PRECALCULUS MATHEMATICS 12

TABLE OF CONTENTS

COMBINING FUNCTIONS
Review of basic functions;
Addition, subtraction, multiplication and division of functions; composition of functions.
- **48 PROBLEMS**

TRANSFORMATIONS
Transformations of graphs and equations (vertical and horizontal translations, stretches, and reflections) of parent functions and relations; inverses.
- **118 PROBLEMS**

POLYNOMIALS
Factoring (Factor Theorem and the Remainder Theorem), the Rational Zero Theorem and polynomial division;
Graphing and polynomial characteristics;
Solving equations and inequalities algebraically and graphically.
- **183 PROBLEMS**

RADICAL & RATIONAL FUNCTIONS
Characteristics and behaviour of graphs, including asymptotes (horizontal, vertical and oblique), intercepts and point discontinuities.
- **51 PROBLEMS**

LOGARITHMIC and EXPONENTIAL FUNCTIONS
Graphing and characteristics of exponential and logarithmic functions including transformations;
Applying laws of logarithms;
Solving equations with the same and with different bases, including base e;
Applications.
- **235 PROBLEMS**
- **LAWS OF LOGARITHMS WORKSHEET**
- **REVIEW PACKAGE**

TRIGONOMETRY I

Examining angles in standard position in both radians and degrees;
Reference and coterminal angles; arc length;
Reciprocal trigonometric functions;
Solving first-degree equations;
Graphing and characteristics of primary trigonometric functions, including transformations.

- **165 PROBLEMS**
- **REVIEW PACKAGE**

TRIGONOMETRY II (EQUATIONS and IDENTITIES)

Trigonometric identities (Pythagorean, quotient, double angle, reciprocal, and sum and difference identities) and two-column proofs;
Solving second-degree equations (over restricted domains and all real numbers).

- **135 PROBLEMS**
- **TRIGONOMETRIC IDENTITIES PACKAGE**
- **REVIEW OF SUM and DIFFERENCE & DOUBLE ANGLE IDENTITIES WORKSHEET**

CONIC SECTIONS

The parabola, circle, ellipse and hyperbola: equations and graphs;
General to Standard Form;
Transformations and applications.

- **194 PROBLEMS**

GEOMETRIC SEQUENCES and SERIES

Common ratio, first term, general term;
Infinite geometric series and summation notation.

- **121 PROBLEMS**

TRANSFORMATIONS

SPECS 2001

1. If the graph of $y = f(x)$ is translated 5 units to the left, determine the resulting equation.

   A. $y - 5 = f(x)$ B. $y + 5 = f(x)$ C. $y = f(x - 5)$ D. $y = f(x + 5)$

2. How is the graph $5y = \sqrt{x}$ related to the graph $y = \sqrt{x}$?

   A. $y = \sqrt{x}$ has been vertically translated 5 units up

   B. $y = \sqrt{x}$ has been expanded vertically by a factor of 5

   C. $y = \sqrt{x}$ has been compressed vertically by a factor of 5

   D. $y = \sqrt{x}$ has been compressed horizontally by a factor of 5

3. Simplify: $f^{-1}(f(x))$

   A. x B. $-x$ C. $\frac{1}{x}$ D. $-\frac{1}{x}$

4. Given the function $f(x) = (x - 1)^3 + 2$, determine $f^{-1}(x)$, the inverse function.

   A. $f^{-1}(x) = \sqrt[3]{x + 2} + 1$ B. $f^{-1}(x) = \sqrt[3]{x - 2} + 1$

   C. $f^{-1}(x) = \sqrt[3]{x + 2} - 1$ D. $f^{-1}(x) = \sqrt[3]{x - 2} - 1$

5. The function $y = f(x)$ is transformed to $y = f(2x + 4)$. Identify the horizontal expansion or compression factor, then the translation to the graph of the function.

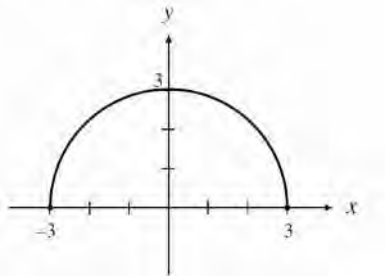
   A. horizontal expansion by a factor of 2, then a translation of 4 units left.

   B. horizontal compression by a factor of 2, then a translation of 4 units left.

   C. horizontal expansion by a factor of 2, then a translation of 2 units left.

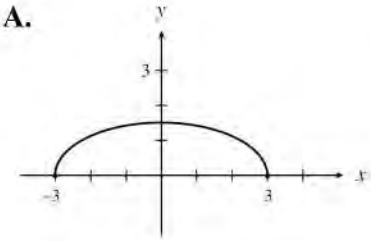
   D. horizontal compression by a factor of 2, then a translation of 2 units left.

6. The graph of $y = \sqrt{9 - x^2}$ is shown.

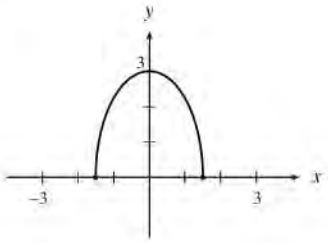


Which of the following graphs represents $2y = \sqrt{9 - x^2}$?

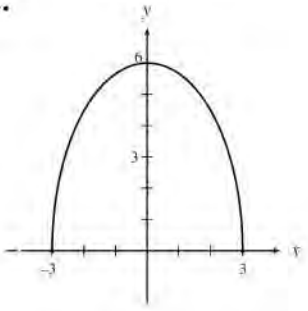
A.



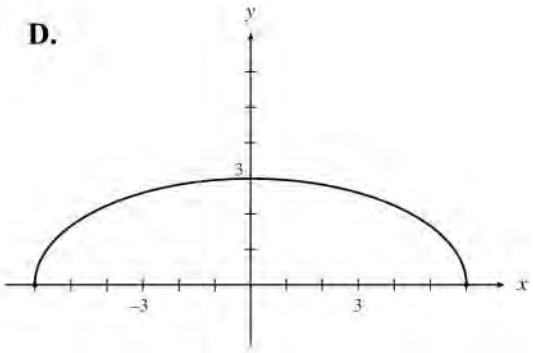
B.



C.



D.



SAMPLE 2001

7. If $(6, -5)$ is a point on the graph of $y = f(x)$, what must be a point on the graph of $y = -f(2(x + 2)) - 3$?

A. $(-1,\ 2)$ B. $(1,\ -2)$ C. $(1,\ 2)$ D. $(10,\ 2)$

8. Given the function $y_1 = f(x)$, describe how the graph of the new function, $y_2 = 4f(x - 2)$, is related to the graph of y_1.

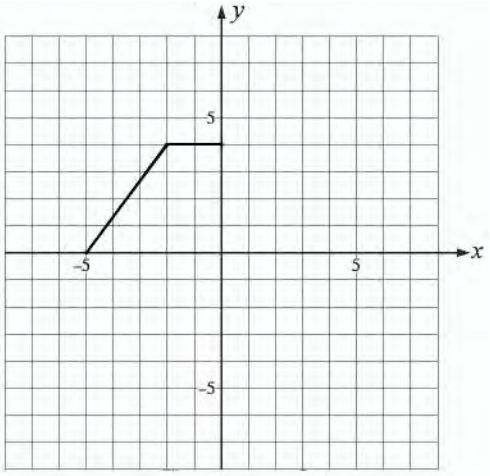
A. The graph of y_1 has been vertically compressed by a factor of 4 then translated 2 units right to form the graph of y_2.

B. The graph of y_1 has been vertically expanded by a factor of 4 then translated 2 units right to form the graph of y_2.

C. The graph of y_1 has been vertically compressed by a factor of 4 then translated 2 units left to form the graph of y_2.

D. The graph of y_1 has been vertically expanded by a factor of 4 then translated 2 units left to form the graph of y_2.

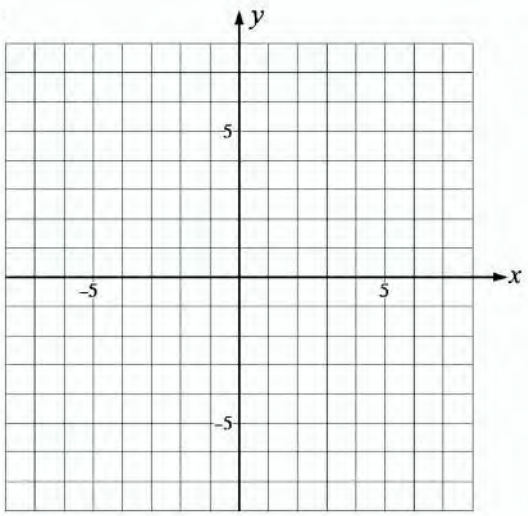
9. The graph of the function $y = f(x)$ is shown below.



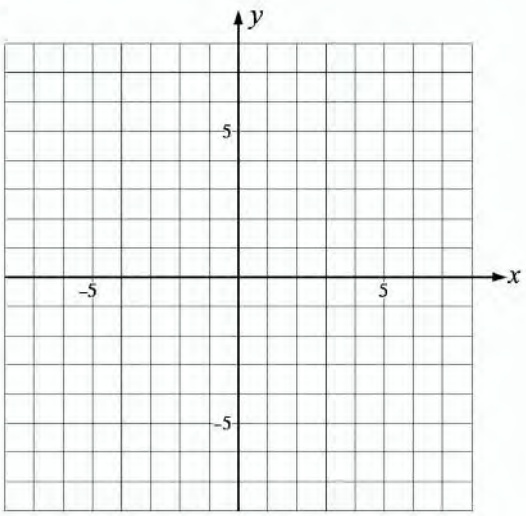
Sketch the graphs of:

A. $y = f(-x)$ B. $y = f(x - 3)$ C. $y = 2f(x)$ D. $x = f(y)$

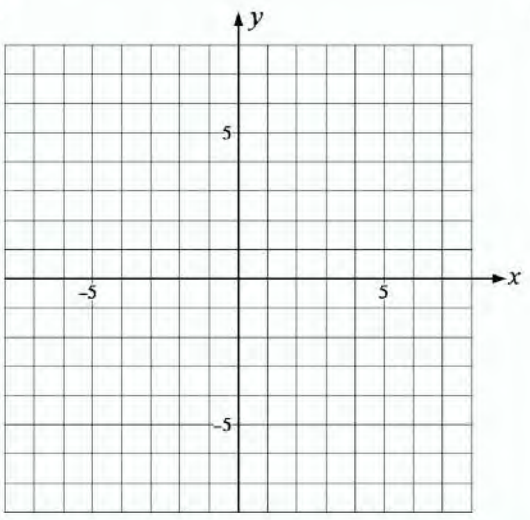
A.



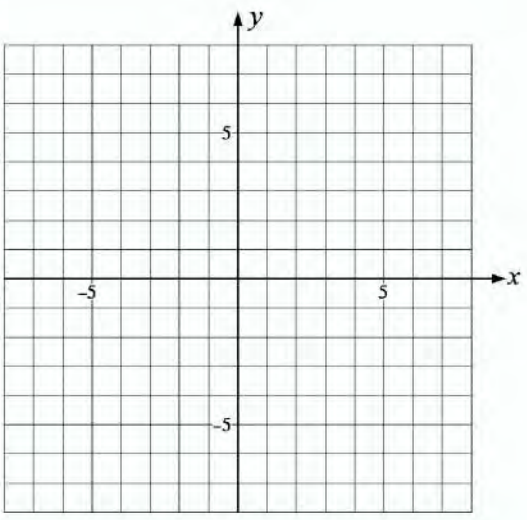
B.



C.



D.



JAN 2002

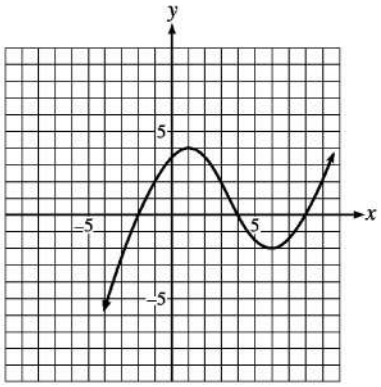
10. Which equation represents the graph of $y = \sqrt{x}$ after it is translated 4 units to the right?

A. $y = \sqrt{x} - 4$ B. $y = \sqrt{x - 4}$ C. $y = \sqrt{x + 4}$ D. $y = \sqrt{x} + 4$

11. If $y = 5x - 1$, determine the equation of $f^{-1}(x)$, the inverse of $f(x)$.

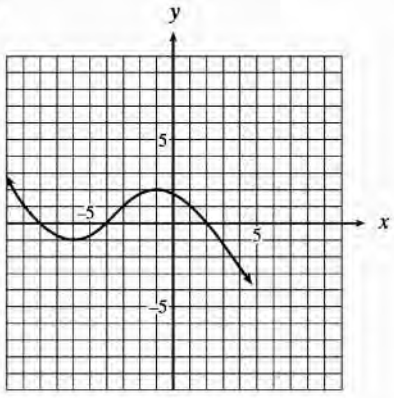
A. $f^{-1}(x) = \dfrac{1}{5x - 1}$ B. $f^{-1}(x) = \dfrac{1}{5}x - 1$ C. $f^{-1}(x) = \dfrac{x + 1}{5}$ D. $f^{-1}(x) = \dfrac{x - 1}{5}$

12. The graph of $y = f(x)$ is shown.

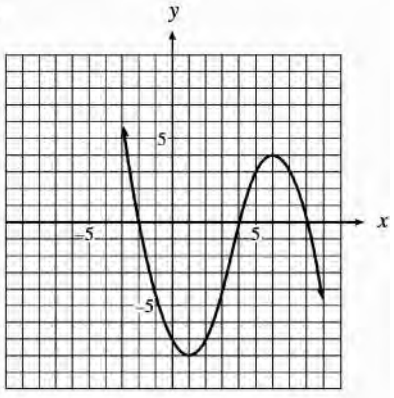


Which of the following graphs represents $y = -2f(x)$?

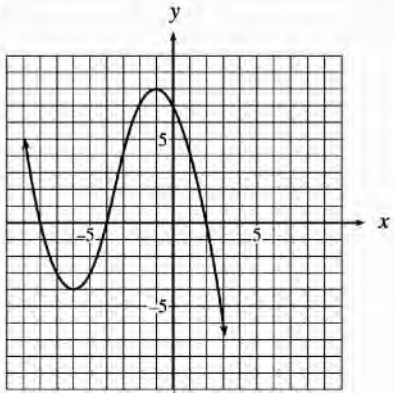
A.



