

PRECALCULUS

MATHEMATICS 12

TABLE OF CONTENTS

COMBINING FUNCTIONS

Review of basic functions;

Addition, subtraction, multiplication and division of functions; composition of functions.

- **48 PROBLEMS**

TRANSFORMATIONS

Transformations of graphs and equations (vertical and horizontal translations, stretches, and reflections) of parent functions and relations; inverses.

- **118 PROBLEMS**

POLYNOMIALS

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Graphing and polynomial characteristics;

Solving equations and inequalities algebraically and graphically.

- **183 PROBLEMS**

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- **51 PROBLEMS**

LOGARITHMIC and EXPONENTIAL FUNCTIONS

Graphing and characteristics of exponential and logarithmic functions including transformations;

Applying laws of logarithms;

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Applications.

- **235 PROBLEMS**
- **LAWS OF LOGARITHMS WORKSHEET**
- **REVIEW PACKAGE**

TRIGONOMETRY I

Examining angles in standard position in both radians and degrees;
Reference and coterminal angles; arc length;
Reciprocal trigonometric functions;
Solving first-degree equations;
Graphing and characteristics of primary trigonometric functions, including transformations.

- **165 PROBLEMS**
- **REVIEW PACKAGE**

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Solving second-degree equations (over restricted domains and all real numbers).

- **135 PROBLEMS**
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- **REVIEW OF SUM and DIFFERENCE & DOUBLE ANGLE IDENTITIES WORKSHEET**

CONIC SECTIONS

The parabola, circle, ellipse and hyperbola: equations and graphs;
General to Standard Form;
Transformations and applications.

- **194 PROBLEMS**

GEOMETRIC SEQUENCES and SERIES

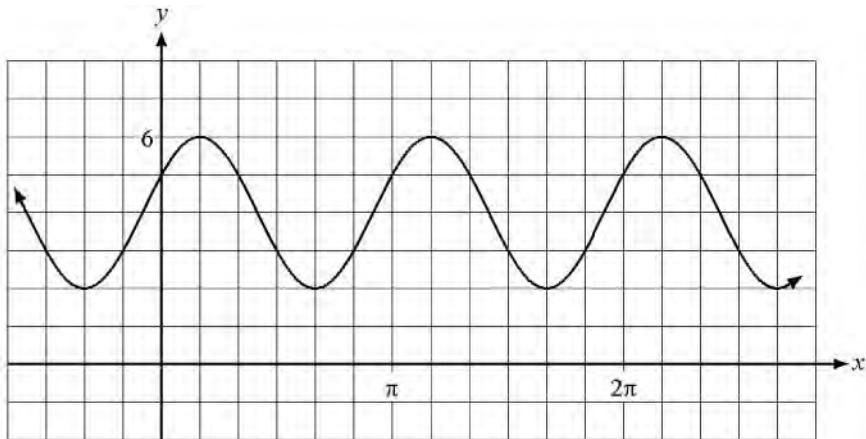
Common ratio, first term, general term;
Infinite geometric series and summation notation.

- **121 PROBLEMS**

TRIGONOMETRY I

JAN 1997

1. Convert 256° to radians.
- A. 2.23 B. 3.39 C. 4.47 D. 8.93
2. Determine the period of the function $f(x) = 3 \sin 4x + 1$.
- A. $\frac{\pi}{2}$ B. $\frac{2\pi}{2}$ C. 6π D. 8π
3. If the point $(-4, 2)$ lies on the terminal arm of an angle θ in standard position, determine the exact value of $\csc \theta$.
- A. $-\sqrt{5}$ B. $-\frac{\sqrt{5}}{2}$ C. $\frac{\sqrt{5}}{2}$ D. $\sqrt{5}$
4. Solve: $2 \cot x + 3 = 0$, where $0 \leq x < 2\pi$
- A. 0.59, 3.73 B. 0.98, 4.12 C. 2.16, 5.30 D. 2.55, 5.70
5. Which equation describes the following graph?



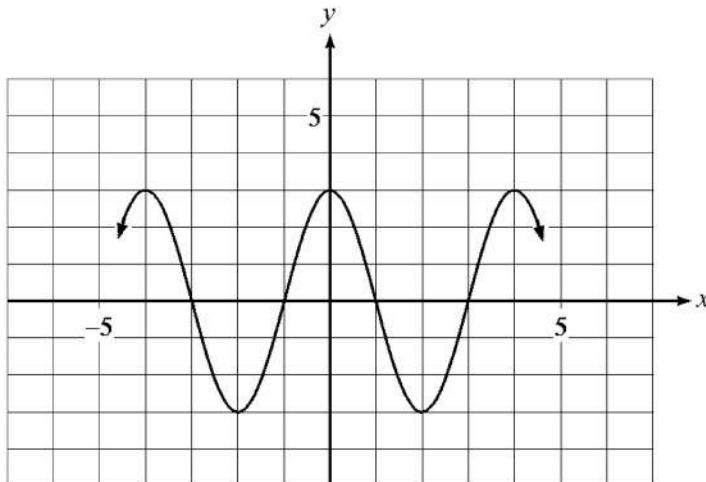
- A. $2 \cos 2\left(x - \frac{\pi}{6}\right) + 4$
- B. $2 \cos 2\left(x + \frac{\pi}{6}\right) + 4$
- C. $2 \cos 2\left(x + \frac{\pi}{6}\right) + 2$
- D. $2 \cos 2\left(x - \frac{\pi}{6}\right) + 2$

JUN 1997

6. Convert $\frac{5\pi}{6}$ radians to degrees.
- A. 108° B. 150° C. 216° D. 300°

7. Determine the period of the trigonometric function graphed below:

- A. 2
B. 3
C. 4
D. 6



8. Evaluate $\sec 0.156$ to 3 decimal places.

- A. 0.992 B. 1.012 C. 1.414 D. 6.436

9. Given two functions, $f(x) = \sin\left(x - \frac{\pi}{4}\right)$ and $g(x) = \cos(x - a)$, determine the smallest positive value for a so that the graphs are identical.

- A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $\frac{3\pi}{4}$ D. $\frac{5\pi}{4}$

JAN 1998

10. Convert 162° to radian measure.

- A. 0.90 B. 2.83 C. 508.94 D. 9281.92

11. Determine the amplitude of the graph of: $y = -2 \cos 3x$

- A. -2 B. 2 C. 3 D. $\frac{2\pi}{3}$

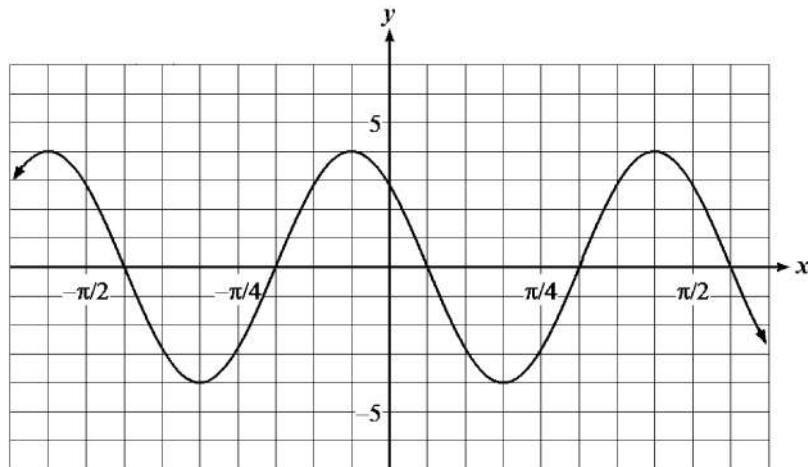
12. Evaluate: $\csc 1.2$

- A. 0.67 B. 0.74 C. 1.07 D. 2.76

13. If $\sin x = \frac{3}{4}$, determine the smallest positive angle x , in radians.

- A. 0.68 B. 0.72 C. 0.85 D. 1.47

14. Which equation describes the following graph?



A. $y = -4 \sin 4\left(x - \frac{\pi}{16}\right)$

B. $y = 4 \sin 4\left(x - \frac{\pi}{16}\right)$

C. $y = 4 \sin 4\left(x - \frac{3\pi}{16}\right)$

D. $y = -4 \sin 4\left(x + \frac{3\pi}{16}\right)$

15. If $\sin \theta = a$ and $0 \leq \theta \leq \frac{\pi}{2}$, determine an expression for: $\cos(\pi + \theta)$

- A. $1 - a$ B. $a - 1$ C. $\sqrt{1 - a^2}$ D. $-\sqrt{1 - a^2}$

JUN 1998

16. Convert 4 radians to degrees.

- A. 13° B. 115° C. 229° D. 720°

17. Determine the amplitude of the graph of $y = -4 \cos 2x$.

A. -4

B. 2

C. 4

D. 8

18. Solve: $3 \cos x + 2 = 0, 0 \leq x < 2\pi$

A. 0.84, 2.30

B. 0.84, 5.44

C. 2.30, 3.98

D. 2.36, 3.93

19. Evaluate: $\sec \frac{3\pi}{8}$

A. 0.75

B. 1.08

C. 1.18

D. 2.61

20. Determine the number of asymptotes of $y = \tan x$ over the interval $-2\pi \leq x \leq 2\pi$.

A. 2

B. 4

C. 6

D. 8

21. Find the period of the function which has a minimum point at $\left(\frac{\pi}{3}, 1\right)$ and its nearest maximum point to the right at $\left(\frac{2\pi}{3}, 5\right)$.

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{4\pi}{3}$

D. 2π

JAN 1999

22. Convert 322° to radians.

A. 0.66

B. 2.81

C. 5.62

D. 11.24

23. Which expression is equivalent to $2 \csc \frac{\pi}{7}$?

A. $2 \sin \frac{7}{\pi}$

B. $\frac{1}{2 \sin \frac{\pi}{7}}$

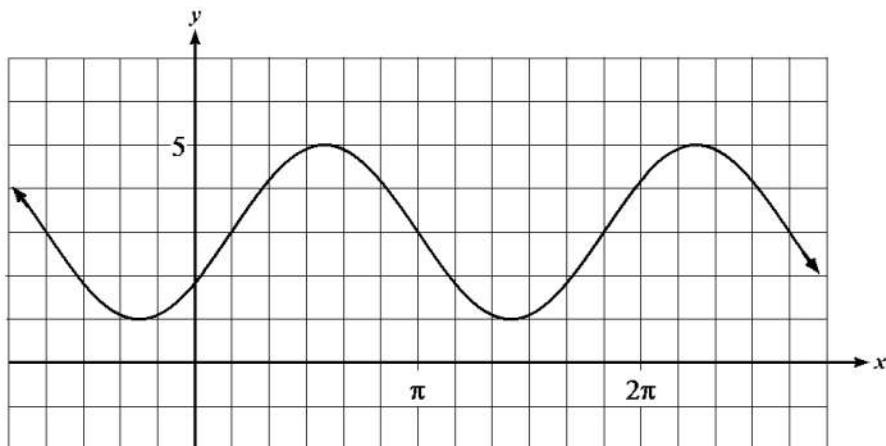
C. $\frac{2}{\sin \frac{7}{\pi}}$

D. $\frac{2}{\sin \frac{\pi}{7}}$

24. Solve: $3 \tan x + \sqrt{15} = 0$, where $0 \leq x < 2\pi$

- A. 0.91, 4.05 B. 2.23, 4.05 C. 2.23, 5.37 D. 4.05, 5.37

25. Determine the amplitude of the sine function shown.



- A. 2 B. 3 C. 4 D. 5

26. Determine the period of the sine function shown in the graph above.

- A. $\frac{5\pi}{6}$ B. π C. $\frac{5\pi}{3}$ D. $\frac{11\pi}{6}$

27. If $\sec \theta = -\frac{5}{3}$ and angle θ terminates in quadrant III, which point must be on the terminal arm of θ ?

