = 0.7$ are $x = 0.258$ and $x = 0.789$. Determine the general solution.

121. Solve: $6\sin^2 x - \sin x - 2 = 0$ over the set of real numbers.

122. Solve: $\sin 2x - 2\cos^2 x = 0$ over the set of real numbers.

123. Determine all restrictions for: $\dfrac{\tan\theta}{2\cos\theta - 1}$

A. $\cos\theta \neq \dfrac{1}{2}$

B. $\sin\theta \neq 0$

C. $\sin\theta \neq 0,\ \cos\theta \neq \dfrac{1}{2}$

D. $\cos\theta \neq 0,\ \cos\theta \neq \dfrac{1}{2}$

124. Determine an expression equivalent to: $\dfrac{\tan\theta\csc^2\theta}{\sec^2\theta}$

A. $\tan\theta$

B. $\cot\theta$

C. $\tan^2\theta$

D. $\tan^3\theta$

125. Determine an equivalent expression for $\sin 3x \cos x + \cos 3x \sin x$

A. $4 \sin x$ B. $2 \sin x \cos x$ C. $4 \sin x \cos x$ D. $2 \sin 2x \cos 2x$

126. Prove: $\dfrac{\cos x + \cot x}{\sec x + \tan x} = \cos x \cot x$

127. Prove: $\dfrac{2\cos x + 2\cos^2 x}{\sin 2x} = \dfrac{\sin x}{1 - \cos x}$

128. Determine an expression equivalent to $\cos(\pi + 2A)$

A. $-\cos 2A$ B. $\cos 2A$ C. $-\sin 2A$ D. $\sin 2A$

129. Simplify: $\cos 2x \cos x + \sin 2x \sin x$

A. $\cos x$ B. $\sin x$ C. $\cos 3x$ D. $\sin 3x$

130. Simplify: $\dfrac{2\sin\theta}{\sin 2\theta}$

A. 1 B. $\cos\theta$ C. $\csc\theta$ D. $\sec\theta$

131. Prove: $\dfrac{\tan x + \sin x}{1 + \cos x} = \dfrac{1}{\csc 2x} - \dfrac{\tan x}{\sec 2x}$

A FEW MORE QUESTIONS

132. Solve: $2\cos^2 x = -3\sin x, \ 0 \le x < 2\pi$

133. Prove: $\dfrac{1}{1 + \cos\theta} = \csc^2\theta - \dfrac{\cot\theta}{\sin\theta}$

134. Given $\cos\alpha = \frac{3}{5}$, where α is in quadrant IV, and $\sin\beta = \frac{2}{3}$, where β is in quadrant II, determine the exact values for:

a) $\sin(\alpha - \beta)$

b) $\cos(2\beta)$

c) $\sin\left(\alpha + \frac{\pi}{3}\right)$

135. Given $\sin\gamma = \frac{5}{13}$, where γ is in quadrant I, and $\tan\delta = 2$, where δ is in quadrant III, determine the exact values for:

a) $\sin(\gamma + \delta)$

b) $\cos(\delta - \gamma)$

c) $\tan 2\gamma$

TRIGONOMETRY II ANSWER KEY

(Note: Blank answers indicate a trigonometric identity proof, for which solutions are not provided here)

1 A
2 A
3 B
4 1.91, 4.37
5 B
6 B
7 C
8 D
9 B
10 B
11 B
12
13 D
14 D
15 B
16
17 D
18 B
19 A
20
21 D
22 D
23 C
24
25 A
26 D
27 C
28 A
29 B
30
31 B
32 C
33 A
34
35 B
36 C
37 D
38 A
39
40 D
41 C
42 C
43
44
45 A
46 A
47 A
48
49 D
50 C
51 A
52 B
53 $\frac{\pi}{3}+2n\pi, \frac{5\pi}{3}+2n\pi, \pi+2n\pi, \quad n\in I$
54 A
55 D
56 C
57 B
58
59 A
60 A
61 C
62 D
63
64 A
65 D
66 A
67 C
68 A
69 D
70 0.84, 3.14, 5.44
71
72 B
73 C
74 C
75 D
76 a) $0, \frac{\pi}{6}, \pi, \frac{5\pi}{6}$
b) $\frac{\pi}{6}+2n\pi, \frac{5\pi}{6}+2n\pi, n\pi, \quad n\in I$
77 A
78 C
79 C
80 $0, \frac{\pi}{6}, \pi, \frac{5\pi}{6}$
81 D
82 D
83
84 B
85 a) $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$
b) $\frac{2\pi}{3}+2n\pi, \frac{4\pi}{3}+2n\pi, \pi+2n\pi, \quad n\in I$
86
87 B
88 A
89 $0.84+2n\pi, 5.44+2n\pi, n\in I$
90
91 D
92 B
93 C
94 B
95 C
96 $0.23+\frac{n\pi}{2}, 1.34+\frac{n\pi}{2}, n\in I$
97
98 D
99 B
100 B
101 C
102 $0, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$
103 $0.18+\frac{n\pi}{3}, n\in I$
104
105 A
106 A
107 D
108 B
109 C
110 A
111 B
112 A
113 D
114 B
115 $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
116 $-\frac{\pi}{6}, 0, \frac{\pi}{6}$
117 a) π
b) $\pi+6n\pi, 2\pi+6n\pi, \ n\in I$
118 B
119 $\frac{\pi}{2}+2n\pi, \frac{3\pi}{2}+2n\pi, 2n\pi, \quad n\in I$
120 $0.26+\frac{2n\pi}{3}, 0.79+\frac{2n\pi}{3}, n\in I$
121 $0.73+2n\pi, \ 2.41+2n\pi, \frac{7\pi}{6}+2n\pi, \ \frac{11\pi}{6}+2n\pi, n\in I$
122 $\frac{\pi}{2}+2n\pi, \ \frac{3\pi}{2}+2n\pi$
$\frac{\pi}{4}+2n\pi, \ \frac{5\pi}{4}+2n\pi, n\in I$
123 D
124 B
125 D
126
127
128 A
129 A
130 D
131
132 $\frac{7\pi}{6}, \frac{11\pi}{6}$
133
134 a) $\frac{4\sqrt{5}-6}{15}$
b) $\frac{1}{9}$
c) $\frac{3\sqrt{3}-4}{10}$
135 a) $\frac{-29}{13\sqrt{5}}$
b) $\frac{-22}{13\sqrt{5}}$
c) $\frac{120}{119}$