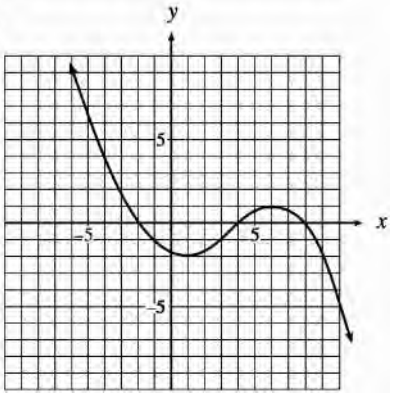
B.



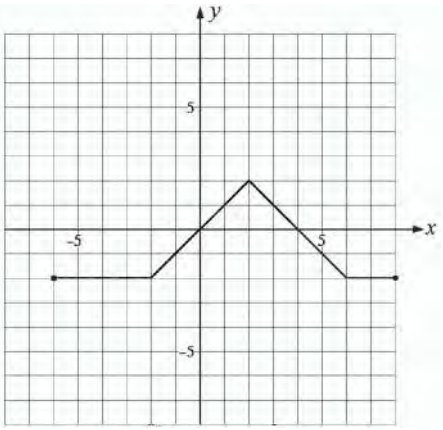
C.



D.



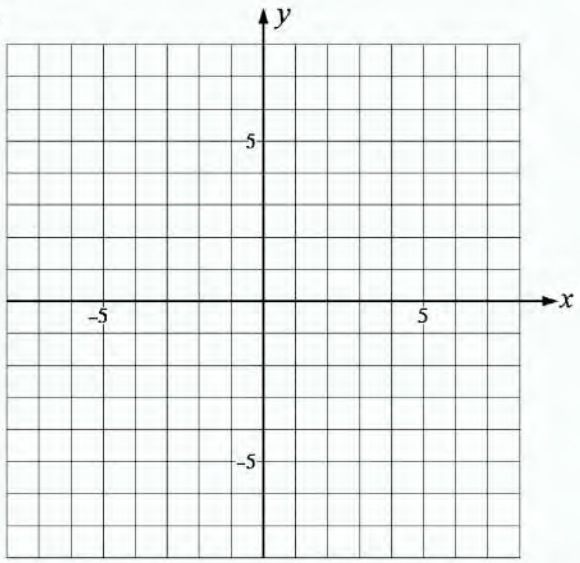
13. The graph of $y = f(x)$ is shown.



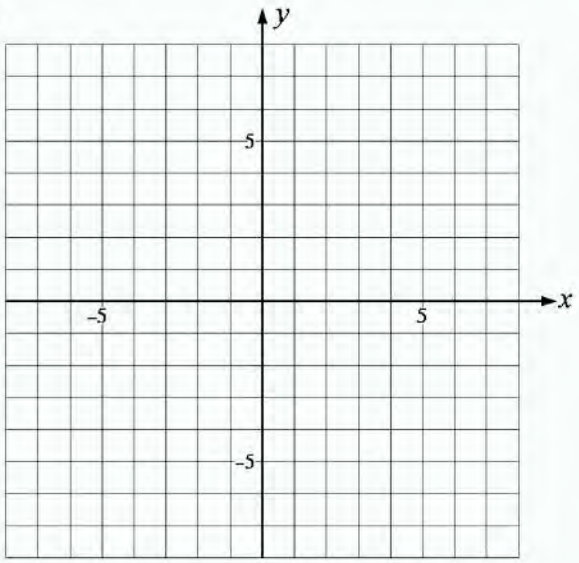
On the grids provided, sketch the graphs of:

A. $y = f(x+2) - 3$ B. $y = f(2x)$ C. $y = |f(2x)|$

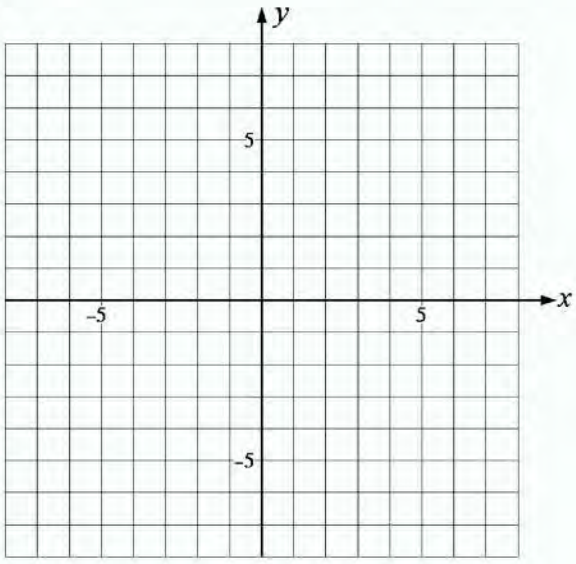
A.



B.



C.



14. Which equation represents the graph of $y = x^3 + x^2$ after it is reflected in the y-axis?

A. $y = -x^3 + x^2$ B. $y = -x^3 - x^2$ C. $y = \frac{1}{x^3 + x^2}$ D. $y = y^3 + y^2$

APR 2002

15. Given the function $y = f(x)$, which of the following represents its reflection in the y-axis?

A. $y = f(-x)$ B. $y = -f(x)$ C. $y = f(y)$ D. $y = \frac{1}{f(x)}$

16. How is the graph of $y = \frac{1}{7}f(x)$ related to the graph of $y = f(x)$?

A. $y = f(x)$ has been compressed vertically by a factor of 7
B. $y = f(x)$ has been compressed horizontally by a factor of 7
C. $y = f(x)$ has been expanded vertically by a factor of 7
D. $y = f(x)$ has been expanded horizontally by a factor of 7

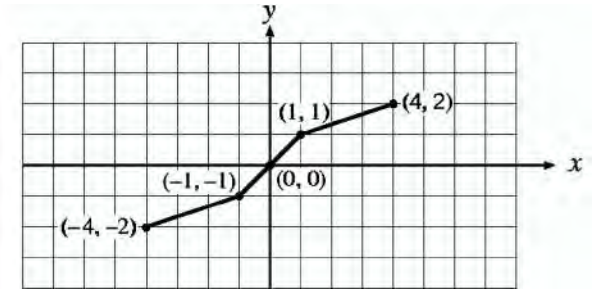
17. Given $f(x) = x^3 - 27$, determine $f^{-1}(x)$, the inverse of $f(x)$.

A. $f^{-1}(x) = \sqrt[3]{x + 27}$ B. $f^{-1}(x) = \sqrt[3]{x - 27}$ C. $f^{-1}(x) = \sqrt[3]{x} + 3$ D. $f^{-1}(x) = x^3 + 27$

18. If $(4, -3)$ is a point on the graph of $y = f(x)$, what must be a point on the graph of $y = f(2x + 10)$?

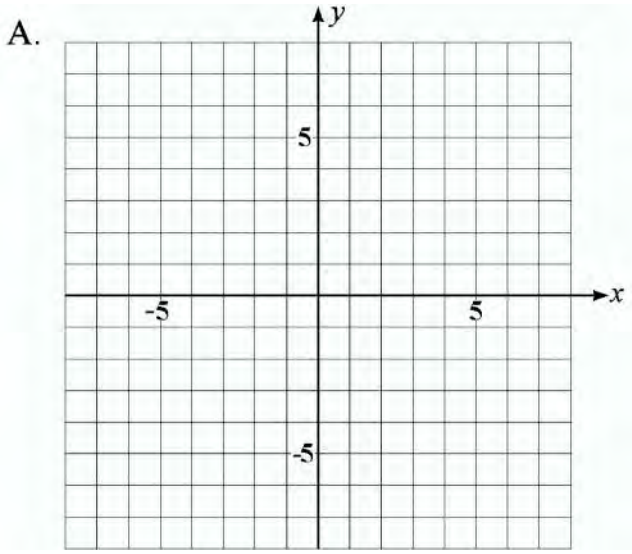
A. $(-8, -3)$ B. $(-3, -3)$ C. $(3, -3)$ D. $(18, -3)$

19. The graph of the function $y = f(x)$ is shown below.

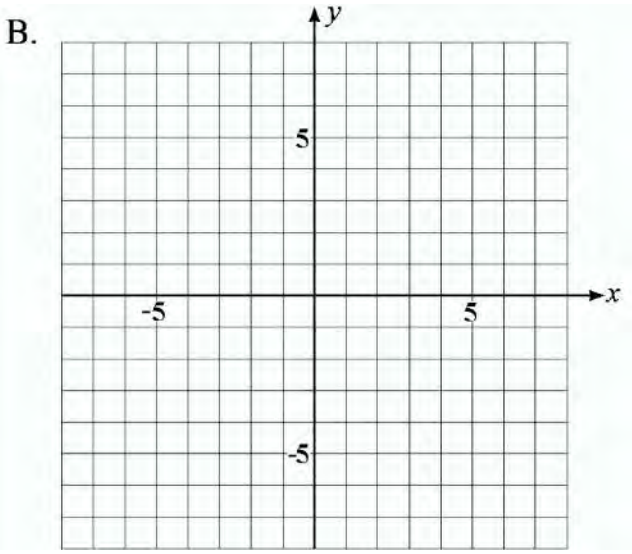


Sketch the graphs of: A. $y = 3f(x-2)$ B. $y = -f\left(\frac{x}{2}\right)$

A.

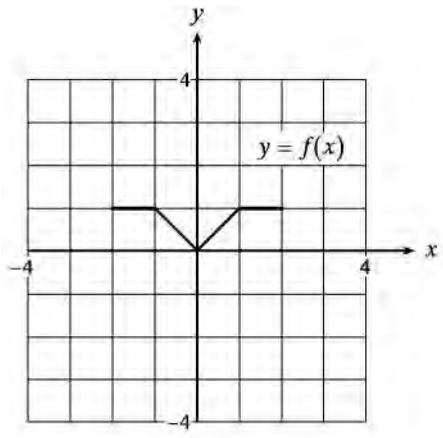


B.

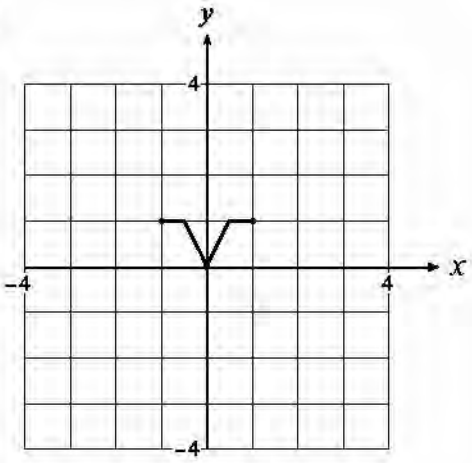


JUNE 2002

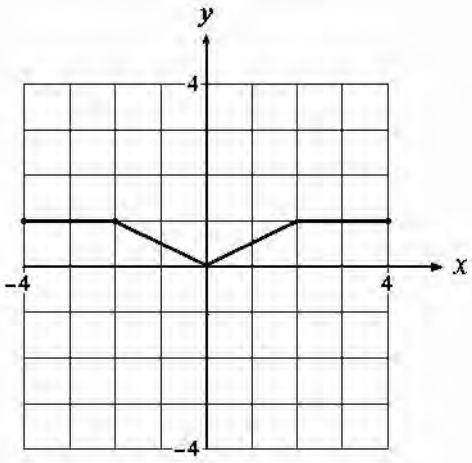
20. Given the graph of $y = f(x)$, select the graph of $y = \frac{1}{2}f(x)$.



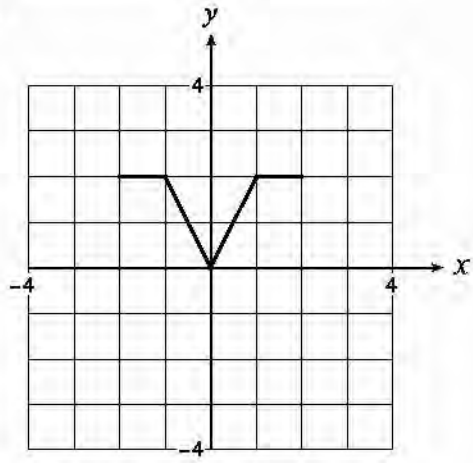
A.



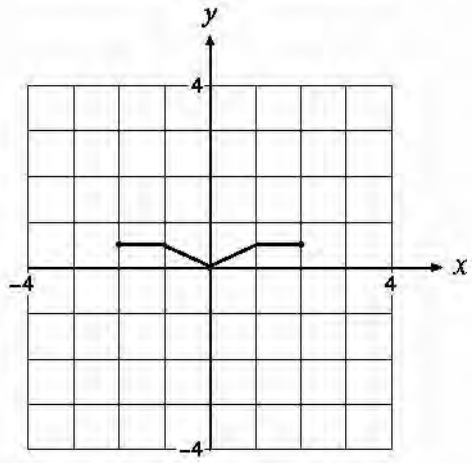
B.



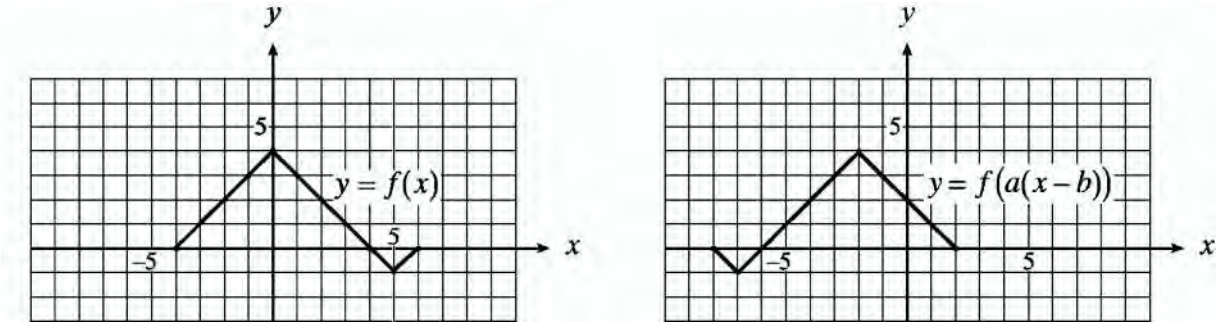
C.



D.