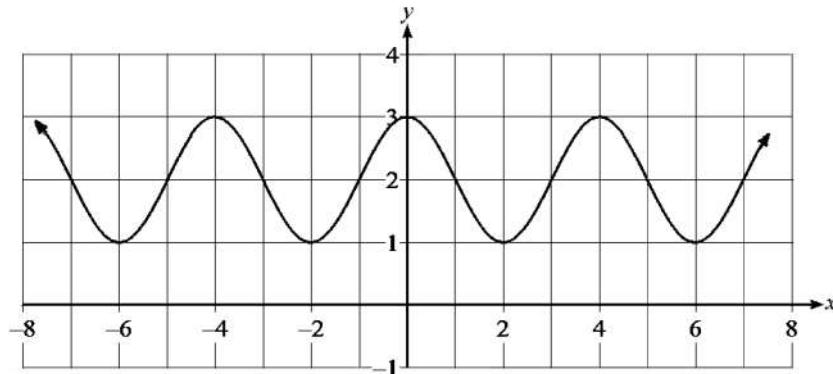
- A. $(-6, -10)$ B. $(-10, -6)$ C. $(-8, -6)$ D. $(-6, -8)$

JUN 1999

28. Convert 200° to radians.

- A. 2.83 B. 3.49 C. 3.83 D. 4.49

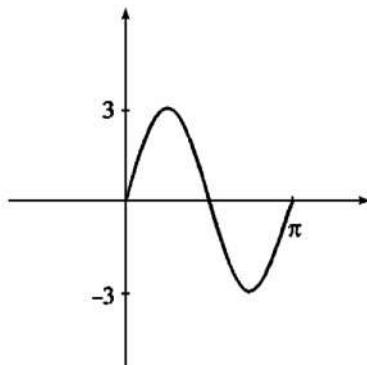
29. Determine the period of the trigonometric function shown.



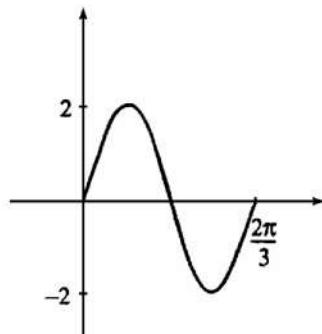
- A. 1 B. 2 C. 3 D. 4

30. Which of the following shows one period of the graph of $f(x) = 2 \sin 3x$?

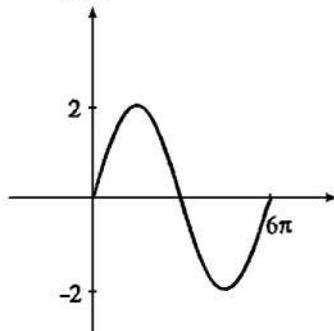
A. $f(x)$



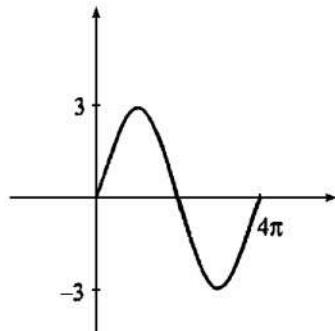
B. $f(x)$



C. $f(x)$



D. $f(x)$



31. If $\cos \theta = \frac{5}{13}$, where θ is in quadrant IV, determine the value of $\cot \theta$.

A. $-\frac{12}{5}$

B. $-\frac{5}{12}$

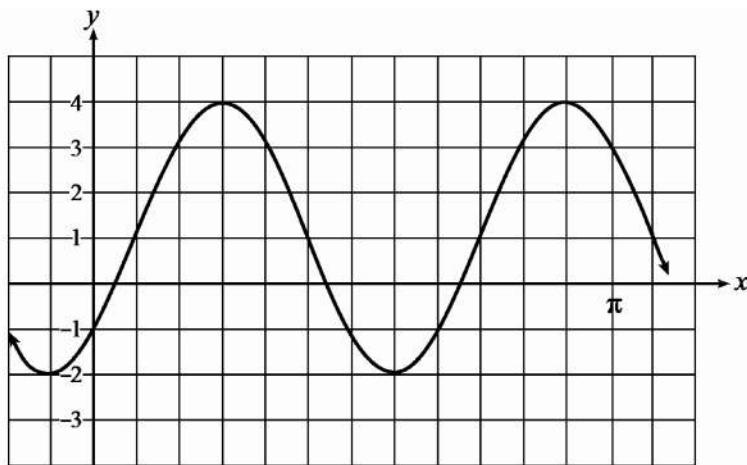
C. $\frac{5}{12}$

D. $\frac{12}{5}$

32. Evaluate: $\sec \frac{2\pi}{5}$
- A. 0.70 B. 1.05 C. 1.43 D. 3.24
33. The range of the trigonometric function $y = a \cos x + b$ is $-2 \leq y \leq 8$. Determine the value of b .
- A. 3 B. 5 C. 6 D. 10
- JAN 2000**
34. Convert 5 radians to degrees.
- A. 0.09° B. 286.48° C. 291.39° D. 318.31°
35. Evaluate: $\cot 4.47$
- A. -0.24 B. 0.23 C. 0.25 D. 4.04
36. Determine the maximum value of the function $y = 3 \cos 2x - 4$.
- A. -2 B. -1 C. 3 D. 7
37. Determine $\csc \theta$ if $(-10, 24)$ lies on the terminal arm of angle θ in standard position.
- A. $-\frac{13}{5}$ B. $-\frac{13}{12}$ C. $\frac{13}{12}$ D. $\frac{13}{5}$
38. A sine curve has a zero at -2 . The nearest zero to the right is at 3 . A maximum point is located between these zeros. If the range of the function is $-1 \leq y \leq 1$, determine an equation of this function.
- A. $y = \sin \frac{\pi}{5}(x - 2)$ B. $y = \sin \frac{2\pi}{5}(x - 2)$ C. $y = \sin \frac{2\pi}{5}(x + 2)$ D. $y = \sin \frac{\pi}{5}(x + 2)$

JUN 2000

The equation of the graph shown is $y = a \cos[b(x - c)] + d$, where a , b , c , and d are all positive.



39. In the graph above, determine the value of d .
- A. 1 B. 2 C. 3 D. 4
40. In the graph above, determine the value of a .
- A. 1 B. 2 C. 3 D. 4
41. In the graph above, determine the value of b .
- A. $\frac{2}{3}$ B. 3 C. 4 D. 8
42. Evaluate: $\csc \frac{2\pi}{7}$
- A. -1.00 B. 0.90 C. 1.28 D. 1.60
43. Convert 3 radians to degrees.
- A. 150° B. 172° C. 180° D. 540°
44. Solve: $\cot \theta = -3$, $0 \leq \theta < 2\pi$
- A. 2.34, 5.48 B. 2.80, 5.94 C. 2.82, 5.94 D. 2.82, 5.96

45. The height of a piston in an engine can be determined by the function $h=20 \sin\left[\frac{\pi t}{0.025}\right]+20$, where height, h , is in centimetres, and time, t , is in seconds. Determine the period of this function.

A. 0.025 B. 0.05 C. 0.25 D. 0.5

JAN 2001

46. Convert 210° to radians.

A. $\frac{7\pi}{12}$ B. $\frac{6\pi}{7}$ C. $\frac{7\pi}{6}$ D. $\frac{12\pi}{7}$

47. Determine the phase shift of the function $y=4 \cos 2\left(x - \frac{\pi}{4}\right) + 5$.

A. $\frac{\pi}{4}$ to the right B. $\frac{\pi}{2}$ to the right C. $\frac{\pi}{4}$ to the left D. $\frac{\pi}{2}$ to the left

48. Solve: $\tan x = 3.2$, $0 \leq x < 2\pi$

A. 0.06, 3.20 B. 1.27, 1.87 C. 1.27, 4.41 D. 1.87, 5.02

49. If the point $(-7, -24)$ is on the terminal arm of angle θ in standard position, determine the value of $\csc \theta$.

A. $-\frac{25}{7}$ B. $-\frac{25}{24}$ C. $\frac{7}{25}$ D. $\frac{24}{25}$

50. A sine function has a maximum point at $(4, 32)$ and the nearest minimum point to the right is $(16, 18)$. Determine an equation for this function.

A. $y = 7 \sin\left[\frac{\pi}{6}(x - 4)\right] + 25$	B. $y = 7 \sin\left[\frac{\pi}{6}(x + 4)\right] + 25$
C. $y = 7 \sin\left[\frac{\pi}{12}(x - 2)\right] + 25$	D. $y = 7 \sin\left[\frac{\pi}{12}(x + 2)\right] + 25$

51. Which expression below is equivalent to $2 \cot \frac{\pi}{5}$?

A. $2 \tan \frac{\pi}{5}$	B. $\frac{1}{2 \tan \frac{5}{\pi}}$	C. $\frac{2}{\tan \frac{5}{\pi}}$	D. $\frac{2}{\tan \frac{\pi}{5}}$
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JUN 2001

52. Convert 2.1 radians to degrees.

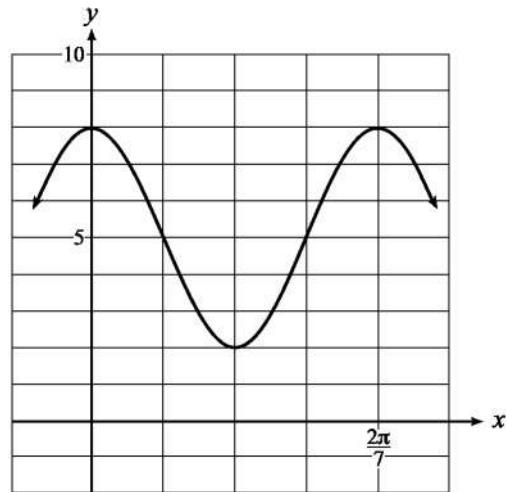
- A. 60.16° B. 120.32° C. 126.35° D. 240.64°

53. Solve: $\tan \theta = -1.25$, $0 \leq \theta < 2\pi$

- A. 0.90, 4.04 B. 2.25, 5.39 C. 2.25, 6.15 D. 3.01, 6.15

54. Determine an equation of the cosine function shown.

- A. $y = 3 \cos 7x + 2$
 B. $y = 3 \cos 7x + 5$
 C. $y = 6 \cos 7x + 5$
 D. $y = 8 \cos 7x + 2$



55. A and B are complementary angles. If $\sin A = \frac{3}{5}$, find the value of $\sec B$.

- A. $\frac{3}{5}$ B. $\frac{4}{5}$ C. $\frac{5}{4}$ D. $\frac{5}{3}$

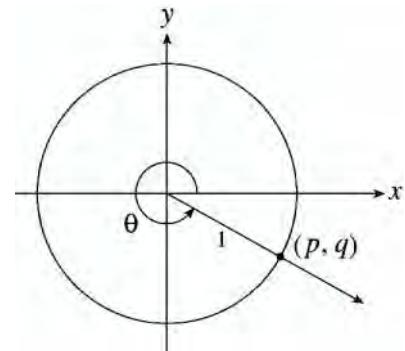
SAMPLE 2001

56. Convert 150° to radians.

- A. $\frac{2\pi}{3}$ B. $\frac{3\pi}{2}$ C. $\frac{5\pi}{6}$ D. $\frac{6\pi}{5}$

57. If the diagram below shows a unit circle, determine $\cos \theta$.

- A. p
- B. q
- C. $-p$
- D. $-q$



58. Determine the period of the function $y = \tan \frac{\pi}{5}x$.

- A. 5
- B. 10
- C. $\frac{\pi}{5}$
- D. $\frac{\pi}{10}$

59. Solve: $\sqrt{3} + 2 \sin x = 0$, $0 \leq x < 2\pi$

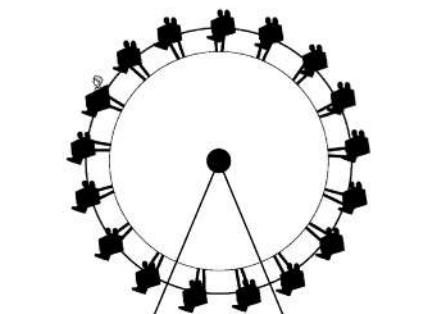
- A. $\frac{\pi}{3}, \frac{2\pi}{3}$
- B. $\frac{4\pi}{3}, \frac{5\pi}{3}$
- C. $\frac{\pi}{6}, \frac{5\pi}{6}$
- D. $\frac{7\pi}{6}, \frac{11\pi}{6}$

60. In the function $y = a \sin(x - c) + d$ where a , c , and d are positive constants, determine the range of the new function formed if a is doubled.