PRE-CALCULUS 12 **REVIEW OF SUM AND DIFFERENCE and DOUBLE ANGLE IDENTITIES**

1. **Simplify:**

a. $\sin^2(2.5)+\cos^2(2.5)$

b. $\cos(-\theta)\sec(-\theta)-\csc(\theta)\sin(-\theta)$

c. $\sin 160^\circ \cos 20^\circ + \cos 160^\circ \sin 20^\circ$

d. $\dfrac{\sin 4\theta}{2\sin 2\theta}$

e. $\sin\theta\csc\theta+\dfrac{\sin\theta}{\cos\theta\cot\theta}$

2. **Use identities to simplify:**

a. $1-2\sin^2(1.5)$

b. $\sin(0.8)\cos(0.8)$

c. $2\sin^2(0.75)-1$

3. If θ is a 2nd quadrant angle with $\sin\theta=\dfrac{4}{5}$ and β is a 3rd quadrant angle with $\sec\beta=-\dfrac{13}{5}$, determine:

a. $\sin(2\beta)$

b. $\cos(\theta+\pi)$

c. $\sin(\beta-\theta)$

4. **Prove**

a. $\cos\theta+\sin\theta=\dfrac{\cos 2\theta}{\cos\theta-\sin\theta}$

b. $\sin 2A=\dfrac{2\tan A}{1+\tan^2 A}$

c. $\sin 3\theta = 3\sin\theta\cos^2\theta-\sin^3\theta$

d. $\dfrac{2\tan x}{1-\tan^2 x}=\tan 2x$

REVIEW OF SUM AND DIFFERENCE AND DOUBLE ANGLE IDENTITIES ANSWER KEY

1a. $\sin^2(2.5)+\cos^2(2.5)$ = 1 (Pythagorean Identity, θ = 2.5 radians)

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b. $\cos(-\theta)\sec(-\theta)-\csc(\theta)\sin(-\theta)$

$\cos(-\theta)=\cos\theta$, by even symmetry $\therefore \sec(-\theta)=\sec(\theta)$

and $\sin(-\theta)=-\sin(\theta)$, by odd symmetry

$\therefore \cos(-\theta)\sec(-\theta)-\csc(\theta)\sin(-\theta)=\cos(\theta)\sec(\theta)-\csc(\theta)(-\sin(\theta))=1-(-1)=2$

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c. $\sin 160^\circ\cos 20^\circ+\cos 160^\circ\sin 20^\circ=\sin(160^\circ+20^\circ)=\sin(180^\circ)=0$

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d. $\dfrac{\sin 4\theta}{2\sin 2\theta}=\dfrac{2\sin 2\theta\cos 2\theta}{2\sin 2\theta}=\cos 2\theta$

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e. $\sin\theta\csc\theta+\dfrac{\sin\theta}{\cos\theta\cot\theta}=1+\dfrac{\sin\theta}{\left(\dfrac{\cos\theta}{1}\right)\left(\dfrac{\cos\theta}{\sin\theta}\right)}=1+\dfrac{\sin\theta}{\left(\dfrac{\cos^2\theta}{\sin\theta}\right)}=1+\dfrac{\sin^2\theta}{\cos^2\theta}=1+\tan^2\theta=\sec^2\theta$

2a. $1-2\sin^2(1.5)=\cos(3)$ (That is the cosine of 3 radians)