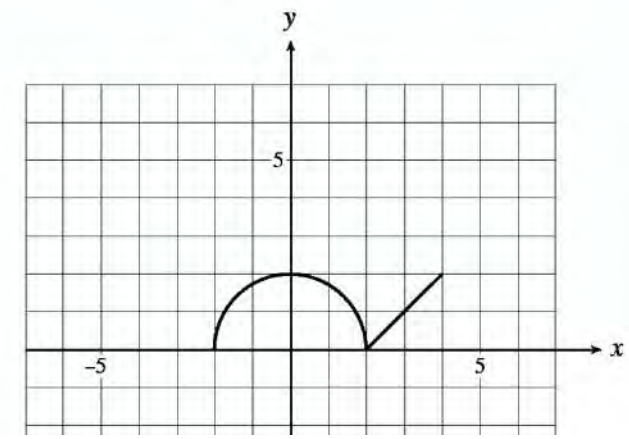


21. Two functions are graphed below, $y = f(x)$ and $y = f(a(x-b))$. Determine the values of a and b.



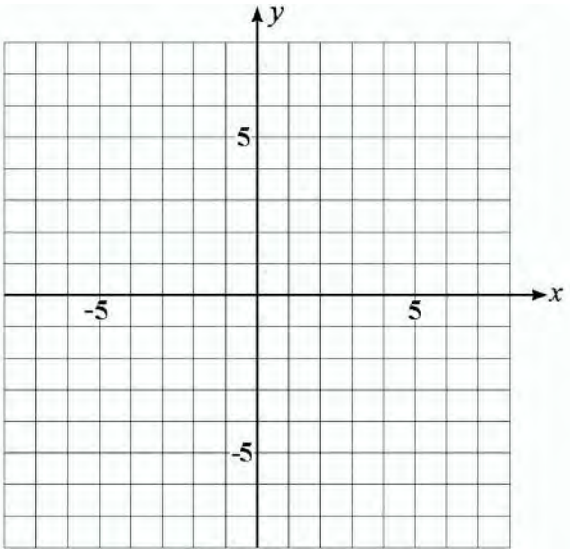
A. $a = -1,\ b = -2$ B. $a = -1,\ b = 2$ C. $a = 1,\ b = -2$ D. $a = 1,\ b = 2$

22. The graph of $y = f(x)$ is shown.

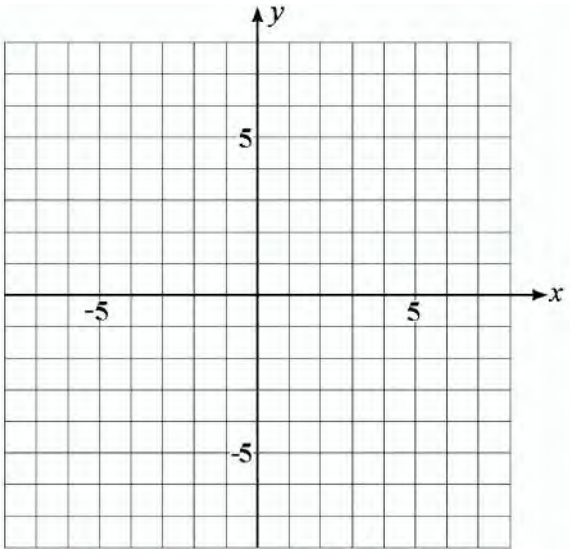


On the grids provided, sketch the graphs of: A. $y = 2f(x+3) - 1$ B. $y = f^{-1}(x)$

A.



B.



AUG 2002

23. How is the graph of $y = \sqrt{x-3}+1$ related to the graph of $y = \sqrt{x}$?

A. $y = \sqrt{x}$ has been translated 3 units right and 1 unit up.

B. $y = \sqrt{x}$ has been translated 3 units right and 1 unit down.

C. $y = \sqrt{x}$ has been translated 3 units left and 1 unit up.

D. $y = \sqrt{x}$ has been translated 3 units left and 1 unit down.

24. Given $f(x) = 3x + 2$, determine $f^{-1}(x)$, the inverse of $f(x)$.

A. $f^{-1}(x) = \frac{x}{3} - 2$ B. $f^{-1}(x) = \frac{x-2}{3}$ C. $f^{-1}(x) = \frac{1}{3x+2}$ D. $f^{-1}(x) = 2 - \frac{x}{3}$

25. Which equation represents a reflection of the graph of $5 - x = 2y^2 + y$ in the y-axis?

A. $5 + x = 2y^2 + y$ B. $5 - x = 2y^2 - y$ C. $5 + y = 2x^2 + x$ D. $-5 - x = 2y^2 + y$

26. In the point $(-3, -6)$ is on the graph of $y = f(x)$, determine a point on the graph of $y = 3|f(x)| + 1$.

A. (3, 3) B. (3, 19) C. (−3, 3) D. (−3, 19)

27. Which equation represents the graph of $y = f(x)$ after it is compressed horizontally by a factor of 2 and then translated 4 units right?

A. $y = f(2x - 8)$ B. $y = f(2x - 4)$ C. $y = f\left(\frac{x-4}{2}\right)$ D. $y = f\left(\frac{x}{2} - 4\right)$

JAN 2003

28. How is the graph of $y = f(x) + 3$ related to the graph of $y = f(x)$?

A. $y = f(x)$ has been translated 3 units up.

B. $y = f(x)$ has been translated 3 units down.

C. $y = f(x)$ has been translated 3 units to the left.

D. $y = f(x)$ has been translated 3 units to the right.

29. Which equation represents the graph of $y = f(x)$ after it is reflected in the line $y = x$?

A. $x = f(y)$ B. $y = f(-x)$ C. $y = -f(x)$ D. $y = \dfrac{1}{f(x)}$

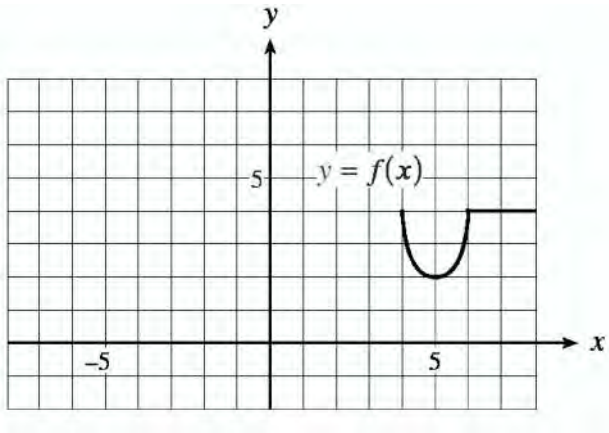
30. If the graph of the function $y = \sqrt{x}$ is horizontally expanded by a factor of 3 and the translated 2 units to the right, determine the equation of this new function.

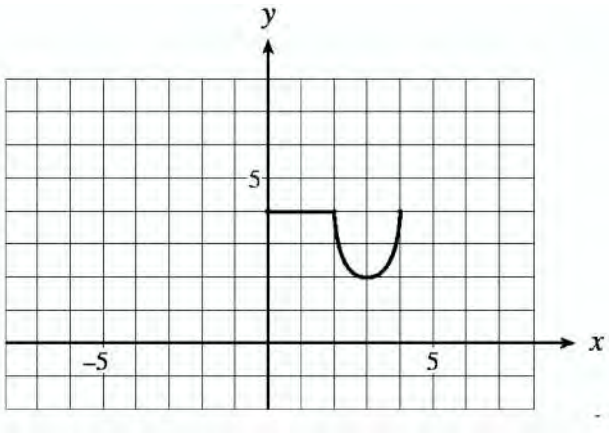
A. $y = \sqrt{3(x-2)}$ B. $y = \sqrt{\frac{1}{3}(x-2)}$ C. $y = \sqrt{3x-2}$ D. $y = \sqrt{\frac{1}{3}x-2}$

31. If $(8, -6)$ is a point on the graph of $y = f(x)$, what must be a point on the graph of $y = -f(2x) + 3$?

A. $(-16, -3)$ B. $(-4, -3)$ C. $(4, 9)$ D. $(16, 9)$

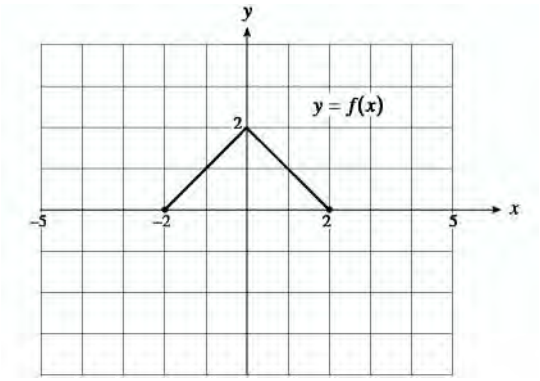
32. The graph of $y = f(x)$ is shown below on the left. Which equation represents the graph shown on the right?





A. $y = f(-(x+8))$ B. $y = f(-(x-8))$ C. $y = -f(x-8)$ D. $y = -f(x+8)$

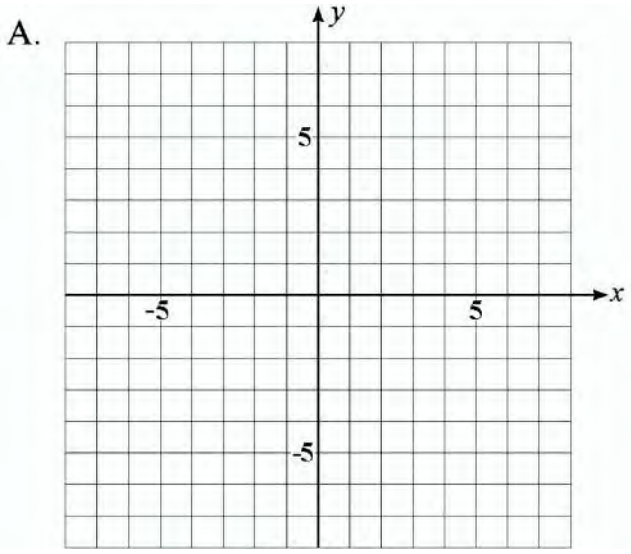
33. The graph of $y = f(x)$ is shown.



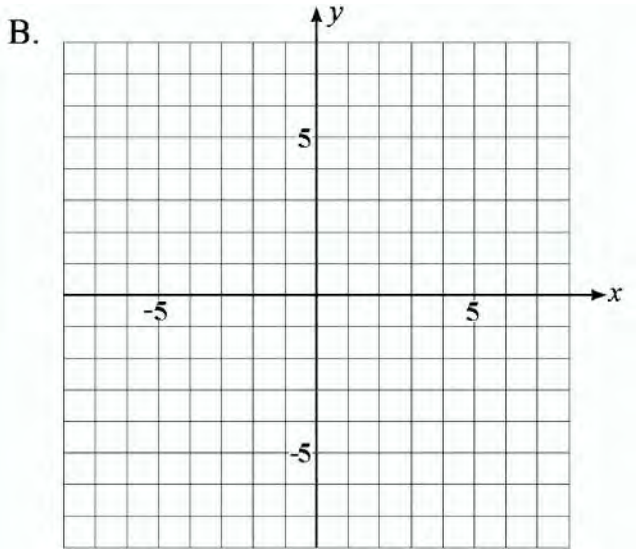
Sketch the graphs of

A. $y = 2f(x+3)$ B. $y = \dfrac{1}{f(x)}$

A.

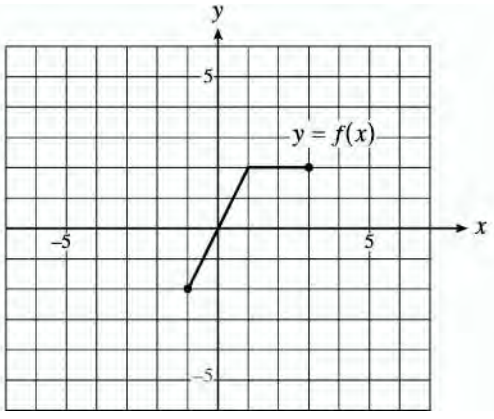


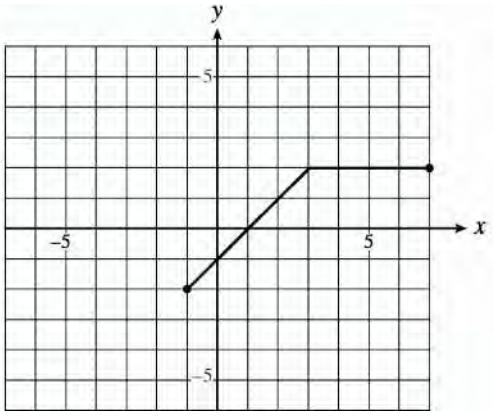
B.



JUNE 2003

34. The function $y = f(x)$ is graphed to the left below. Determine the equation of the function shown to the right.



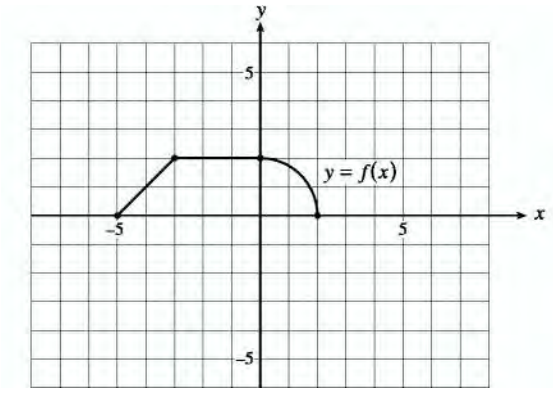


A. $y = f(2(x-1))$ B. $y = f\left(\frac{1}{2}(x-1)\right)$ C. $y = 2f(x-1)$ D. $y = \frac{1}{2}(x-1)$

35. If the point (a, b) is on the graph of $y = f(x)$, which point must be on the graph of $y = \frac{1}{f(x-2)}$? $(a \neq 0,\ b \neq 0)$

A. $\left(a-2,\ \frac{1}{b}\right)$ B. $\left(a+2,\ \frac{1}{b}\right)$ C. $\left(\frac{1}{a},\ b\right)$ D. $(a+2,\ b)$

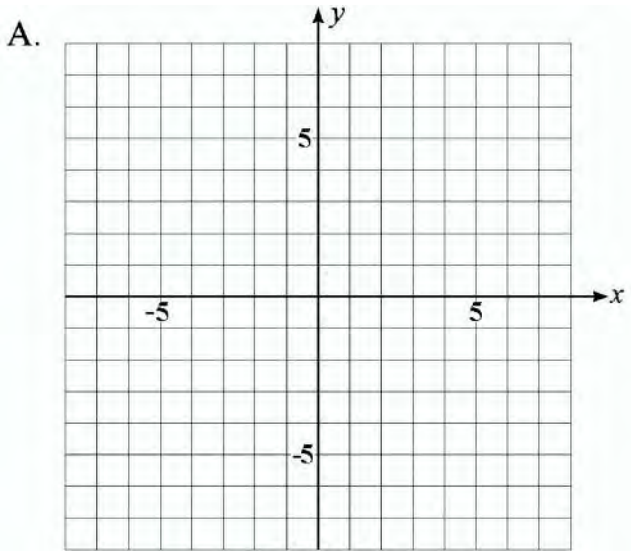
36. The graph of $y = f(x)$ is shown below.