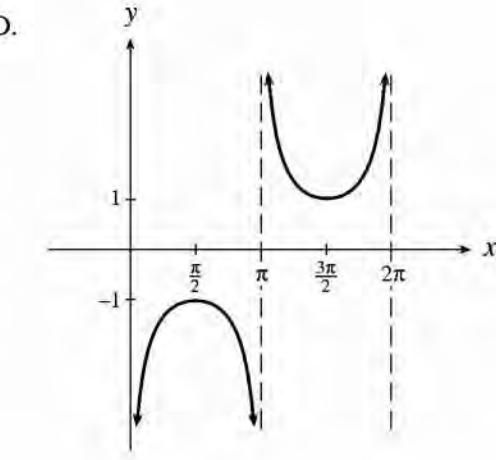
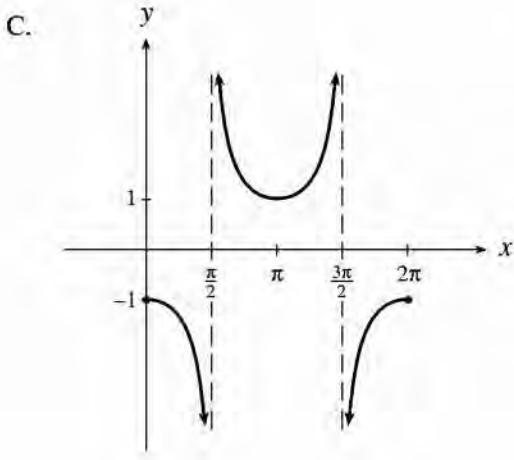
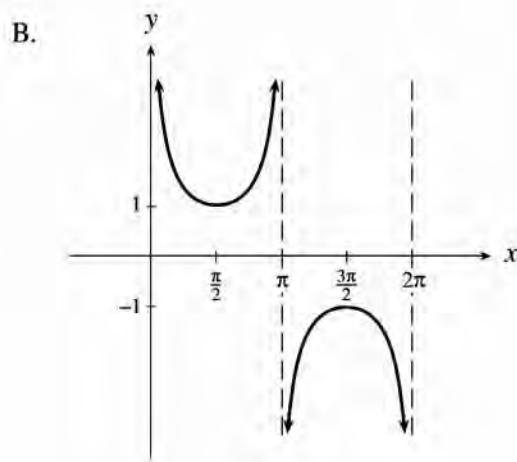
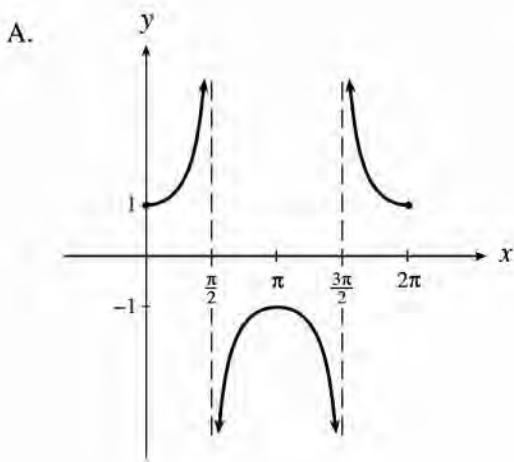
- A. $d - \frac{a}{2} \leq y \leq d + \frac{a}{2}$
- B. $d - 2a \leq y \leq d + 2a$
- C. $-d - \frac{a}{2} \leq y \leq -d + \frac{a}{2}$
- D. $-d - 2a \leq y \leq -d + 2a$

61. The Ferris wheel shown in the diagram has a radius of 16 m and its centre is 18 m above the ground. It rotates once every 60 s. Ethan gets on the Ferris wheel at its lowest point and then the wheel starts to rotate. How long does it take Ethan to reach 29 m above the ground for the first time? 

- A. 11.12 s
- B. 22.24 s
- C. 23.92 s
- D. 37.76 s



62. Which graph best represents $y = \sec x$, $0 \leq x \leq 2\pi$?



JAN 2002

63. Convert 5.3 radians to degrees.

A. 0.09°

B. 0.18°

C. 151.83°

D. 303.67°

64. Determine the period of $y = 6 \cos \frac{2\pi}{15}x + 8$.

A. $\frac{2}{15}$

B. $\frac{15}{2}$

C. 15

D. 30

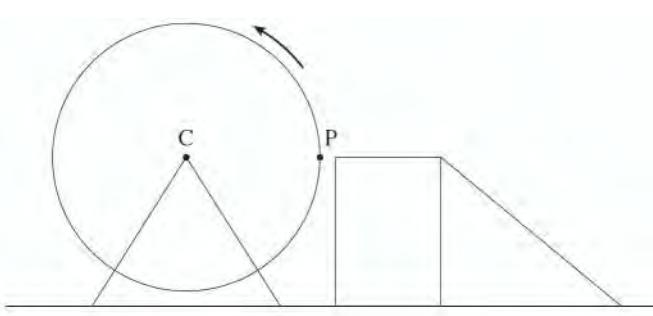
65. Determine the exact value of $\tan \frac{5\pi}{6}$.

A. $-\frac{\sqrt{3}}{2}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{\sqrt{3}}{2}$

66. The point $P(m, n)$ is the intersection point of the terminal arm of angle θ in standard position and the unit circle $x^2 + y^2 = 1$. Which expression represents $\sin \theta$?
- A. m B. n C. $\frac{m}{n}$ D. $\frac{n}{m}$
67. Which of the following is an asymptote of $y = \sec x$?
- A. $x = 0$ B. $x = \frac{\pi}{4}$ C. $x = \frac{\pi}{2}$ D. $x = \pi$
68. A Ferris wheel has a radius of 18 metres and a centre C which is 20 metres above the ground. It rotates once every 32 seconds in the direction shown in the diagram. A platform allows a passenger to get on the Ferris wheel at a point P which is 20 metres above the ground. If the ride begins at point P , when the time $t = 0$ seconds, determine a sine function that gives the passenger's height, h , in metres, above the ground as a function of t .
- A. $h(t) = 18 \sin\left[\frac{\pi}{16}t\right] + 20$
 B. $h(t) = 18 \sin\left[\frac{\pi}{32}t\right] + 20$
 C. $h(t) = 20 \sin\left[\frac{\pi}{16}t\right] + 18$
 D. $h(t) = 20 \sin\left[\frac{\pi}{32}t\right] + 18$
- 
- APR 2002
69. Determine the period of $y = \tan x$.
- A. 1 radian B. $\frac{\pi}{2}$ radians C. π radians D. 2π radians
70. Given a circle with radius 10 cm, calculate the length of arc which contains a sector angle of 2 radians.
- A. 5π cm B. 10π cm C. 10 cm D. 20 cm
71. Find the exact value of $\tan \theta = \frac{5\pi}{3}$.
- A. $-\sqrt{3}$ B. $-\frac{1}{\sqrt{3}}$ C. $\frac{1}{\sqrt{3}}$ D. $\sqrt{3}$

72. Determine the maximum value of the function $f(x) = a \cos x + d$, where $a > 0$ and $d > 0$.
- A. a B. $d - a$ C. $a + d$ D. $2a + d$
73. The terminal arm of angle θ in standard position passes through point (m, n) where $m > 0$, $n > 0$. Determine the value of $\sin(\pi + \theta)$.
- A. $\frac{-n}{\sqrt{m^2 + n^2}}$ B. $\frac{-m}{\sqrt{m^2 + n^2}}$ C. $\frac{n}{\sqrt{m^2 + n^2}}$ D. $\frac{m}{\sqrt{m^2 + n^2}}$
74. A wheel of radius 30 cm has its centre 36 cm above the ground. It rotates once every 12 seconds. Determine an equation for the height, h , above the ground of a point on the wheel at time t seconds if this point has a minimum height at $t = 0$ seconds.
- A. $h = -30 \cos\left[\frac{\pi}{12}t\right] + 6$ B. $h = -30 \cos\left[\frac{\pi}{6}t\right] + 6$
 C. $h = -30 \cos\left[\frac{\pi}{12}t\right] + 36$ D. $h = -30 \cos\left[\frac{\pi}{6}t\right] + 36$

JUN 2002

75. Convert 210° to radians.
- A. 1.83 B. 2.69 C. 3.49 D. 3.67
76. Determine the exact value of $\sec \frac{7\pi}{4}$.
- A. $-\sqrt{2}$ B. $-\frac{1}{\sqrt{2}}$ C. $\frac{1}{\sqrt{2}}$ D. $\sqrt{2}$
77. Determine the period of the function $y = 3 \cos 4x$.
- A. $\frac{\pi}{2}$ B. $\frac{2\pi}{3}$ C. 6π D. 8π
78. Determine the range of the function $y = -2 \sin 3x + 4$.
- A. $-6 \leq y \leq -2$ B. $-2 \leq y \leq 2$ C. $0 \leq y \leq 4$ D. $2 \leq y \leq 6$

79. Solve: $2 \cos x + \sqrt{3} = 0$, $0 \leq x < 2\pi$

A. $\frac{5\pi}{6}, \frac{7\pi}{6}$

B. $\frac{4\pi}{3}, \frac{5\pi}{3}$

C. $\frac{2\pi}{3}, \frac{4\pi}{3}$

D. $\frac{7\pi}{6}, \frac{11\pi}{6}$

80. The function $h(t) = 3.9 \sin[0.16\pi(t - 3)] + 6.5$ gives the depth of water, h metres, at any time, t hours, during a certain day. A cruise ship needs at least 8 metres of water to dock safely. Use the graph of the function to estimate the number of hours in the 24 hour interval starting at $t = 0$ during which the cruise ship can dock safely. 