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b. $\sin(0.8)\cos(0.8)=\dfrac{\sin(1.6)}{2}$

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c. $2\sin^2(0.75)-1=-\cos(1.5)$

3a. $\sin(2\beta)=2\sin\beta\cos\beta=2\left(-\dfrac{12}{13}\right)\left(-\dfrac{5}{13}\right)=\dfrac{120}{169}$

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b. $\cos(\theta+\pi)=\cos\theta\cos\pi-\sin\theta\sin\pi=(\cos\theta)(-1)-(\sin\theta)(0)=-\cos\theta$

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c. $\sin(\beta-\theta)=\sin\beta\cos\theta-\cos\beta\sin\theta=\left(-\dfrac{12}{13}\right)\left(-\dfrac{3}{5}\right)-\left(-\dfrac{5}{13}\right)\left(\dfrac{4}{5}\right)=\dfrac{56}{65}$

4c. $\sin 3\theta$ $\qquad\qquad$ $3\sin\theta\cos^2\theta-\sin^3\theta$

$=\sin(\theta+2\theta)$

$=\sin\theta\cos 2\theta+\cos\theta\sin 2\theta$

$=\sin\theta\left(\cos^2\theta-\sin^2\theta\right)+\cos\theta(2\sin\theta\cos\theta)$

$=\sin\theta\cos^2\theta-\sin^3\theta+2\sin\theta\cos^2\theta$

$=3\sin\theta\cos^2\theta-\sin^3\theta$

TRIGONOMETRIC IDENTITIES

1. $\dfrac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

2. $\dfrac{\cos 2\theta + 1}{\sin 2\theta} = \dfrac{\cos\theta}{\sin\theta}$

3. $\dfrac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$

4. $\dfrac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$

5. $\dfrac{1}{1+\sin\theta} = \sec^2\theta - \dfrac{\tan\theta}{\cos\theta}$

6. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

7. $\sin\theta + \cos\theta\cot\theta = \csc\theta$

8. $\dfrac{\sin 2\theta}{2-2\cos^2\theta} = \cot\theta$

9. $\dfrac{1-\cos\theta}{\sin^2\theta} = \dfrac{1}{1+\cos\theta}$

10. $\dfrac{\csc\theta}{\tan\theta + \cot\theta} = \cos\theta$

11. $\dfrac{\cot\theta}{\csc\theta - 1} = \dfrac{\csc\theta + 1}{\cot\theta}$

12. $\dfrac{\sin\theta + \tan\theta}{1+\cos\theta} = \dfrac{\sin 2\theta}{2\cos^2\theta}$

13. $\dfrac{\sin 2\theta}{\cos\theta} + \dfrac{\cos 2\theta}{\sin\theta} = \csc\theta$

14. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

15. $\dfrac{1}{\sec\theta + \tan\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

16. $\dfrac{\cos 2\theta}{\sin\theta} = \dfrac{\cot^2\theta - 1}{\csc\theta}$

17. $(1-\sin\theta)(\sec\theta + \tan\theta) = \dfrac{1}{\sec\theta}$

18. $\dfrac{\sin\theta\cos\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\tan\theta}$

19. $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$

20. $\sin 2x\left(\tan x + \cot x\right) = 2$

21. $\dfrac{2\cos x + 2\cos^2 x}{\sin 2x} = \dfrac{\sin x}{1-\cos x}$

22. $\left(\csc\theta - \sin\theta\right)\tan\theta = \dfrac{\sin 2\theta}{2\sin\theta}$

23. $\dfrac{\cot\theta}{\sin\theta - \csc\theta} = -\sec\theta$

24. $\dfrac{\sin x}{1-\sin x} - \dfrac{\sin x}{1+\sin x} = 2\tan^2 x$

25. $\dfrac{\cos\theta + \cot\theta}{1+\sin\theta} = \cot\theta$

26. $\dfrac{\cos x + \cot x}{\sec x + \tan x} = \cos x \cot x$

27. $\tan\theta\cos 2\theta + \tan\theta = \sin 2\theta$

28. $\csc\theta\sin 2\theta - \sec\theta\cos 2\theta = \sec\theta$

29. $\dfrac{1-\cos 2x}{\sin 2x} = \dfrac{1+\tan x}{1+\cot x}$

30. $\cos 2x = \dfrac{\cot x - \sin 2x}{\cot x}$

31. $\dfrac{\tan x\left(\cos x + \cot x\right)}{\sec x + \tan x} = \dfrac{\sin x \sin 2x}{2 - 2\cos^2 x}$

32. $\dfrac{\tan x + \sin x}{1+\cos x} = \dfrac{1}{\csc 2x} - \dfrac{\tan x}{\sec 2x}$

33. $\dfrac{\tan x}{\sec + 1} = \dfrac{2\cos x - 2\cos^2 x}{\sin 2x}$

34. $\dfrac{\sin x}{1-\sin x} + \dfrac{\sin x}{1+\sin x} = \sin 2x \sec^3 x$