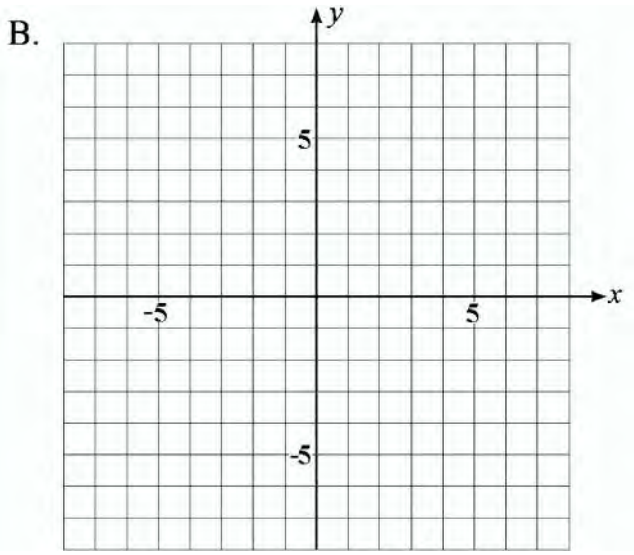
Sketch the graphs of

A. $y = 2f(x) - 3$ B. $y = f^{-1}(x)$

A.



B.

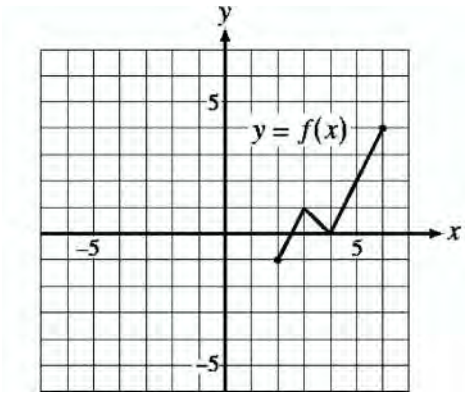


JAN 2004

37. Which equation represents the graph of $y = g(x)$ after it is translated 5 units up?

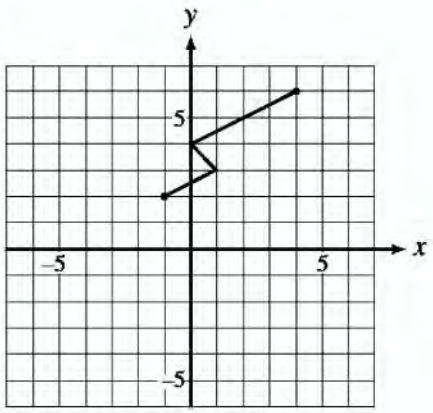
A. $y = g(x) + 5$ B. $y = g(x) - 5$ C. $y = g(x+5)$ D. $y = g(x-5)$

38. The graph of $y = f(x)$ is shown below.

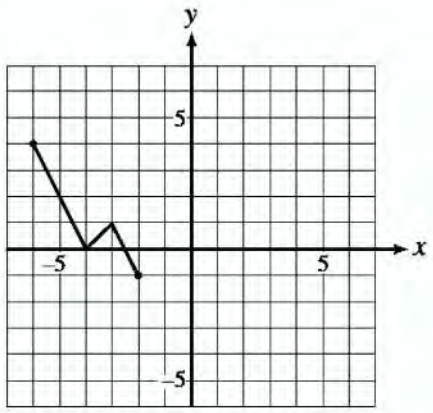


Which graph represents $x = f(y)$?

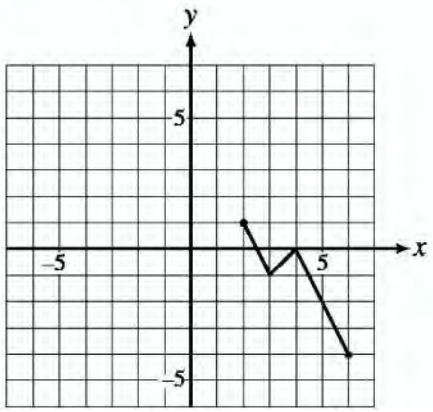
A.



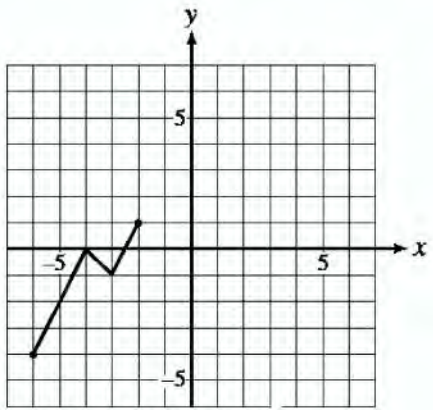
B.



C.



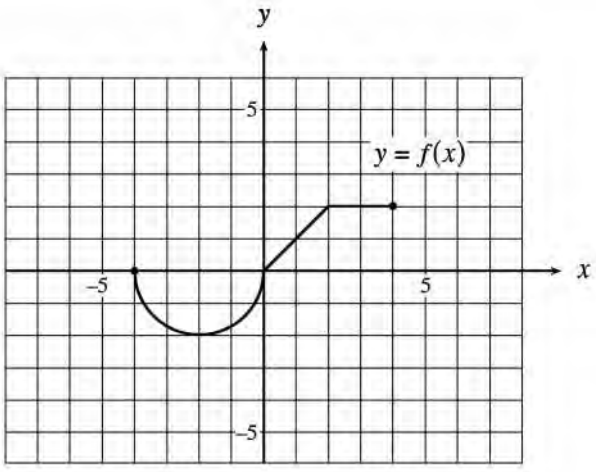
D.



39. If the point $(4, 6)$ is on the graph of $y = f(x)$, what point must be on the graph of $y = 3\left(\frac{1}{f(x)}\right)$?

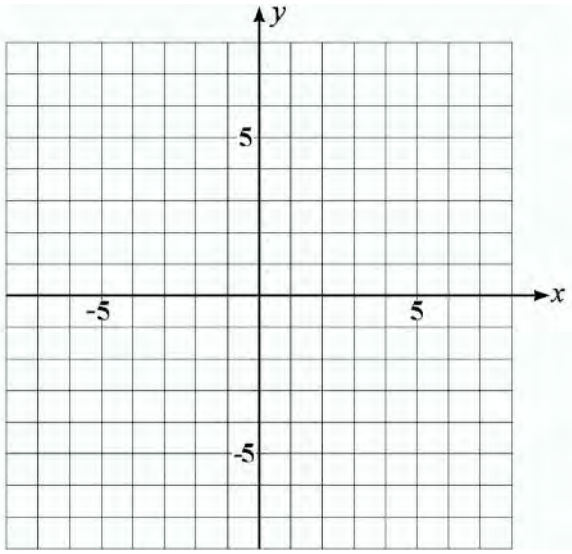
A. $\left(12, \frac{1}{6}\right)$ B. $\left(4, \frac{1}{18}\right)$ C. $\left(4, \frac{1}{2}\right)$ D. $(2, 18)$

40. The graph of $y = f(x)$ is shown below. Sketch the graphs of:

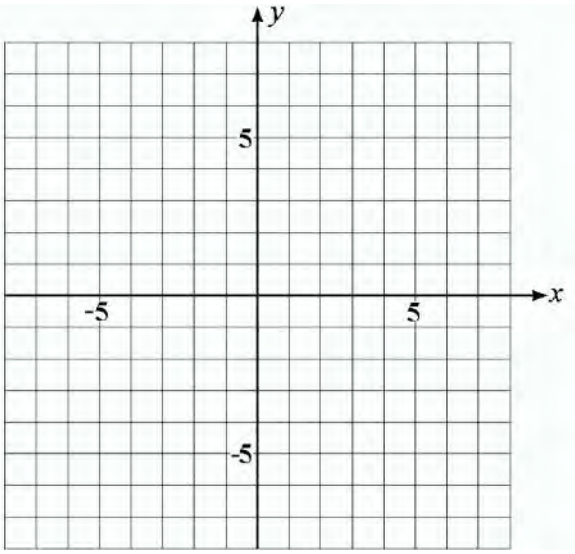


A. $y = -2f(x+3)$ B. $y = \left| f\left(\frac{x}{2}\right) \right|$

A.



B.



JUNE 2004

41. Which equation represents the graph of $y = \tan x$ after it has been translated 4 units up and 7 units left?

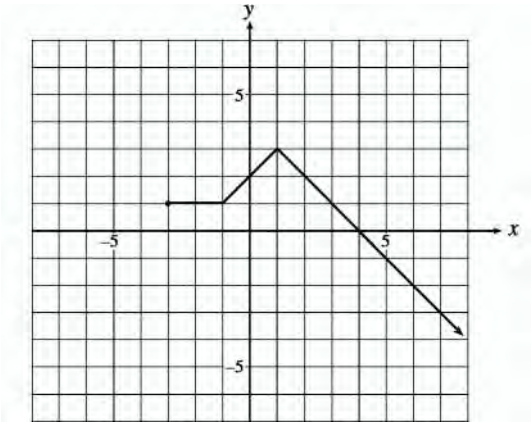
A. $y = \tan(x+7)+4$ B. $y = \tan(x+7)-4$ C. $y = \tan(x-7)+4$ D. $y = \tan(x-7)-4$

42. The point $(9,-12)$ is on the graph of a function. What will the coordinates of this point be after all of the following transformations are performed on the function, in the order given?

-horizontal expansion by a factor of 3
-reflection in the x-axis
-vertical translation of 5 downward
-reflection in the line $y = x$

A. $(-27,7)$ B. $(-17,-27)$ C. $(7,3)$ D. $(7,27)$

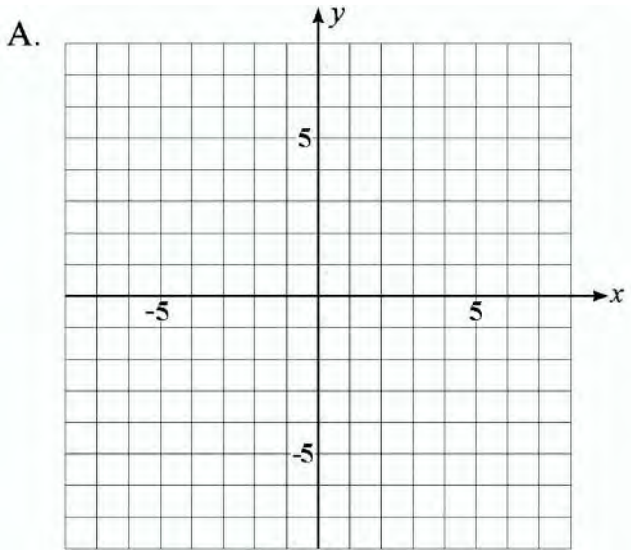
43. The graph of $y = f(x)$ is shown below.



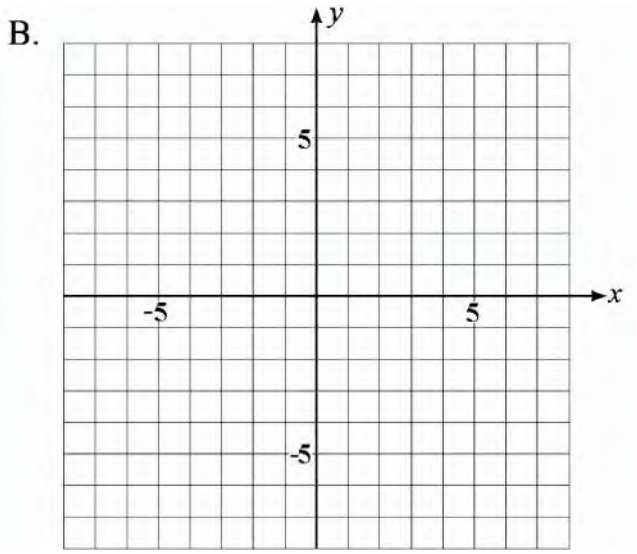
On the grids provided, sketch the graphs of:

A. $y = f(-x) - 3$ B. $y = \frac{1}{f(x)}$

A.



B.