A. 3.79

B. 4.68

C. 7.57

D. 9.36

AUG 2002

81. Determine the amplitude of $y = -5 \sin \pi(x - 3) + 4$.

A. -5

B. 3

C. 4

D. 5

82. Convert 135° to radians.

A. 1.18

B. 1.92

C. 2.36

D. 4.71

83. Determine the period of $y = \tan 4x$.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 2π

D. 4π

84. Determine the exact value of $\sec \frac{11\pi}{6}$.

A. -2

B. 2

C. $-\frac{2}{\sqrt{3}}$

D. $\frac{2}{\sqrt{3}}$

85. Solve: $\sqrt{2} \sin x + 1 = 0$, $0 \leq x < 2\pi$.

A. $\frac{\pi}{4}, \frac{3\pi}{4}$

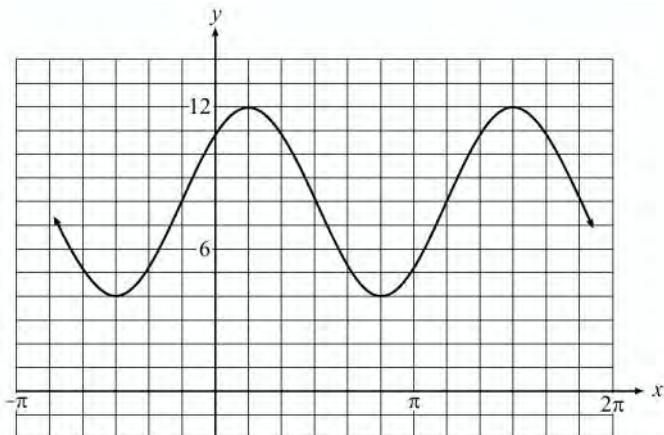
B. $\frac{\pi}{4}, \frac{7\pi}{4}$

C. $\frac{3\pi}{4}, \frac{5\pi}{4}$

D. $\frac{5\pi}{4}, \frac{7\pi}{4}$

86. Which equation represents the sine function shown?

- A. $y = 4 \sin\left[\frac{4}{3}\left(x + \frac{\pi}{6}\right)\right] + 8$
- B. $y = 4 \sin\left[\frac{4}{3}\left(x - \frac{\pi}{6}\right)\right] + 8$
- C. $y = 4 \sin\left[\frac{3}{2}\left(x - \frac{\pi}{6}\right)\right] + 8$
- D. $y = 4 \sin\left[\frac{3}{2}\left(x + \frac{\pi}{6}\right)\right] + 8$



87. A wheel rolling along the ground has a radius of 32 cm and rotates once every 8 seconds. At time $t = 0$ seconds, a point P on the outside edge of the wheel is touching the ground. Determine a cosine function that gives the height, h , of point P above the ground at any time, t , where h is in cm and t is in seconds.

- A. $h(t) = -32 \cos\left[\frac{\pi}{4}t\right]$
- B. $h(t) = -32 \cos[2\pi t]$
- C. $h(t) = -32 \cos\left[\frac{\pi}{4}t\right] + 32$
- D. $h(t) = -32 \cos[2\pi t + 32]$

JAN 2003

88. Determine the range of the function $y = 4 \cos x - 2$.

- A. $-4 \leq y \leq 4$
- B. $-2 \leq y \leq 6$
- C. $-6 \leq y \leq 2$
- D. $2 \leq y \leq 6$

89. Determine the exact value of $\cot \frac{5\pi}{3}$.

- A. $\sqrt{3}$
- B. $-\sqrt{3}$
- C. $\frac{1}{\sqrt{3}}$
- D. $-\frac{1}{\sqrt{3}}$

90. Determine the period of the function $f(x) = -\frac{1}{2} \sin \frac{x}{3}$.

- A. $\frac{2\pi}{3}$
- B. π
- C. 4π
- D. 6π

91. Solve: $2 \sin x + 1 = 0$, $0 \leq x < 2\pi$

- A. $-\frac{\pi}{6}, -\frac{5\pi}{6}$
- B. $\frac{\pi}{6}, \frac{5\pi}{6}$
- C. $\frac{7\pi}{6}, \frac{11\pi}{6}$
- D. $\frac{4\pi}{3}, \frac{5\pi}{3}$

JUN 2003

92. Convert 120° to radians.

A. $\frac{2\pi}{3}$

B. $\frac{5\pi}{6}$

C. $\frac{3\pi}{2}$

D. $\frac{6\pi}{5}$

93. Determine the amplitude of $y = -2 \sin 4\left(x - \frac{\pi}{3}\right) + 3$.

A. -2

B. 2

C. 3

D. 4

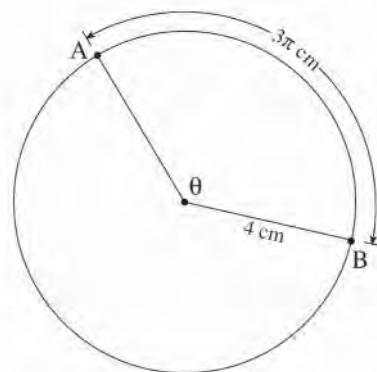
94. A circle has a radius of 4 cm. If the length of arc AB shown on the diagram is 3π cm, determine the measure of the central angle θ in radians.

A. $\frac{3\pi}{4}$

B. $\frac{4}{3\pi}$

C. $\frac{3\pi}{2}$

D. 3π



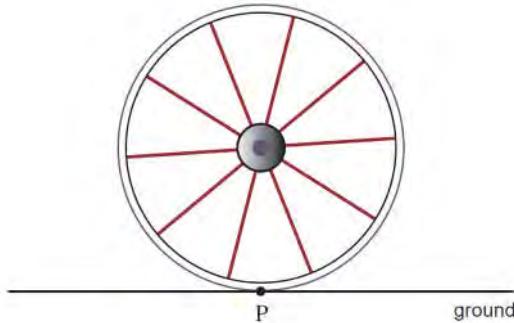
95. A wheel with diameter 10 cm is rolling along the ground. Point P on the edge of the wheel is on the ground as shown in the diagram at the time $t = 0$ seconds. Which equation gives the height, h , of point P above the ground at time t seconds, if the wheel rotates once every 12 seconds?

A. $h = -5 \cos \frac{\pi}{12}t$

B. $h = -5 \cos \frac{\pi}{6}t$

C. $h = -5 \cos \frac{\pi}{12}t + 5$

D. $h = -5 \cos \frac{\pi}{6}t + 5$



96. The point (p, q) is the point of intersection of the terminal arm of angle θ in standard position and the unit circle centred at $(0, 0)$. Which expression represents $\sec \theta$?

A. q

B. p

C. $\frac{1}{q}$

D. $\frac{1}{p}$

97. Determine the equations of the asymptotes of the function $y = \tan bx$, where $b > 0$.

- A. $x = \frac{n\pi}{b}$, n is an integer
 C. $x = \frac{\pi}{b} + \frac{n\pi}{b}$, n is an integer

- B. $x = \frac{n\pi}{2b}$, n is an integer
 D. $x = \frac{\pi}{2b} + \frac{n\pi}{b}$, n is an integer

JAN 2004

98. Convert $\frac{8\pi}{3}$ radians to degrees.

A. 60°

B. 120°

C. 240°

D. 480°

99. Determine the minimum value of the function $y = -3 \sin 2x + 4$.

A. -7

B. -3

C. -1

D. 1

100. Determine the exact value of $\sec \frac{5\pi}{4}$.

A. $-\sqrt{2}$

B. $-\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\sqrt{2}$

101. The terminal arm of angle θ , in standard position, passes through the point $(-2, 9)$. Determine the value of $\sin \theta$.

A. $\frac{-2}{\sqrt{77}}$

B. $\frac{9}{77}$

C. $\frac{-2}{\sqrt{85}}$

D. $\frac{9}{\sqrt{85}}$

102. In a seaport, the function $d(t) = 2.6 \sin 0.25(t - 5) + 3.3$ can be used to approximate the depth of the water, d metres, at time t hours after midnight. Estimate the number of hours in the 24-hour interval at $t = 0$ when the depth of the water is at least 3.5 metres.

A. 5.31 h

B. 11.95 h

C. 17.26 h

D. 23.90 h

JUN 2004

103. Determine the exact value of $\cos \frac{7\pi}{4}$.

A. $\frac{1}{\sqrt{2}}$

B. $\frac{\sqrt{3}}{2}$

C. $-\frac{\sqrt{3}}{2}$

D. $-\frac{1}{\sqrt{2}}$

104. In a circle with radius 12 cm an arc of length 20 cm subtends a central angle of θ . Determine the measure of θ in radians.

A. 0.60 B. 1.20 C. 1.67 D. 3.33

105. Solve: $7 \tan x = -3$, $0 \leq x < 2\pi$

A. 2.74, 3.55 B. 2.74, 5.88 C. 0.40, 3.55 D. 0.40, 5.88

106. Give the period of $f(x) = 3 \csc x$.

A. $\frac{\pi}{3}$ B. π C. 2π D. 3π

107. Let θ be an angle in standard position such that $\tan \theta = \frac{2}{3}$ and $\sin \theta < 0$. Determine the exact value of $\sec \theta$.

A. $-\frac{\sqrt{13}}{2}$ B. $-\frac{\sqrt{13}}{3}$ C. $\frac{\sqrt{13}}{3}$ D. $\frac{\sqrt{13}}{2}$

108. For the function $f(x) = 3 \sin bx + d$ where b and d are positive constants, determine an expression for the smallest positive value of x that produces the maximum value of $f(x)$.

A. $\frac{2\pi}{b}$ B. $\frac{\pi}{2b}$ C. $\frac{4\pi}{b}$ D. $\frac{\pi}{4b}$

AUG 2005

109. Determine the amplitude of $y = -2 \cos x - 3$.

A. -3 B. -2 C. 2 D. 3

110. Evaluate: $\sec \frac{4\pi}{3}$

A. -2 B. $-\frac{2}{\sqrt{3}}$ C. $\frac{2}{\sqrt{3}}$ D. 2

111. If $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$ and $\tan \theta = -\frac{4}{3}$, determine the exact value of $\sin \theta$

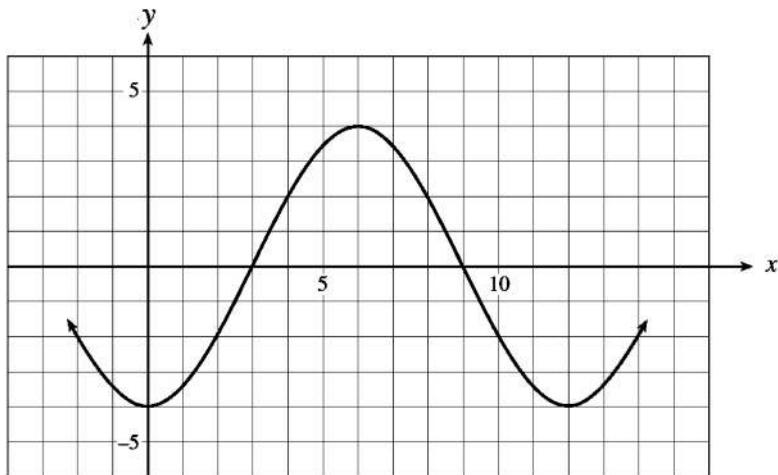
A. $-\frac{4}{5}$

B. $-\frac{3}{5}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

112. Given the graph below, determine an equation of this function.



A. $y = -4 \cos \frac{\pi}{6}x$

B. $y = 4 \cos \frac{\pi}{6}x$

C. $y = -4 \cos \frac{\pi}{12}x$

D. $y = 4 \cos \frac{\pi}{12}x$

113. Determine the reference angle for $\frac{7\pi}{5}$.

A. $\frac{\pi}{5}$

B. $\frac{2\pi}{5}$

C. $\frac{3\pi}{5}$

D. $\frac{4\pi}{5}$

AUG 2006

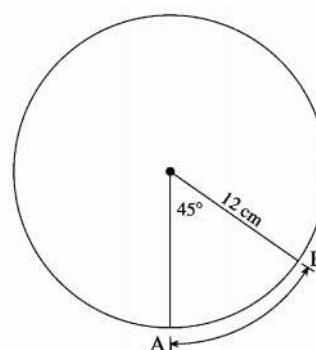
114. A circle has a radius of 12 cm. If the central angle is 45° , as shown in the diagram, determine the length of arc AB.