AUG 2005

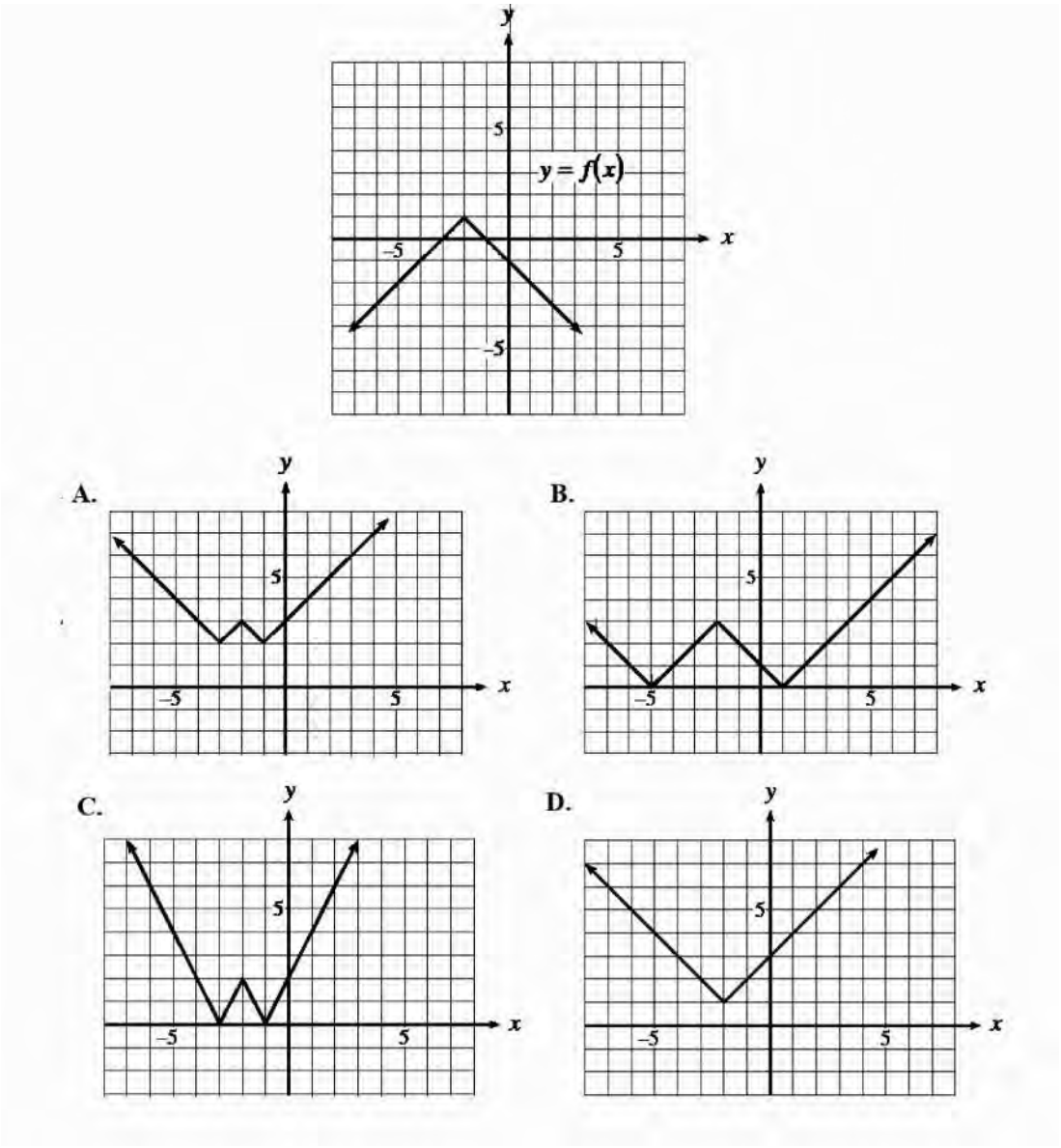
44. If the function $y = 3^x$ is expanded vertically by a factor of 9 to produce a new function, which of the following is an equation of the new function?

A. $y = 3^{2x}$ B. $y = 3^{3x}$ C. $y = 3^{x+2}$ D. $y = 3^{x-2}$

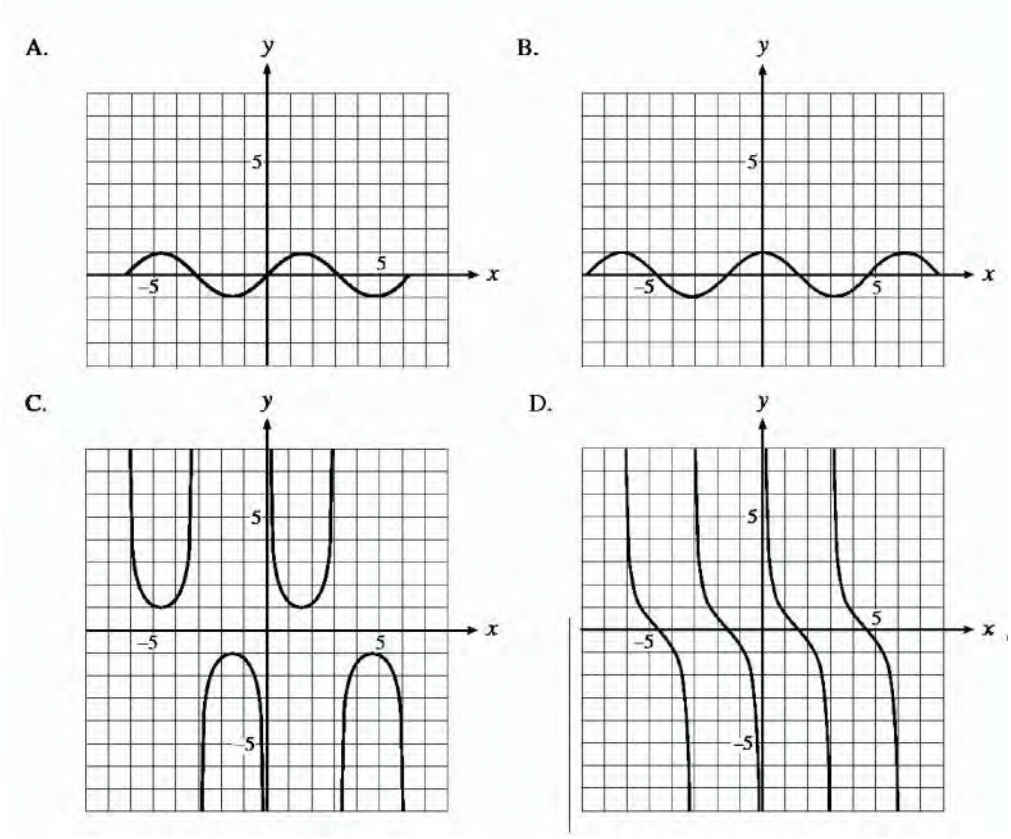
45. Which equation represents the graph of $y = g(x)$ after it is translated 3 units to the right?

A. $y = g(x) + 3$ B. $y = g(x) - 3$ C. $y = g(x + 3)$ D. $y = g(x - 3)$

46. The graph of $y = f(x)$ is shown below. Which graph represents $y = |f(x)| + 2$?



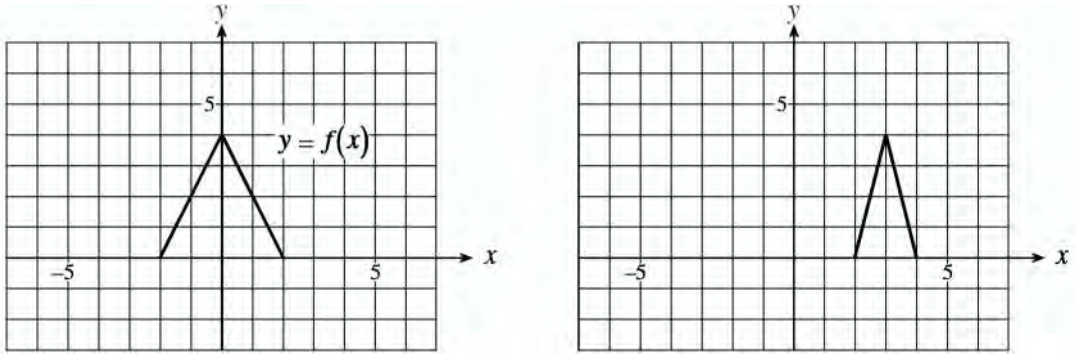
47. For which of the following functions is $f(-x) = f(x)$?



48. If the point $(6, 10)$ is on the graph of $y = f(x)$, which point must be on the graph of $y = \dfrac{1}{2f(x)}$?

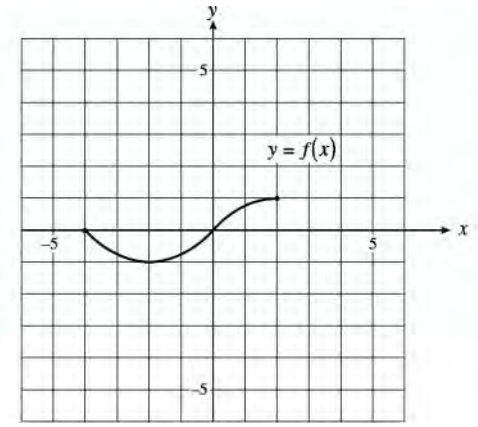
A. $\left(3, \dfrac{1}{10}\right)$ B. $\left(6, \dfrac{1}{5}\right)$ C. $\left(6, \dfrac{1}{10}\right)$ D. $\left(6, \dfrac{1}{20}\right)$

49. Given the graph of the function $y = f(x)$ on the left, determine the equation of the function on the right.



A. $y = f\left(\dfrac{x}{2} - 3\right)$ B. $y = f\left(\dfrac{x-3}{2}\right)$ C. $y = f(2x - 3)$ D. $y = f(2x - 6)$

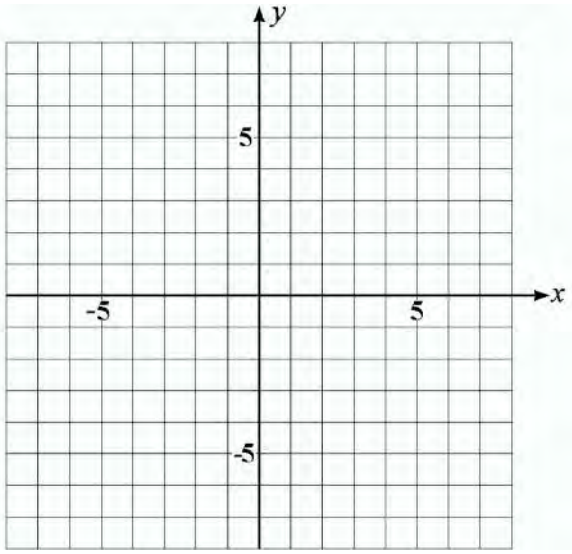
50. The graph of $y = f(x)$ is shown below. On the grids provided, sketch the graphs of



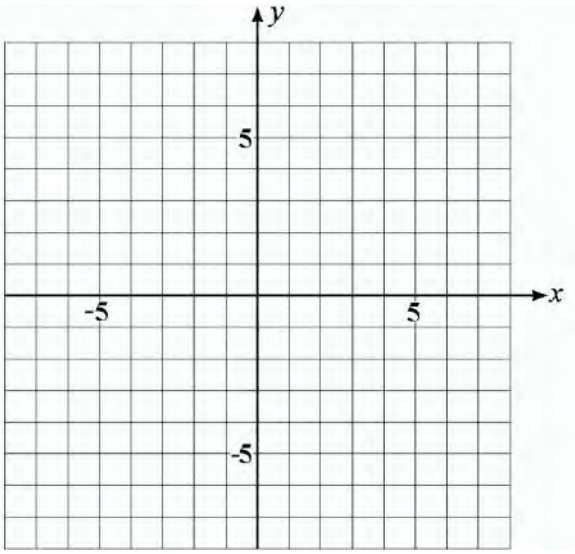
A. $y = 3f(x) + 1$

B. $y = \dfrac{1}{f(x)}$

A.



B.



51. Which equation represents the graph of $\dfrac{(x-2)^2}{4} + \dfrac{(y-3)^2}{9} = 1$ after it is translated 5 units to the right and 1 unit up?

A. $\dfrac{(x-7)^2}{4} + \dfrac{(y-4)^2}{9} = 1$

B. $\dfrac{(x-7)^2}{4} + \dfrac{(y-2)^2}{9} = 1$

C. $\dfrac{(x+3)^2}{4} + \dfrac{(y-4)^2}{9} = 1$

D. $\dfrac{(x+3)^2}{4} + \dfrac{(y-2)^2}{9} = 1$

52. Which equation represents the graph of $y = 2^x$ after it is reflected in the x-axis?

A. $y = 2^{-x}$ B. $y = -2^x$ C. $y = \log_2 x$ D. $y = -\log_2 x$

AUG 2006

53. How is the graph of $y = f(4x)$ related to the graph of $y = f(x)$?

A. $y = f(x)$ has been compressed vertically by a factor of 4.
B. $y = f(x)$ has been compressed horizontally by a factor of 4.
C. $y = f(x)$ has been expanded vertically by a factor of 4.
D. $y = f(x)$ has been expanded horizontally by a factor of 4.

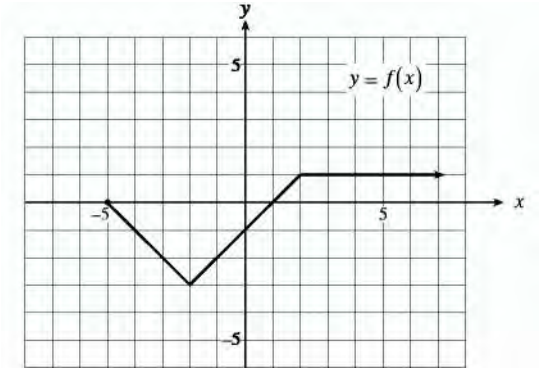
54. If the maximum value of the function $y = f(x)$ is 6, determine the maximum value of $y = \frac{1}{3}f\left(\frac{1}{2}x\right)$.

A. 2 B. 3 C. 12 D. 18

55. If the point $(-2, -5)$ is on the graph of $y = f(x)$, which point must be on the graph of $y = |f(x-1)| - 3$?

A. $(-3, 2)$ B. $(-1, 2)$ C. $(1, -8)$ D. $(3, -8)$

56. The graph of $y = f(x)$ is shown below.

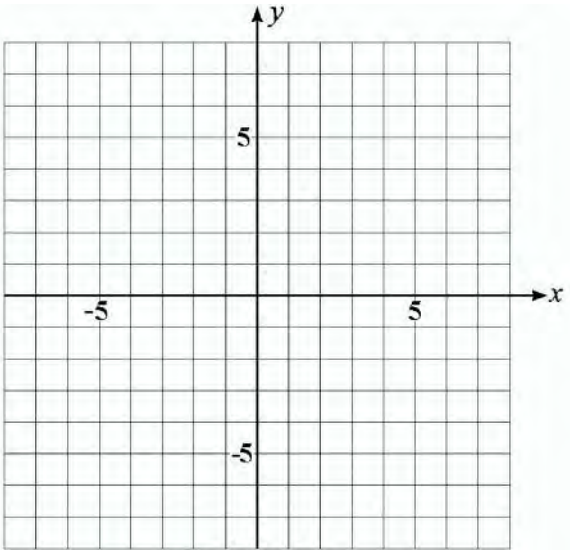


On the grids provided, sketch the graphs of:

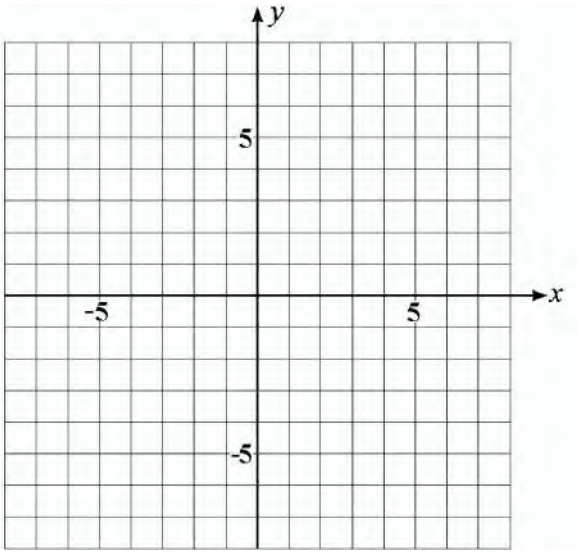
A. $y = 2\,|f(x)| + 1$

B. $y = \dfrac{1}{f(x)}$

A.



B.



SAMPLE 2008

57. Which equation represents the graph of $y = f(x)$ after it is vertically compressed by a factor of 2 and then translated 2 units to the left?

A. $\dfrac{y}{2} = f(x+2)$ B. $\dfrac{y}{2} = f(x-2)$ C. $2y = f(x+2)$ D. $2y = f(x-2)$

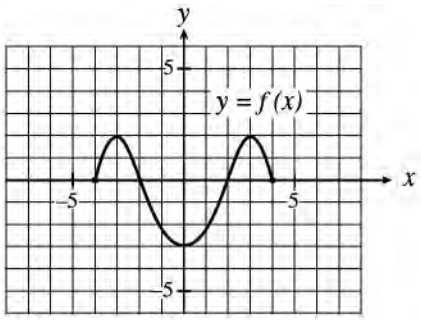
58. Determine the inverse of the function $f(x) = \dfrac{4x+1}{3x}$.

A. $f^{-1}(x) = \dfrac{1}{3X-4}$ B. $f^{-1}(x) = \dfrac{-1}{3x-4}$ C. $f^{-1}(x) = \dfrac{3x}{4x+1}$ D. $f^{-1}(x) = \dfrac{-3x}{4x+1}$

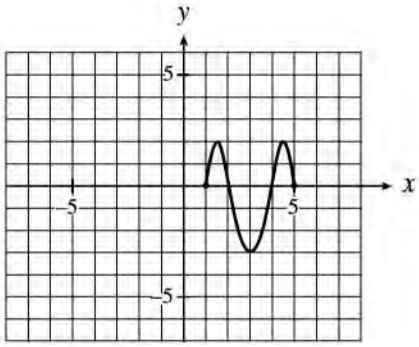
59. The y-intercept of the function $y = f(x)$ is 5. Determine the y-intercept of $y = -f(x) + 3$

A. -2 B. -8 C. 8 D. 2

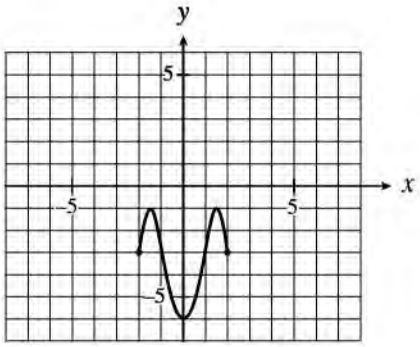
60. The graph of function $y = f(x)$ is shown. Which of the following is the graph of $y = f(2x) - 3$?



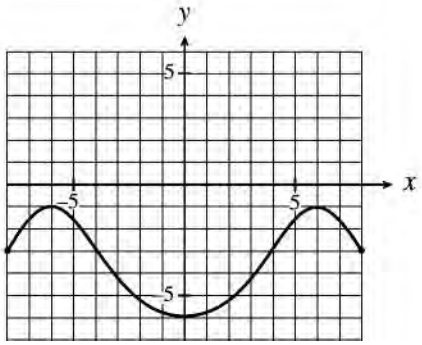
A.



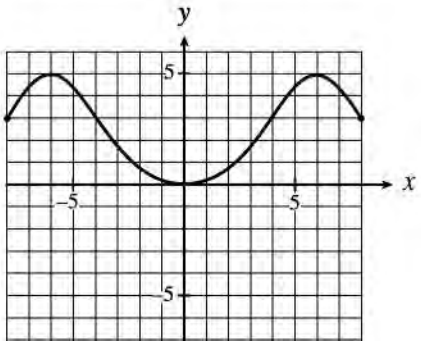
B.



C.



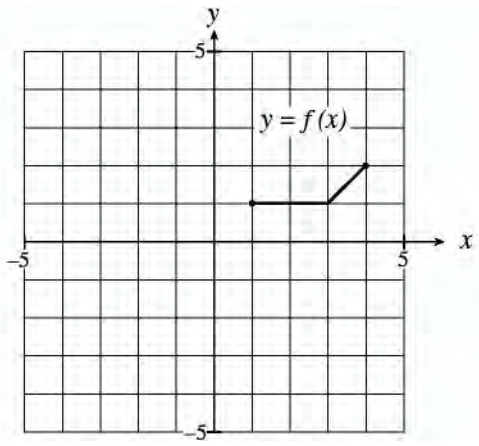
D.

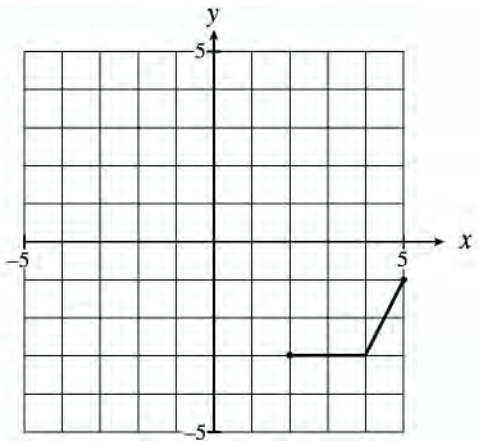


61. The point (10, 6) is on the graph of $y = f(x)$, what point must be on the graph of $y = f(-2x - 4)$?

A. $(-7, 6)$ B. $(-9, 6)$ C. $(-22, 6)$ D. $(-24, 6)$

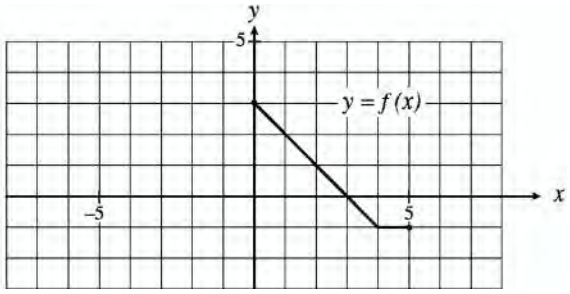
62. The graph of $y = f(x)$ is shown on the left. Determine an equation of the function graphed on the right.





A. $y = \frac{1}{2}f(x-1)-5$ B. $y = \frac{1}{2}f(x-1)-4$ C. $y = 2f(x-1)-5$ D. $y = 2f(x-1)-4$

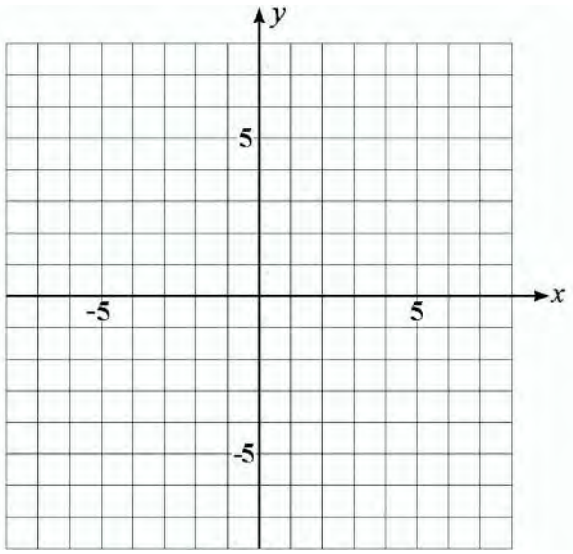
63. The graph of $y = f(x)$ is shown below.



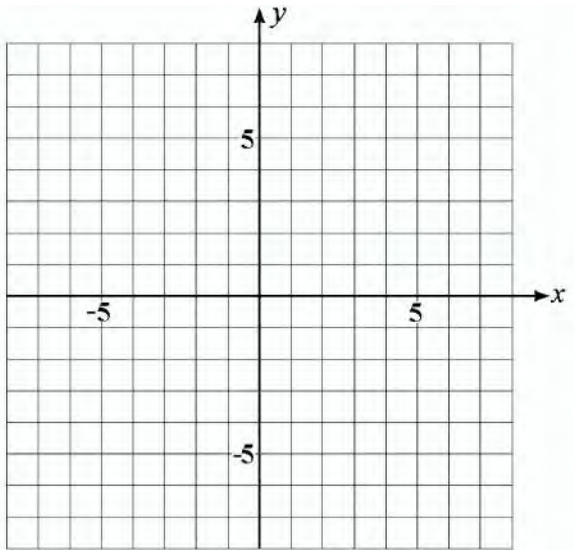
On the grids provided, sketch the graphs of:

A. $y = 2\,|f(x)-1|$ B. $y = \frac{1}{f(x)}$

A.



B.



JAN 2008

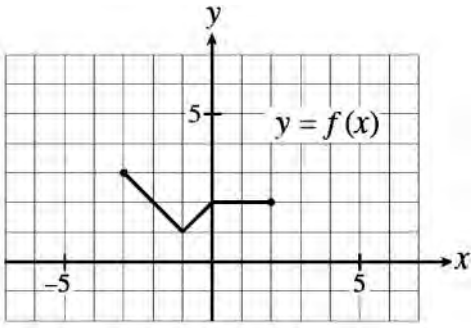
64. If $y = (x+4)(x-2)$, determine the zeros of the function $y = f(2x)$.

A. $-8, 4$ B. $-4, 2$ C. $-2, 1$ D. $-1, 2$

65. Which equation represents the graph of $y = f(x)$ after it is expanded vertically by a factor of 5?

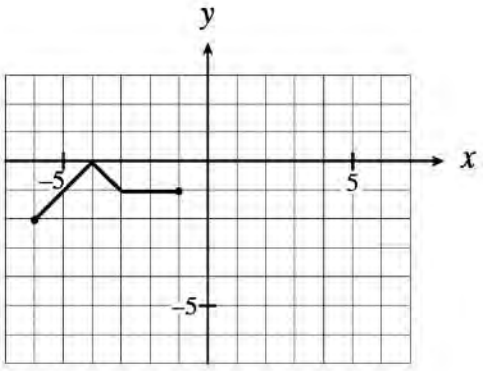
A. $y = \frac{1}{5}f(x)$ B. $y = 5f(x)$ C. $y = f\left(\frac{x}{5}\right)$ D. $y = f(5x)$

66. The graph of $y = f(x)$ is shown:

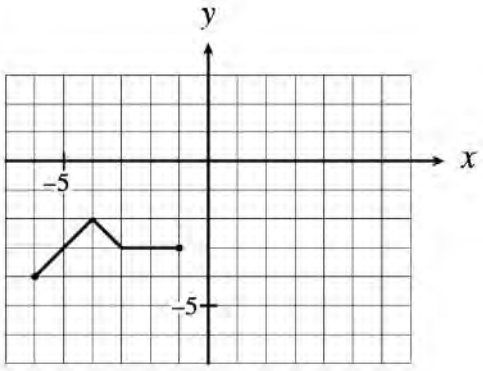


Which graph represents the graph of $y = -f(x+3)+1$?

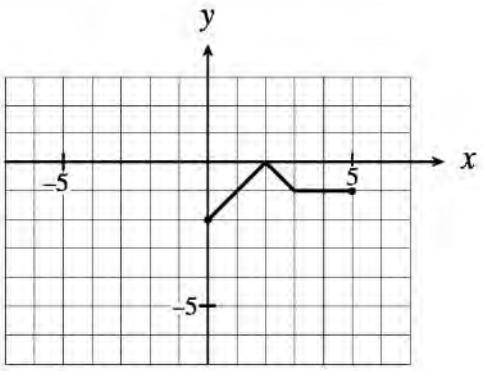
A.



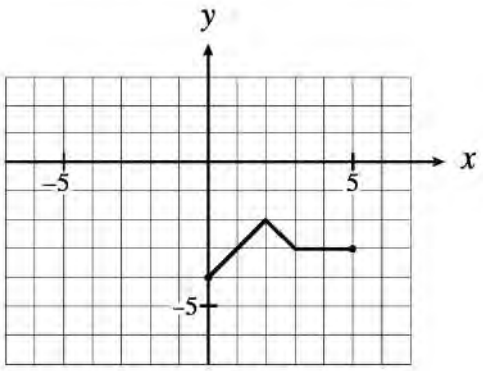
B.



C.



D.



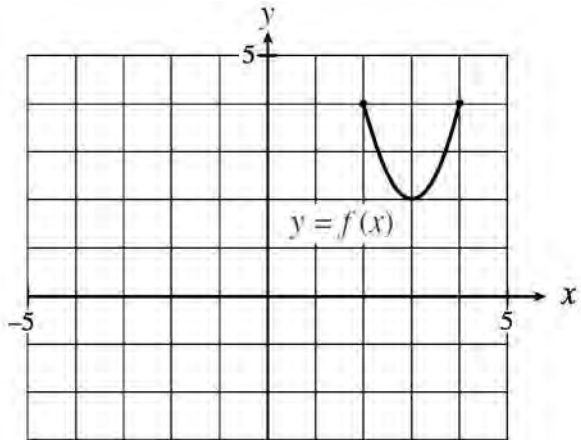
67. Determine the inverse of the function $f(x) = x^3 - 2$.

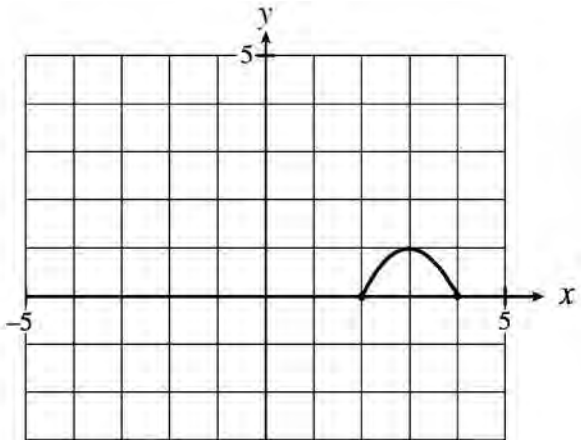
A. $f^{-1}(x) = \sqrt[3]{x+2}$ B. $f^{-1}(x) = \sqrt[3]{x}+2$ C. $f^{-1}(x) = \sqrt[3]{x}-2$ D. $f^{-1}(x) = \sqrt[3]{x-2}$

68. If the point $(6,-12)$ is on the graph of $y = f(x)$, which point must be on the graph of $y = f\left(-\frac{1}{3}x+6\right)$?