A. 2π cm

B. 3π cm

C. 4π cm

D. 6π cm



115. Determine the amplitude of the function $y = -4 \cos(x - 2)$.

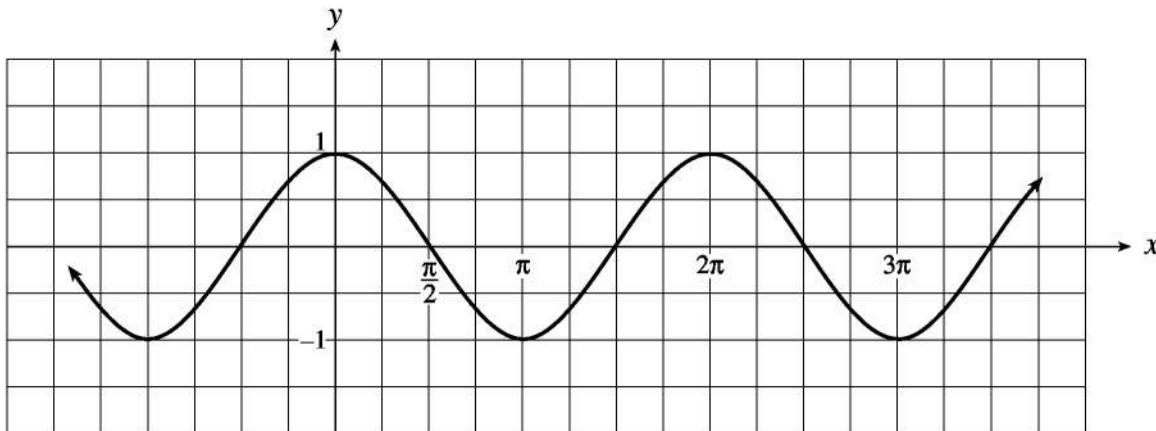
A. -4

B. -2

C. 2

D. 4

116. Which equation represents the function graphed below?



A. $y = \cos\left(x + \frac{\pi}{2}\right)$

B. $y = \sin\left(x - \frac{\pi}{2}\right)$

C. $y = -\cos\left(x - \frac{\pi}{2}\right)$

D. $y = -\sin\left(x - \frac{\pi}{2}\right)$

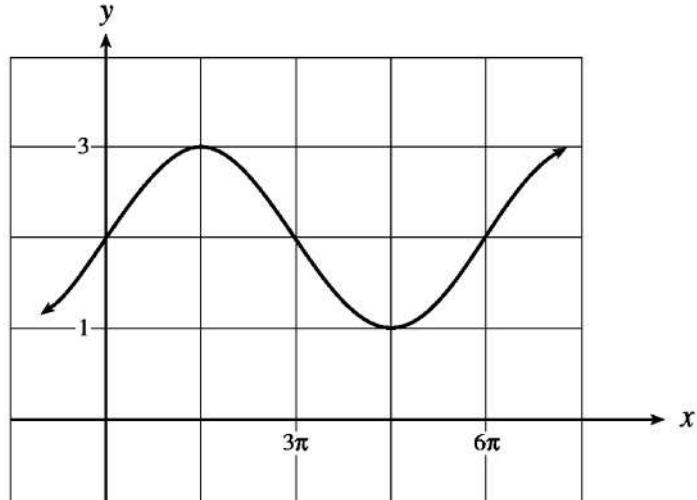
117. If the graph of the function shown below has the equation $y = a \sin bx + d$, determine the value of b . ($b > 0$)

A. $\frac{1}{3}$

B. 3

C. 2π

D. 6π



118. Let θ be an angle in standard position such that $\cot \theta = -\frac{4}{3}$ and $\sin \theta < 0$. Determine the exact value of $\sec \theta$.

A. $-\frac{5}{3}$

B. $-\frac{5}{4}$

C. $\frac{5}{4}$

D. $\frac{5}{3}$

119. A wheel rolling along the ground has a diameter of 16 cm and rotates every 12 seconds. At time $t = 0$ seconds, a point P on the outside edge of the wheel is at its highest point. Determine a cosine function that gives the height, h , of point P above the ground at any time t , where h is in cm and t is in seconds.

A. $h(t) = -8 \cos \frac{\pi}{6}t + 8$

B. $h(t) = 8 \cos \frac{\pi}{12}t + 8$

C. $h(t) = 8 \cos \frac{\pi}{6}t + 8$

D. $h(t) = -8 \cos \frac{\pi}{12}t + 8$

SAMPLE 2008

120. Evaluate: $\tan \frac{5\pi}{3}$

A. $-\frac{1}{\sqrt{3}}$

B. $\frac{1}{\sqrt{3}}$

C. $-\sqrt{3}$

D. $\sqrt{3}$

121. Determine the period of the function $y = 3 \cos \frac{\pi}{4}x$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 4

D. 8

122. The terminal arm of angle θ in standard position intersects the unit circle at (m, n) . Which expression represents $\cot \theta$?

A. m

B. n

C. $\frac{n}{m}$

D. $\frac{m}{n}$

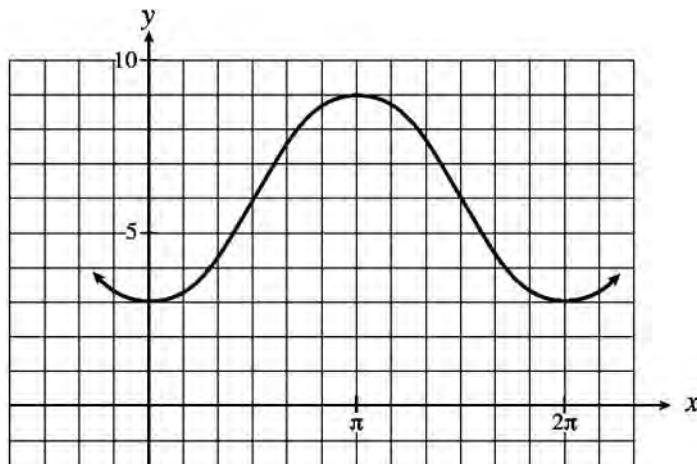
123. If the graph of the function shown below has the equation $y = a \cos b(x - c) + d$, determine the value of d .

A. 3

B. 5

C. 6

D. 9



124. Determine an equation of an asymptote of $y = \sec 3x$.

A. $x = \frac{\pi}{6}$

B. $x = \frac{\pi}{3}$

C. $x = \frac{2\pi}{3}$

D. $x = \pi$

125. The height above the ground, h metres, of a person on a Ferris Wheel at time t seconds, is given by formula $h(t) = -20 \cos \frac{2\pi}{40}t + 23$, where $t \geq 0$. Determine the earliest time, in seconds, at which the person will be 15 m above the ground.

A. 7.38

B. 12.62

C. 32.62

D. 37.14

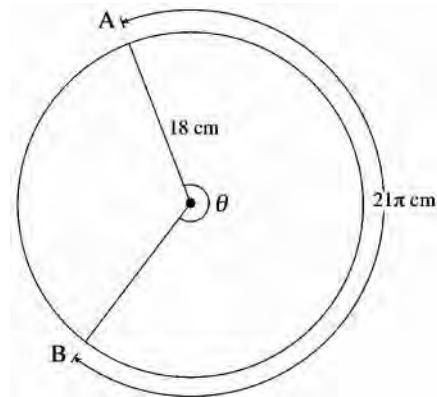
126. A circle has a radius of 18 cm. If the length of arc AB is 21π cm, determine the measure of the central angle θ in degrees.

A. 120°

B. 150°

C. 210°

D. 240°



127. A minimum value of a sinusoidal function is at $\left(\frac{\pi}{4}, 3\right)$. The nearest maximum value to the right of this point is at $\left(\frac{7\pi}{12}, 7\right)$. Determine an equation of this function.

128. Determine the amplitude of the function $y = -4 \cos(x - 2)$

A. -4

B. -2

C. 2

D. 4

129. The terminal arm of angle θ in standard position intersects the unit circle at the point (m, n) . Which expression represents $\tan \theta$?

A. $\frac{n}{m}$

B. $\frac{m}{n}$

C. $\frac{1}{m}$

D. $\frac{1}{n}$

130. Determine the exact value of $\csc \frac{7\pi}{4}$.

A. $-\sqrt{2}$

B. $\sqrt{2}$

C. $-\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{2}}$

131. In a circle, an arc of length 30 cm contains a central angle of 120° . Determine the radius of this circle.

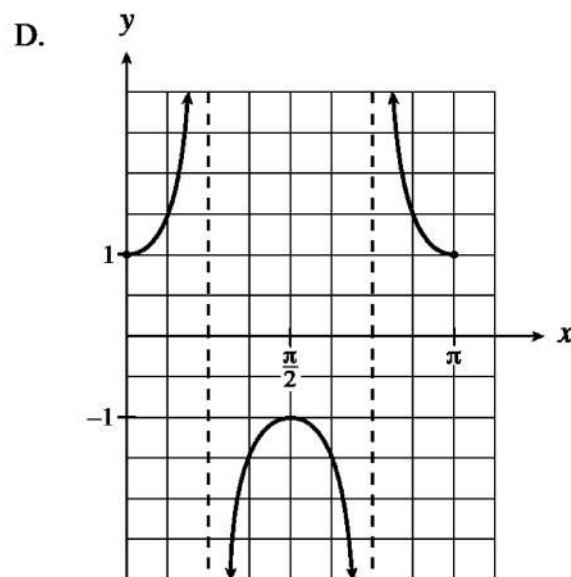
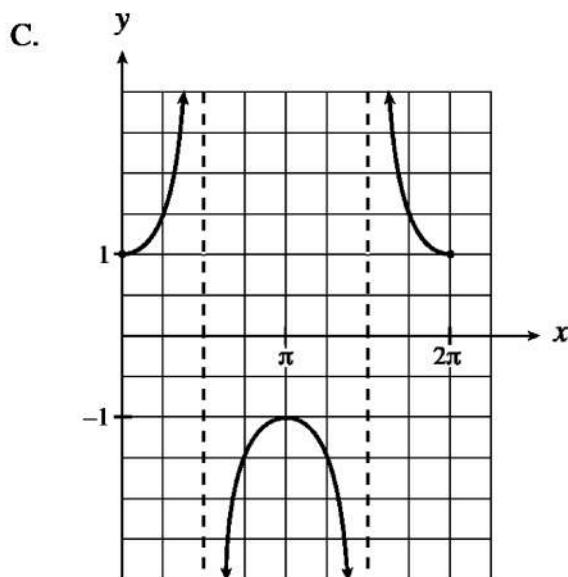
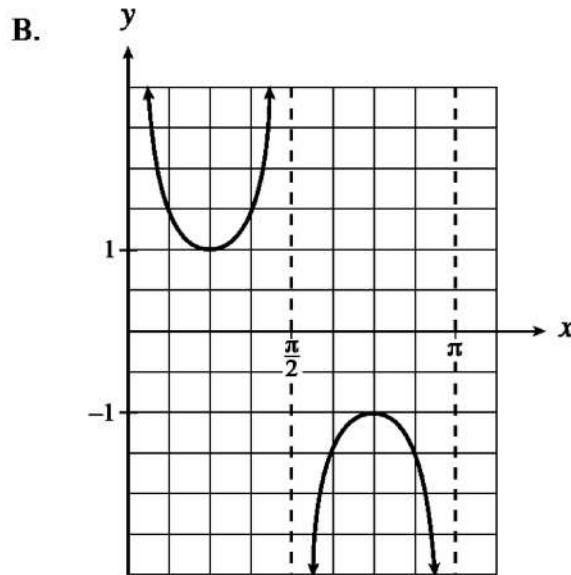
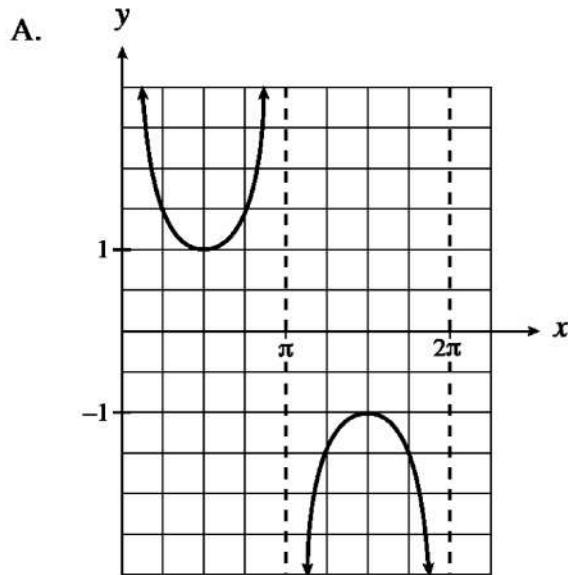
A. $\frac{36}{\pi}$ cm