A. $(-36,-12)$ B. $(-24,-12)$ C. $(0,-12)$ D. $(16,-12)$

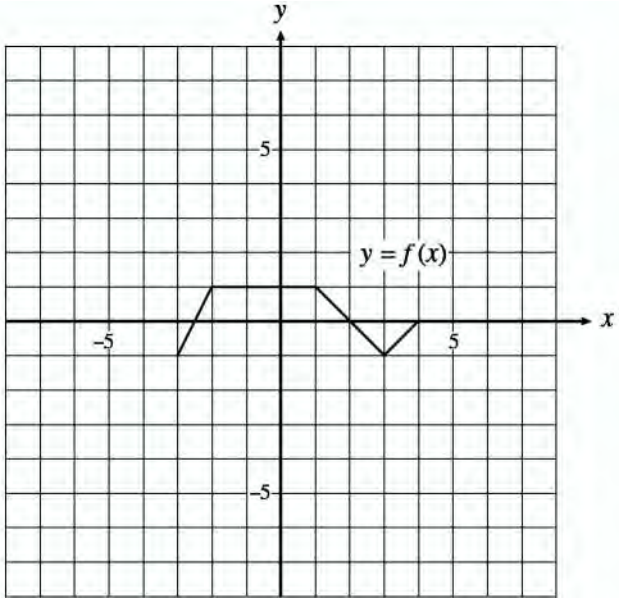
69. The graph of $y = f(x)$ is shown below on the left. Determine an equation of the function graphed on the right.



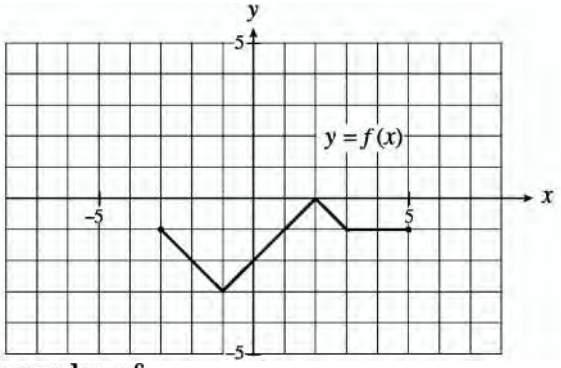


A. $y = -\frac{1}{2}f(x)$ B. $y = -\frac{1}{2}f(x)+2$ C. $y = -\frac{1}{2}f(x)+3$ D. $y = -\frac{1}{2}f(x)+4$

70. Given the graph of $y = f(x)$ below, sketch $g(x) = 3|f(x)| - 2$.



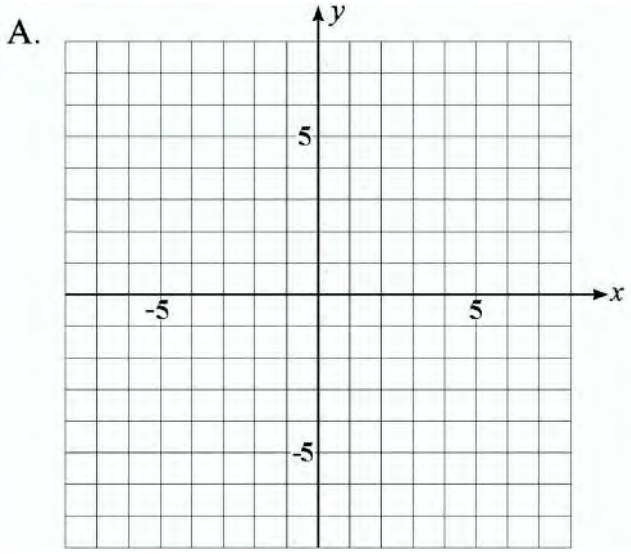
71. The graph of $y = f(x)$ is shown:



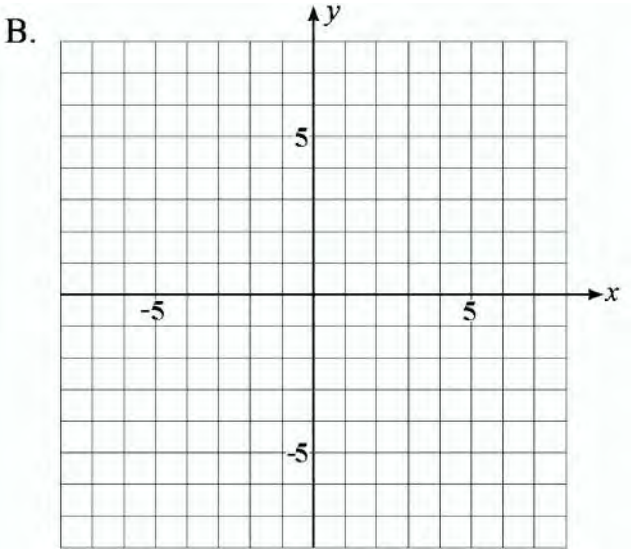
On the grids provided, sketch the graphs of:

A. $y = -|f(x) + 2|$ B. $y = \frac{1}{f(x)}$

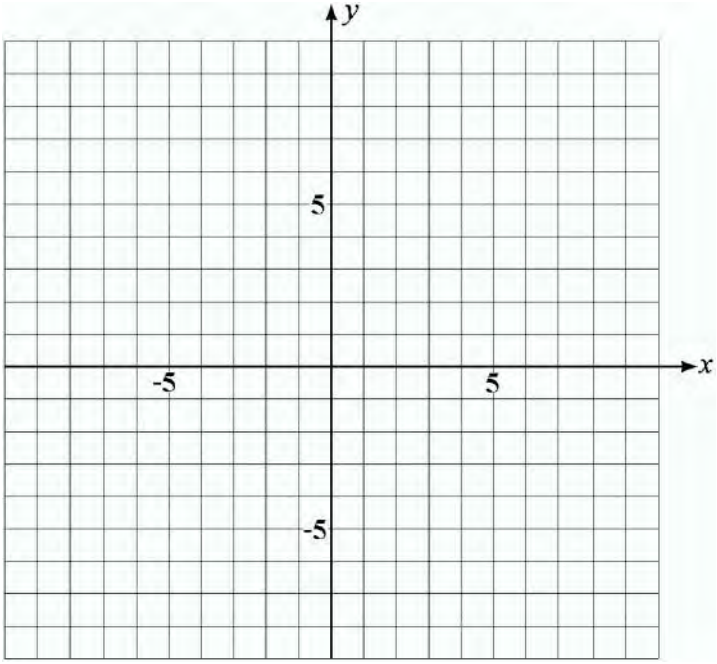
A.



B.



72. For the function $f(x) = \frac{1}{x+3}$: Determine the equation that defines the inverse function, $f^{-1}(x)$, and sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the grid provided.



2009 SAMPLE QUESTIONS

73. If the graph of $2x + 3y = 5$ is translated 4 units up, determine an equation of the new graph.

A. $2x + 3y = 1$ B. $2x + 3y = 9$ C. $2x + 3(y + 4) = 5$ D. $2x + 3(y - 4) = 5$

74. If (a, b) is a point on the graph of $y = f(x)$, determine a point on the graph of $y = f(x - 2) + 3$.

A. $(a - 2, b + 3)$ B. $(a - 2, b - 3)$ C. $(a + 2, b + 3)$ D. $(a + 2, b - 3)$

75. If the point $(2, -8)$ is on the graph of $y = f(x - 3) + 4$, what point must be on the graph of $y = f(x)$.

A. $(-1, -12)$ B. $(-1, -4)$ C. $(5, -12)$ D. $(5, -4)$

76. How is the graph of $y = 7^{3x}$ related to the graph of $y = 7^x$?

A. the graph of $y = 7^x$ has been expanded vertically by a factor of 3
B. the graph of $y = 7^x$ has been compressed vertically by a factor of 3
C. the graph of $y = 7^x$ has been expanded horizontally by a factor of 3
D. the graph of $y = 7^x$ has been compressed horizontally by a factor of 3

77. If the graph of $x^2 + y^2 = 4$ is vertically compressed by a factor of 5, then reflected in the y-axis, determine an equation for the new graph.

A. $x^2 + \frac{y^2}{25} = 4$ B. $-x^2 + 25y^2 = 4$ C. $x^2 + 25y^2 = 4$ D. $-x^2 + \frac{y^2}{25} = 4$

78. The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in

A. the y-axis. B. the x-axis. C. the line $y = x$. D. the line $y = -x$.

79. What is the inverse of the relation $y = x^3$?

A. $y = \frac{1}{x^3}$ B. $x = y^3$ C. $y = (-x)^3$ D. $x = y^{\frac{1}{3}}$

80. The point $(6, -12)$ is on the graph of the function $y = f(x)$. Which point must be on the graph of the function $y = 3f(-x)$?

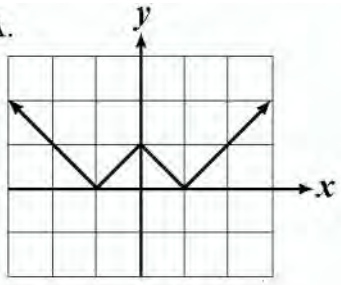
A. $(-6, -36)$ B. $(6, 36)$ C. $(-6, -4)$ D. $(6, 4)$

81. If $f(x) = \frac{2x}{x-1}$, determine the equation of $f^{-1}(x)$, the inverse of $f(x)$.

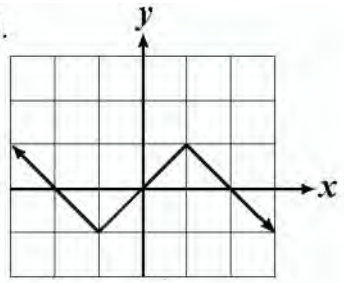
A. $f^{-1}(x) = \frac{x}{x-2}$ B. $f^{-1}(x) = \frac{2x}{2x-1}$ C. $f^{-1}(x) = \frac{x-1}{2x}$ D. $f^{-1}(x) = \frac{1}{x-2}$

82. For which graph of $y = f(x)$ would $f(-x) = -f(x)$?

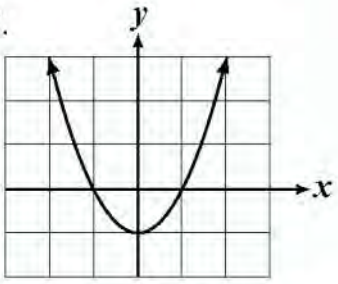
A.



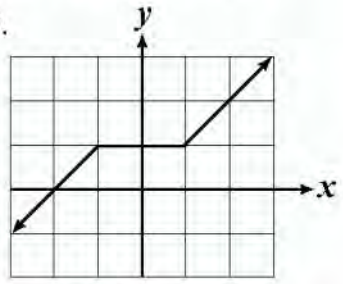
B.



C.



D.



83. When the graph of $y = f(x)$ is transformed to the graph of $y = f(-x)$, on which line(s) will the invariant points lie?

A. $y = 0$ B. $x = 0$ C. $y = x$ D. $y = 1, y = -1$

84. If the range of $y = f(x)$ is $-1 \le y \le 2$, what is the range of $y = \frac{1}{f(x)}$?

A. $-1 \le y \le \frac{1}{2}$ B. $-1 \le y \le \frac{1}{2}, y \neq 0$ C. $y \ge \frac{1}{2}$ or $y \le -1$ D. $y \ge 2$ or $y \le -1$

85. The range of $y = f(x)$ is transformed to the graph of $y = \frac{1}{f(x)}$. If the following points are on the graph of $y = f(x)$, which point would be invariant?

A. $(1, 2)$ B. $(2, 1)$ C. $(3, 0)$ D. $(0, 3)$

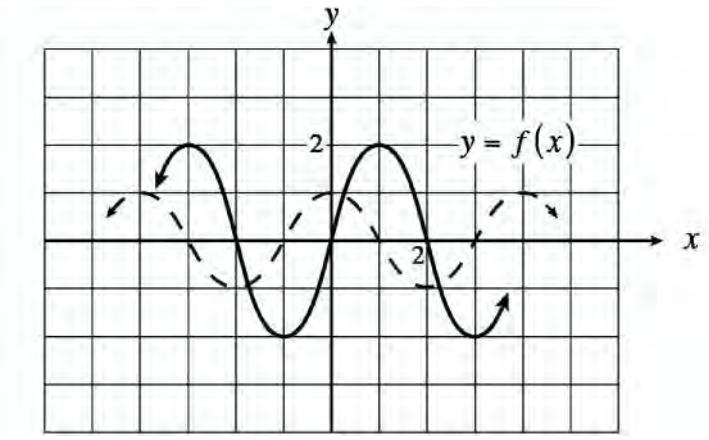
86. If the range of $y = f(x)$ is $-3 \le y \le 5$, what is the range of $y = |f(x)|$?

A. $-3 \le y \le 5$ B. $0 \le y \le 3$ C. $0 \le y \le 5$ D. $3 \le y \le 5$

87. Determine an equation that will cause the graph of $y = f(x)$ to expand vertically by a factor of 4 and then translate 3 units up.

A. $y = \frac{1}{4}f(x) + 3$ B. $y = \frac{1}{4}f(x) - 3$ C. $y = 4f(x) + 3$ D. $y = 4f(x) - 3$

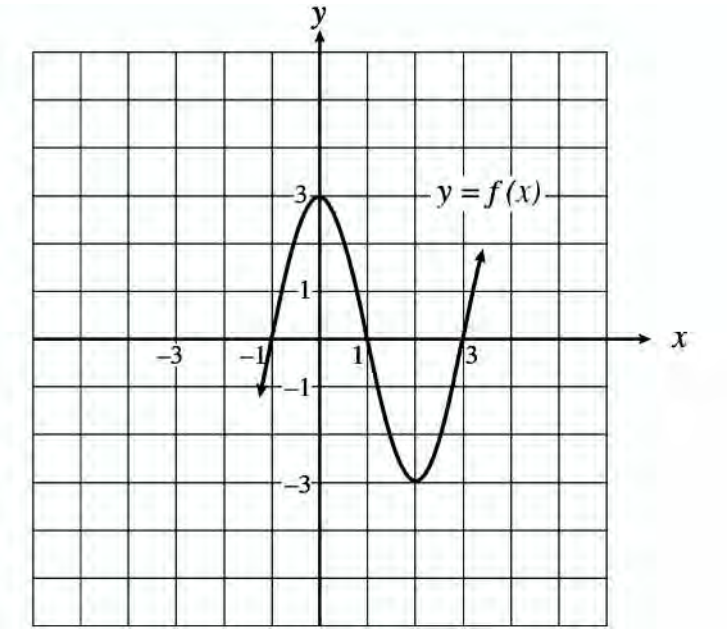
88. In the diagram below, $y = f(x)$ is graphed as a broken line.