B. $\frac{45}{\pi}$ cm

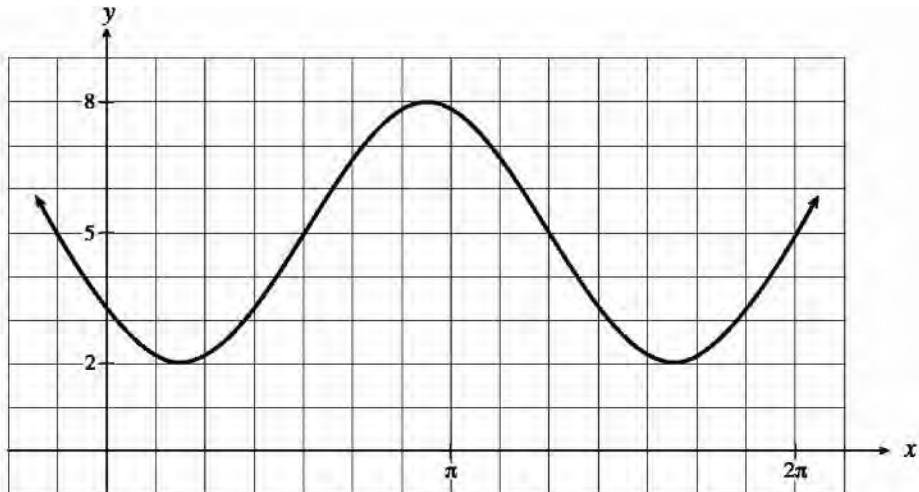
C. 20π cm

D. 45π cm

132. Which of the following is the graph of one period of the function $y = \sec 2x$?



133. Given the graph of the function $y = a \sin[b(x - c)] + d$ below, determine the value of b .



- A. $\frac{10\pi}{7}$ B. $\frac{7}{5}$ C. $\frac{5\pi}{7}$ D. $\frac{14}{5}$
134. The point $(5, -6)$ is on the terminal arm of standard position angle θ . Determine the smallest positive measure of θ in radians.
- A. 0.69 B. 0.88 C. 5.41 D. 5.59
135. At a seaport, the water has a maximum depth of 16 m at midnight. After the maximum depth, the first minimum depth of 4 m occurs 5.8 h later. Assume that the relation between the depth in metres and the time in hours is a sinusoidal function. How many hours after midnight will the water reach a depth of 8 m for the first time?
- A. 1.76 B. 2.27 C. 3.53 D. 3.67

ADDITIONAL QUESTIONS

136. What is the reference angle of $\frac{16\pi}{3}$ radians?
- A. $\frac{\pi}{6}$ B. $\frac{\pi}{3}$ C. $\frac{2\pi}{3}$ D. $\frac{4\pi}{3}$
137. Determine the smallest positive angle θ , in radians, that $\csc \theta = -\sqrt{2}$.
- A. $\frac{\pi}{4}$ B. $\frac{3\pi}{4}$ C. $\frac{5\pi}{4}$ D. $\frac{7\pi}{4}$

138. A circle has a radius of 12 cm. Determine the area of a sector (pie-shaped region) of the circle which has a central angle of 2.1 radians. (accurate to 1 decimal place)

- A. 25.2 cm^2 B. 151.2 cm^2 C. 215.4 cm^2 D. 302.4 cm^2

139. The smallest positive zero of the function $y = \cos k\left(x + \frac{\pi}{8}\right)$ occurs at $x = \frac{\pi}{2}$. Find the value of k if $k > 0$.

- A. $\frac{\pi}{2}$ B. $\frac{3\pi}{8}$ C. $\frac{4}{5}$ D. $\frac{5}{4}$

140. A cosine curve has a maximum point at $(3, 20)$ and the nearest minimum point to the right of this point is $(8, 4)$. Which of the following is an equation for this curve?

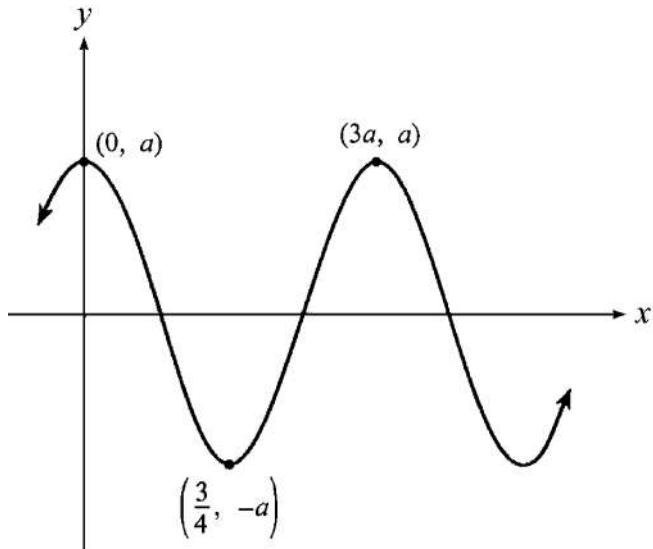
- A. $y = 8 \cos\left[\frac{2\pi}{5}(x+3)\right] + 12$ B. $y = 8 \cos\left[\frac{2\pi}{5}(x-3)\right] + 12$
 C. $y = 8 \cos\left[\frac{\pi}{5}(x+3)\right] + 12$ D. $y = 8 \cos\left[\frac{\pi}{5}(x-3)\right] + 12$

141. If the point $(1, 2)$ lies on the terminal arm of an angle θ in standard position, determine the value of $\cos(\pi + \theta)$.

- A. $\frac{-2}{\sqrt{5}}$ B. $\frac{-1}{\sqrt{5}}$ C. $\frac{1}{\sqrt{5}}$ D. $\frac{2}{\sqrt{5}}$

142. Given the graph of $y = a \cos kx$ as shown, determine a .

- A. $\frac{1}{2}$
 B. 1
 C. $\frac{3}{2}$
 D. 3



143. If $\sum_{k=1}^{\infty} (\sin x)^{k-1} = 6$, determine x to the nearest degree.

A. 36°

B. 46°

C. 56°

D. 66°

SAMPLE 2009

144. Convert $\frac{5\pi}{2}$ radians to degrees.

A. 90°

B. 180°

C. 270°

D. 450°

145. A circle has a radius of 20 cm. Determine the length of the arc subtended by a central angle of 135° .

A. $\frac{3\pi}{4}$ cm

B. 5π cm

C. 15π cm

D. $\frac{80}{3\pi}$ cm

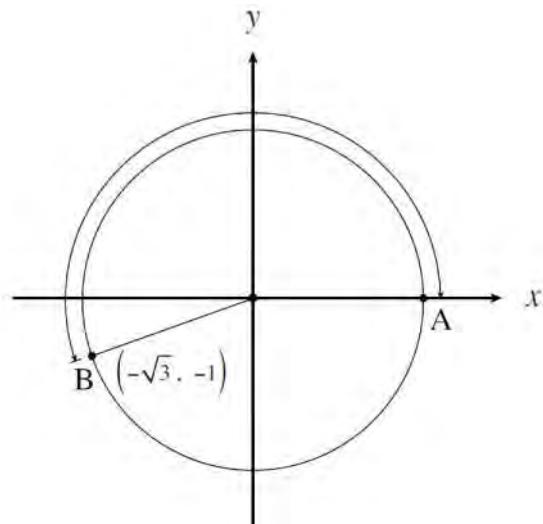
146. The terminal arm of angle θ in standard position passes through the point $(-\sqrt{3}, -1)$. Determine the length of arc AB, as shown below.

A. $\frac{5\pi}{6}$

B. $\frac{7\pi}{6}$

C. $\frac{7\pi}{3}$

D. $\frac{8\pi}{3}$



147. Evaluate: $\sec \frac{4\pi}{3}$

A. -2

B. $-\frac{2}{\sqrt{3}}$

C. $\frac{2}{\sqrt{3}}$

D. 2

148. Determine the exact value of $\tan \frac{8\pi}{3}$.

A. $-\sqrt{3}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

149. Determine the exact value of $\sin\left(-\frac{3\pi}{4}\right)$.

A. $-\sqrt{2}$

B. $-\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\sqrt{2}$

150. Solve: $\csc x = 2$, $0 \leq x < 2\pi$

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

B. $x = \frac{\pi}{6}, \frac{11\pi}{6}$

C. $x = \frac{\pi}{3}, \frac{2\pi}{3}$

D. $x = \frac{\pi}{3}, \frac{4\pi}{3}$

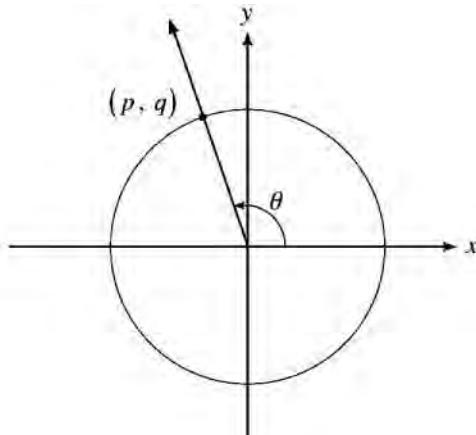
151. The point (p, q) is the point of intersection of the terminal arm of angle of angle θ in standard position and the unit circle as shown in the diagram. Which expression represents $\tan \theta$?

A. p

B. q

C. $\frac{p}{q}$

D. $\frac{q}{p}$



152. The terminal arm of angle θ in standard position passes through the point $(-2, 5)$. Determine the value of $\sec \theta$.

A. $-\frac{\sqrt{21}}{2}$

B. $\frac{\sqrt{21}}{5}$

C. $-\frac{\sqrt{29}}{2}$

D. $\frac{\sqrt{29}}{5}$

153. Point M $(-a, b)$ is in quadrant 2 and lies on the terminal arm of angle θ in standard position. Point N is the point of intersection of the terminal arm of angle θ and the unit circle centred at $(0, 0)$. Determine the x-coordinate of point N in terms of a and b .

A. $\frac{-a}{\sqrt{a^2+b^2}}$

B. $\frac{-b}{\sqrt{a^2+b^2}}$

C. $\frac{a}{\sqrt{a^2+b^2}}$

D. $\frac{b}{\sqrt{a^2+b^2}}$

154. Determine the amplitude of $y = -3 \cos 4x + 2$.

- A. -4 B. -3 C. 3 D. 4

155. Determine the period of $y = \sin\left[\frac{2\pi}{3}(x - 6)\right]$.

- A. 3 B. 6 C. $\frac{2\pi}{3}$ D. $\frac{4\pi}{3}$

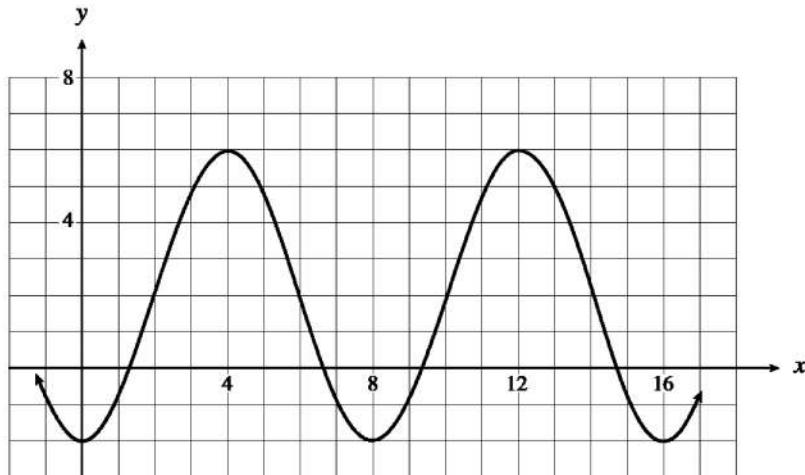
156. Determine the range of the function $y = 6 \cos\left[\frac{1}{2}(x - 3)\right] + 4$.

- A. $-6 \leq y \leq 6$ B. $1 \leq y \leq 7$ C. $-4 \leq y \leq 4$ D. $-2 \leq y \leq 10$

157. Which of the following lines is an asymptote for the graph of $y = \csc 2x$?

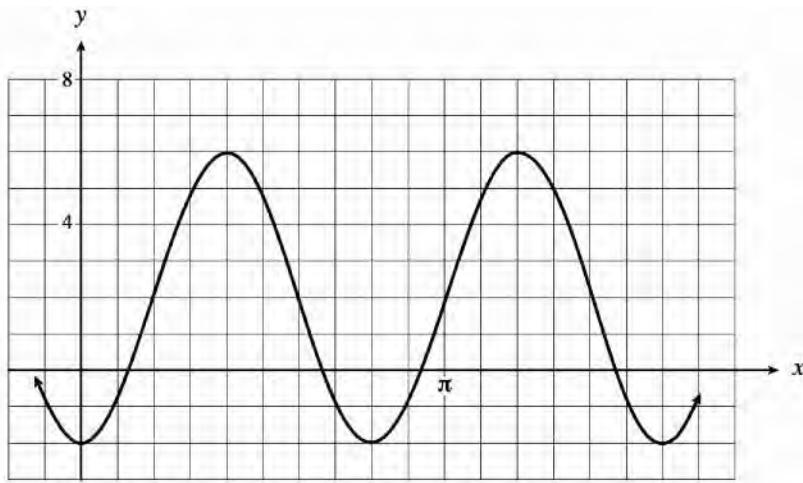
- A. $x = 1$ B. $x = \frac{\pi}{4}$ C. $x = \frac{\pi}{2}$ D. $x = \frac{3\pi}{4}$

158. If the graph of the function shown below has the equation $y = a \sin[b(x - c)] + d$, determine the value of b . ($b > 0$)



- A. 4 B. 8 C. $\frac{\pi}{4}$ D. $\frac{\pi}{8}$

159. If the graph of the function shown below has the equation $y = a \sin b(x - c) + d$, determine the value of b . ($b > 0$)



- A. $\frac{5}{4}$ B. $\frac{5}{2}$ C. $\frac{2\pi}{5}$ D. $\frac{4\pi}{5}$
160. State the phase shift of the function: $y = -\cos\left(4x - \frac{\pi}{2}\right)$
- A. $\frac{\pi}{8}$ to the right B. $\frac{\pi}{8}$ to the left C. $\frac{\pi}{2}$ to the right D. $\frac{\pi}{2}$ to the left
161. Determine the domain of: $f(x) = \tan 2x$
- A. $x \in R$ B. $x \in R, x \neq \frac{\pi}{4} + \frac{n\pi}{2}, n$ is an integer
 C. $x \in R, x \neq \frac{\pi}{2} + n\pi, n$ is an integer D. $x \in R, x \neq \pi + 2n\pi, n$ is an integer
162. At a seaport, the depth of the water, d , in metres, at time t hours, during a certain day is given by: $d = 3.4 \sin\left[2\pi \frac{(t - 7.00)}{10.6}\right] + 4.8$
 On that day, determine the depth of the water at 6:30 pm.
- A. 3.43 m B. 3.81 m C. 4.80 m D. 6.53 m

163. A wheel with radius 20 cm has its centre 30 cm above the ground. It rotates once every 15 seconds. Determine an equation for the height, h , above the ground of a point on the wheel at time, t seconds if this point has a maximum height at $t = 2$ seconds.

A. $h = 20 \cos\left[\frac{2\pi}{15}(t+2)\right] + 30$

B. $h = 20 \cos\left[\frac{2\pi}{15}(t-2)\right] + 30$

C. $h = 30 \cos\left[\frac{2\pi}{15}(t+2)\right] + 20$

D. $h = 30 \cos\left[\frac{2\pi}{15}(t-2)\right] + 20$

164. A Ferris wheel with a diameter of 60 m rotates once every 48 seconds. At time $t = 0$, a rider is at his lowest height which is 2 m above the ground.

a) Determine a sinusoidal equation that gives the height, h , of the rider above the ground as a function of the elapsed time, t , where h is in metres and t is in seconds.

b) Determine the time t when the rider will be 38 m above the ground for the first time after $t = 0$.

165. A mass is supported by a spring so that it rests 50 cm above a table top, as shown in the diagram below. The mass is pulled down to a height of 20 cm above the tabletop and released at time $t = 0$.

It takes 0.8 seconds for the mass to reach a maximum height of 80 cm above the tabletop. As the mass moves up and down, its height h , in cm above the tabletop, is approximated by a sinusoidal function of the elapsed time, t , in seconds, for a short period of time.

Determine an equation for a sinusoidal function that gives h as a function of t .

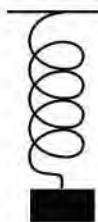


table top

TRIGONOMETRY I ANSWER KEY

1	C	51	D	101	D	150	A
2	A	52	B	102	B	151	D
3	D	53	B	103	A	152	C
4	D	54	B	104	C	153	A
5	A	55	D	105	B	154	C
6	B	56	A	106	C	155	A
7	C	57	C	107	B	156	D
8	B	58	A	108	B	157	C
9	C	59	B	109	C	158	C
10	B	60	B	110	A	159	B
11	B	61	B	111	D	160	A
12	C	62	A	112	A	161	B
13	C	63	D	113	B	162	D
14	A	64	C	114	B	163	B
15	D	65	B	115	D	164	a) $h = -30 \cos\left(\frac{\pi}{24}t\right) + 32$
16	C	66	B	116	D		b) 13.54 s
17	C	67	C	117	A		
18	C	68	A	118	C		
19	D	69	C	119	C	165	$h = -30 \cos\left(\frac{2\pi}{1.6}t\right) + 50$
20	B	70	D	120	C		
21	B	71	A	121	D		
22	C	72	C	122	D		
23	D	73	A	123	C		
24	C	74	D	124	A		
25	A	75	D	125	A		
26	C	76	D	126	C		
27	D	77	A	127	$h = -2 \cos 3\left(x - \frac{\pi}{4}\right) + 5$		
28	B	78	D				
29	D	79	A	128	D		
30	B	80	D	129	A		
31	B	81	D	130	A		
32	D	82	C	131	B		
33	A	83	A	132	D		
34	B	84	D	133	B		
35	C	85	D	134	C		
36	B	86	D	135	C		
37	C	87	C	136	B		
38	D	88	C	137	C		
39	A	89	D	138	B		
40	C	90	D	139	C		
41	B	91	C	140	D		
42	C	92	A	141	B		
43	B	93	B	142	A		
44	D	94	A	143	C		
45	B	95	D	144	D		
46	C	96	D	145	C		
47	A	97	D	146	C		
48	C	98	D	147	A		
49	B	99	D	148	A		
50	D	100	A	149	B		

TRIGONOMETRY I REVIEW

In all possible cases, exact values should be used. Radian measures which are not exact should be to three decimal places. Otherwise answers should be to 2 decimal places.

1. Convert to radians:

a) 105° _____ b) 400° _____ c) 40° _____ d) 10° _____

2. Convert to degrees:

a) $\frac{13\pi}{12}$ _____ b) 0.800 _____ c) -7.500 _____ d) $\frac{7\pi}{5}$ _____

3. Determine the reference angle for:

a) $\frac{7\pi}{5}$ _____ b) -1.000 _____

4. Find 2 coterminal angles for each of the above (in question 3)

a) _____ b) _____

5. A fan at high speed is turning at 220 rpm. Convert this speed to radians per second.

6. The earth has a radius of 6.38×10^6 m, and completes one full rotation about its axis every 24 hours.

a) Determine your angular speed.

b) How far will you travel in one minute?

7. Find the diameter of a pizza slice with central angle of 1.50 radians and area of 170 cm².

8. The terminal angle of an angle θ in standard position passes through the point $(3, -8)$. Determine:

$\sin\theta$	$\csc\theta$
$\cos\theta$	$\sec\theta$
$\tan\theta$	$\cot\theta$
$\theta (0 < \theta < 2\pi)$	

9. If $\csc \theta = -\frac{5}{4}$, and θ lies in quadrant III, determine $\cos \theta$.

10. Complete the table with exact trig ratios:

	$\frac{2\pi}{3}$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
$\sin \theta$				
$\cos \theta$				
$\tan \theta$				
$\csc \theta$				
$\sec \theta$				
$\cot \theta$				

11. Solve, $0 \leq \theta < 2\pi$:

a) $\sin \theta = -\frac{1}{\sqrt{2}}$ _____

b) $\cot \theta = \sqrt{3}$ _____

c) $\sec \theta = -2$ _____

d) $\sin \theta = 0.600$ _____

e) $\tan \theta = 2.5$ _____

f) $\cos \theta = 0.825$ _____

g) $\csc \theta = -\frac{2}{\sqrt{3}}$ _____

h) $\csc \theta = 0$ _____

i) $\cos \theta = -1$ _____

j) $\sec^2 \theta = 2$ _____

12. For each of the following functions, state the amplitude, period, phase shift and vertical displacement:

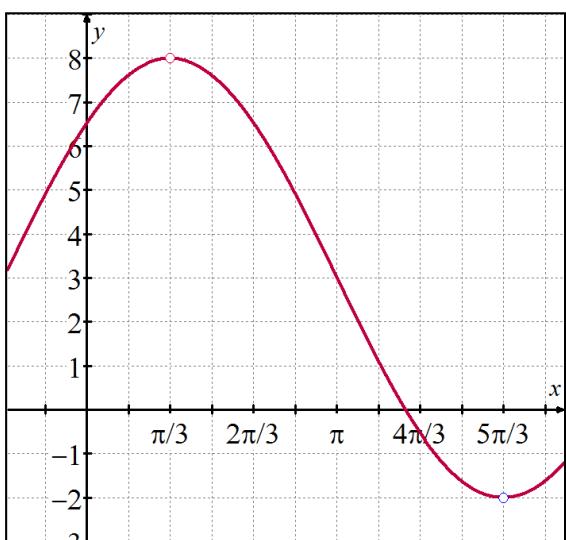
	Amplitude	Period	Phase shift	Vertical displacement
a) $y = -3 \sin\left(2x + \frac{\pi}{3}\right) - 5$				
b) $y = \sin\left(0.75\left(\theta - \frac{\pi}{4}\right)\right)$				
c) $y = -20 \cos\left(\frac{2\pi}{120}(t + 7)\right) + 10$				
d) $y = 2 \tan 2x$				

13. Determine the equation of the sinusoidal function which has a minimum at $(3, -5)$ and rises to the next nearest maximum at $(21, 7)$.

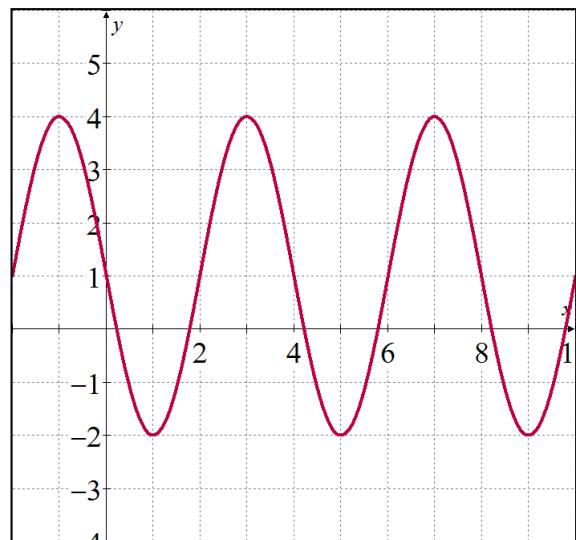
14. Determine the equation of the cosine function with the amplitude 2, period 5π , phase shift $\frac{\pi}{12}$ and vertical displacement -8 .

For the next two questions, determine two possible equations to represent each function shown: (3 marks each)

15.

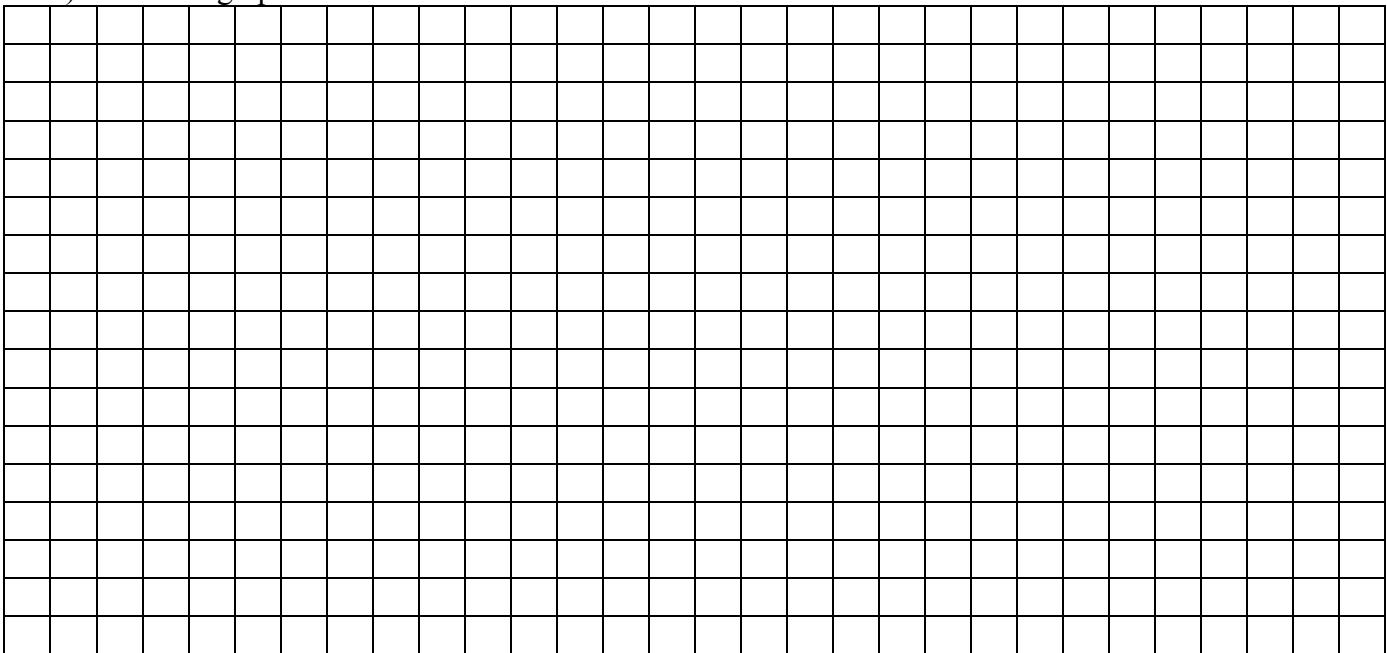


16.



17. The number of people skiing/snowboarding (P) at any time on a ski mountain can be modeled by a sinusoidal function. At 9:00 am there are 150 skiers on the hill, which is the minimum. The number of skiers peaks 7 hours later when there are 1050 skiers on the hill. The function is valid until 9:00 pm, when the hill shuts down for the night.

- a) Sketch a graph of this function.



- b) Determine an equation to represent this function, where t is the time in hours (midnight is zero hours, 9:00 p.m. is 21:00), and the hill is open from 9:00 am to 9:00 pm.
- c) What is the earliest time (after 9:00 am) at which there will be 600 people on the mountain?
- d) How many people are on the mountain at 12:00 noon?
- d) For what time period(s) are there more than 800 skiers on the hill? (intended to require graphing technology).

18. Using graph paper, sketch at least one complete period of the functions in question 12.

TRIGONOMETRY I REVIEW SOLUTIONS

1. Convert to radians:

$$\text{a) } 105^\circ = \frac{7\pi}{12} \quad \text{b) } 400^\circ = \frac{20\pi}{9} \quad \text{c) } 40^\circ = \frac{2\pi}{9} \quad \text{d) } 10^\circ = \frac{\pi}{18}$$

2. Convert to degrees:

$$\text{a) } \frac{13\pi}{12} = 195^\circ \quad \text{b) } 0.800 = 45.84^\circ \quad \text{c) } -7.500 = -429.72^\circ \quad \text{d) } \frac{7\pi}{5} = 252^\circ$$

3. Determine the reference angle for:

$$\text{a) } \frac{7\pi}{5}, \frac{2\pi}{5} \quad \text{b) } -1.000, 1.000$$

4. Find 2 coterminal angles for each of the above (in question 3)

$$\text{a) } \frac{-13\pi}{5}, \frac{-3\pi}{5}, \frac{17\pi}{5}, \frac{27\pi}{5} \quad \text{b) } -13.566, -7.283, 5.283, 11.566$$

5. A fan at high speed is turning at 220 rpm. Convert this speed to radians per second.

$$\frac{220 \text{ rev.}}{1 \text{ min}} \times \frac{2\pi \text{ radians}}{1 \text{ rev.}} \times \frac{1 \text{ min}}{60 \text{ s}} \approx 23.04 \text{ radians per second}$$

6. The earth has a radius of 6.38×10^6 m, and completes one full rotation about its axis every 24 hours.

$$\text{a) Determine your angular speed.} \quad \text{b) How far will you travel in one minute?}$$

$$\frac{2\pi r}{24h} = \frac{2\pi(6.38 \times 10^6)}{24} \frac{m}{h} \approx 1670280 \text{ metres per hour} \quad 27838 \text{ metres}$$

7. Find the diameter of a pizza from which a slice with central angle of 1.50 radians and area of 170 cm^2 is taken

$$A = \frac{r^2 \theta}{2} = \frac{r^2 (1.50)}{2} = 170$$

$$r^2 = \frac{170(2)}{1.50}, \quad r \approx 15.06 \text{ cm}, \quad d \approx 30.11 \text{ cm}$$

8.

$\sin \theta = -\frac{8}{\sqrt{73}}$	$\csc \theta = -\frac{\sqrt{73}}{8}$
$\cos \theta = \frac{3}{\sqrt{73}}$	$\sec \theta = \frac{\sqrt{73}}{3}$
$\tan \theta = -\frac{8}{3}$	$\cot \theta = -\frac{3}{8}$
$\theta (0 < \theta < 2\pi) \quad 5.071$	

9. If $\csc \theta = -\frac{5}{4}$, and θ lies in quadrant III, determine $\cos \theta$. $\sin \theta = -\frac{4}{5}, \therefore \cos \theta = -\frac{3}{5}$

10. Complete the table with exact trig ratios:

(12 marks)	$\frac{2\pi}{3}$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$
$\sin \theta$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	-1	$-\frac{\sqrt{3}}{2}$
$\cos \theta$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{2}$
$\tan \theta$	$-\sqrt{3}$	1	und	$-\sqrt{3}$
$\csc \theta$	$\frac{2}{\sqrt{3}}$	$-\sqrt{2}$	-1	$\frac{2}{\sqrt{3}}$
$\sec \theta$	-2	$-\sqrt{2}$	und	2
$\cot \theta$	$-\frac{1}{\sqrt{3}}$	1	0	$-\frac{1}{\sqrt{3}}$

11. a) $\sin \theta = -\frac{1}{\sqrt{2}}$ $\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$ b) $\cot \theta = \sqrt{3}$ $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$
 c) $\sec \theta = -2$ $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ d) $\sin \theta = 0.600$ $\theta = 0.644, 2.498$
 e) $\tan \theta = 2.5$ $\theta = 1.190, 4.332$ f) $\cos \theta = 0.825$ $\theta = 0.601, 5.683$
 g) $\csc \theta = -\frac{2}{\sqrt{3}}$ $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$ h) $\csc \theta = 0$ no solution
 i) $\cos \theta = -1$ $\theta = \pi$ j) $\sec^2 \theta = 2$ $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

12.

	Amplitude	Period	Phase shift	Vertical displacement
a) $y = -3 \sin\left(2x + \frac{\pi}{3}\right) - 5$	3	π	$-\frac{\pi}{6}$	-5
b) $y = \sin\left(0.75\left(\theta - \frac{\pi}{4}\right)\right)$	1	$\frac{8\pi}{3}$	$\frac{\pi}{4}$	none
c) $y = -20 \cos\left(\frac{2\pi}{120}(t + 7)\right) + 10$	20	120	-7	+10
d) $y = 2 \tan 2x$	----	$\frac{\pi}{2}$	----	none

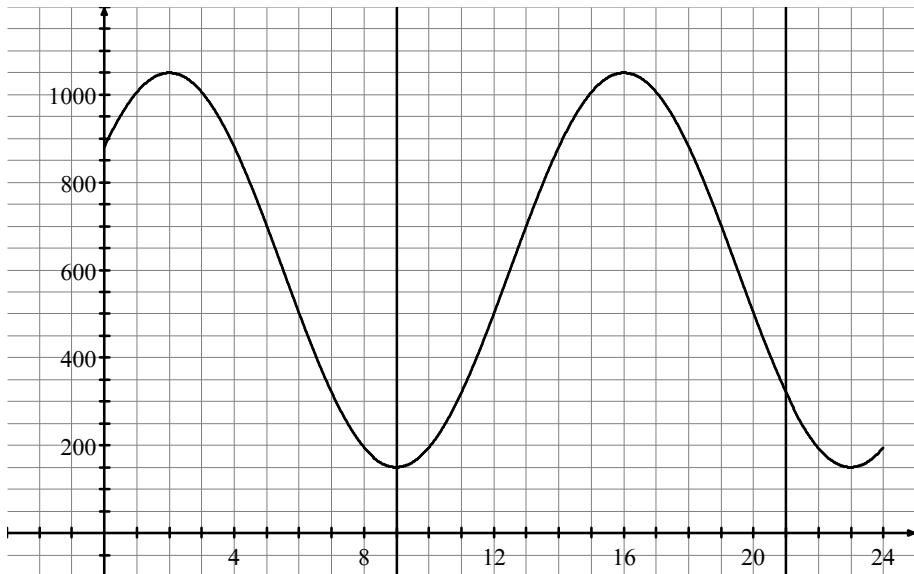
13. $y = -6 \cos\left(\frac{\pi}{18}(x-3)\right) + 1$

14. $y = 2 \cos\left(\frac{2}{5}\left(x-\frac{\pi}{12}\right)\right) - 8$

15. $y = 5 \cos\left(\frac{3}{4}\left(x-\frac{\pi}{3}\right)\right) + 3$ $y = -5 \sin\left(\frac{3}{4}(x-\pi)\right) + 3$

16. $y = 3 \cos\left(\frac{\pi}{2}(x+1)\right) + 1$; $y = 3 \sin\left(\frac{\pi}{2}(x-2)\right) + 1$

17.a)



b) $P(t) = -450 \cos\left[\frac{\pi(t-9)}{7}\right] + 600$

c) 12:30 pm

d) 500

e) 13.526 to 18.474, which is from 1:32 pm to 6:28 pm, a time of 4:57

18a)