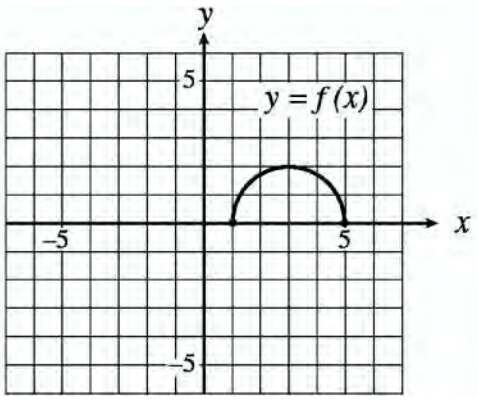
Which equation is defined by the solid line?

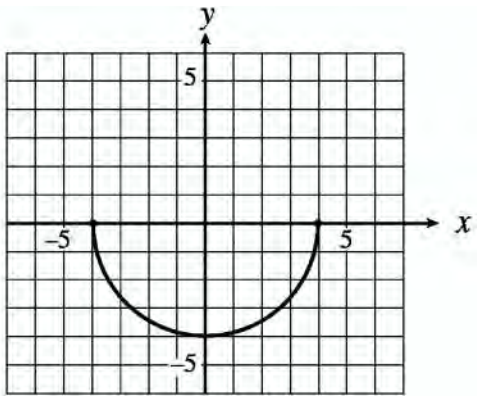
A. $y = 2f(x + 1)$ B. $y = f(2x - 1)$ C. $y = f(2x + 1)$ D. $y = 2f(x - 1)$

89. The graph of $y = f(x)$ is shown below. Sketch the graph of $y = -f\left(\frac{1}{2}(x + 2)\right)$.



90. The graph of $y=f(x)$ is shown below on the left. Which equation represents the graph shown on the right?



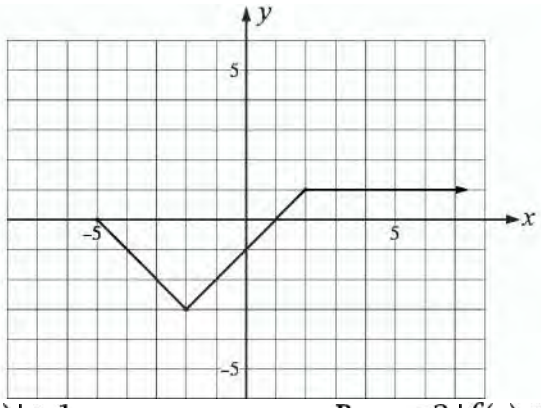


A. $y=-2f(2x+3)$ B. $y=-2f(2x+6)$ C. $y=-2f\left(\frac{1}{2}x+3\right)$ D. $y=-2f\left(\frac{1}{2}x+6\right)$

91. If the point $(6,-2)$ is on the graph $y=f(x)$, which point must be on the graph of $y=\frac{1}{f(-x)+4}$?

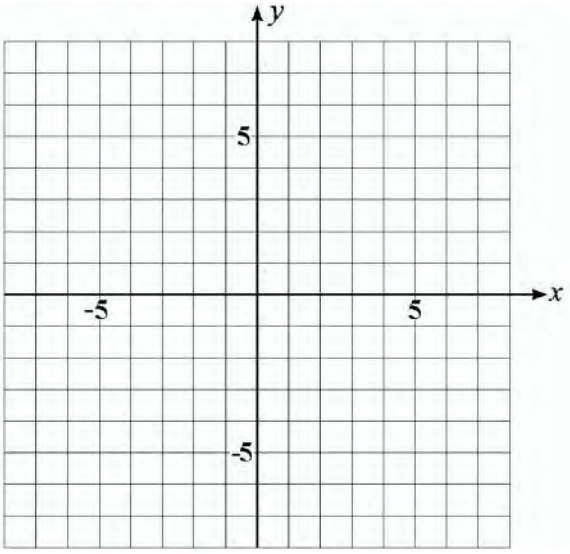
A. $\left(-10,-\frac{1}{2}\right)$ B. $\left(-6,\frac{1}{2}\right)$ C. $\left(-6,\frac{7}{2}\right)$ D. $\left(-\frac{1}{6},2\right)$

92. The graph of $y=f(x)$ is shown:

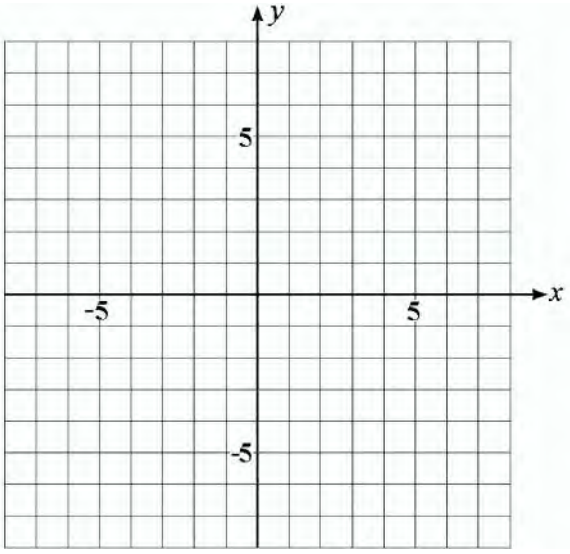


Sketch the graphs of: A. $y=2\,|f(x)|+1$ B. $y=2\,|f(x)+1|$

A.



B.



ADDITIONAL QUESTIONS

93. If $y = f(x)$ is a function with domain $[-8, 12]$, determine the domain of $y = \frac{1}{2}f(x-3)$.

A. $[-5, 6]$ B. $[-7, 12]$ C. $[-5, 15]$ D. $[-11, 9]$

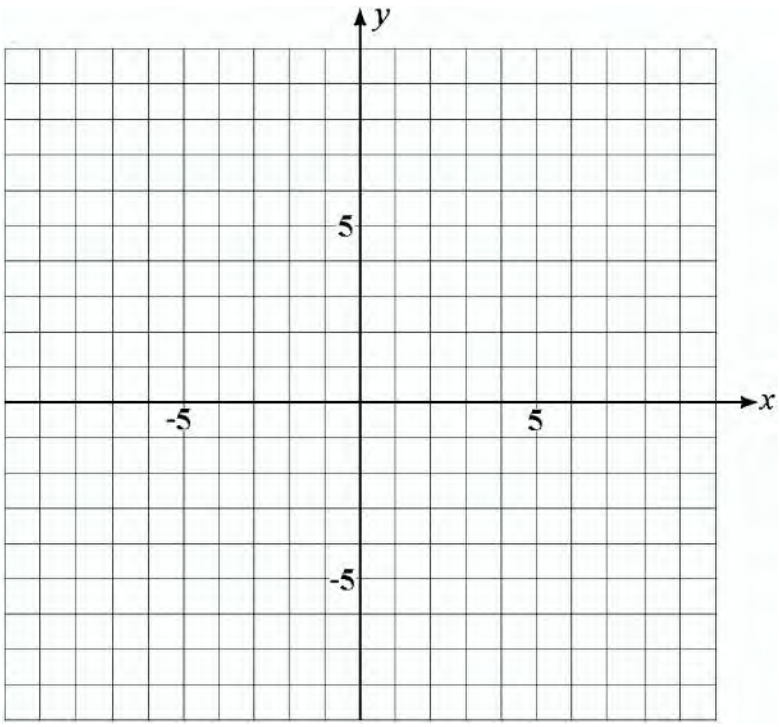
94. The graph of $y = -g(2x)$ is obtained by transforming the graph of $y = g(x)$ in the following way:

A. Shrink horizontally and reflect across the x-axis.
B. Shrink horizontally and reflect across the y-axis.
C. Stretch vertically and reflect across the x-axis.
D. Stretch vertically and reflect across the y-axis.

95. If $(-4, 7)$ is a point on the graph of $y = h(t)$, which of the following must be a point on the graph of $y = h(-t) - 2$?

A. $(-4, -9)$ B. $(-4, -5)$ C. $(4, 5)$ D. $(4, 9)$

96. If $f(x) = |x-2|$, sketch the graph of $y = f\left(\frac{x}{2}\right)$



97. The relation $x = \sqrt{9 - y^2}$ is multiplied vertically by a factor of $\frac{1}{3}$, then translated 1 unit to the right. Determine the equation of the transformed relation.

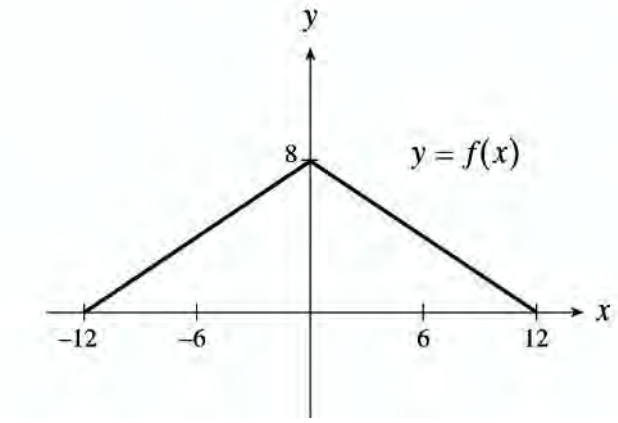
A. $x = \sqrt{9 - 9y^2} + 1$ B. $x = \sqrt{9 - 9y^2} - 1$ C. $x = \sqrt{9 - \frac{y^2}{9}} + 1$ D. $x = \sqrt{9 - \frac{y^2}{9}} - 1$

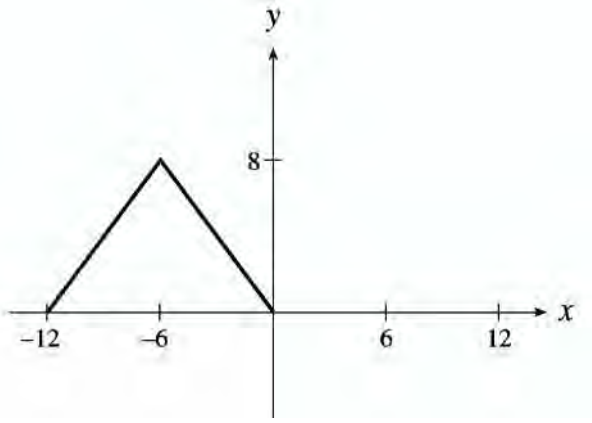
98. The zeros of the function $y = f(x)$ are -4, 1 and 2. Determine the zeros of the new function $y = -f(x - 1)$.

A. $-5,\ 0,\ 1$ B. $-3,\ -2,\ 3$ C. $-3,\ 2,\ 3$ D. $-1,\ 0,\ 5$

99. Given the function $f(x) = \frac{3x}{x+1}$, determine the equation of the inverse function $f^{-1}(x)$.

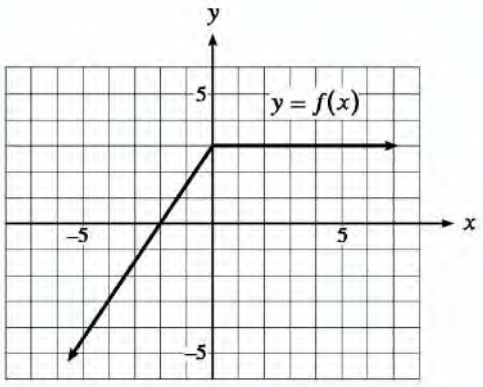
100. The function $y = f(x)$ is graphed to the left below. Determine an equation of the function shown on the right.





A. $y = f(2x)$ B. $y = f(2x + 6)$ C. $y = f(2x - 6)$ D. $y = f(2x + 12)$

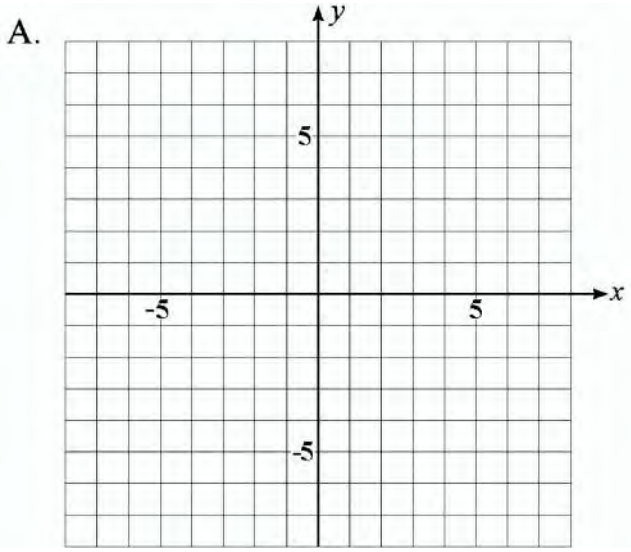
101. The graph of $y = f(x)$ is shown below.



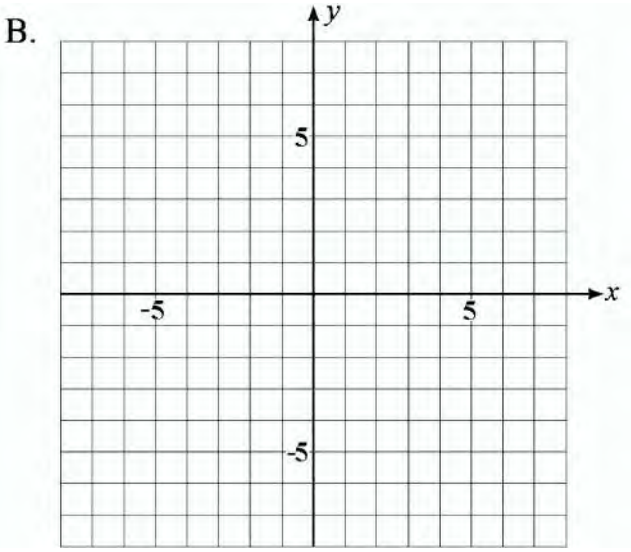
On the grid provided, sketch the graphs of

A. $y = -f(x+3)$ B. $y = 2|f(x)| - 3$

A.



B.



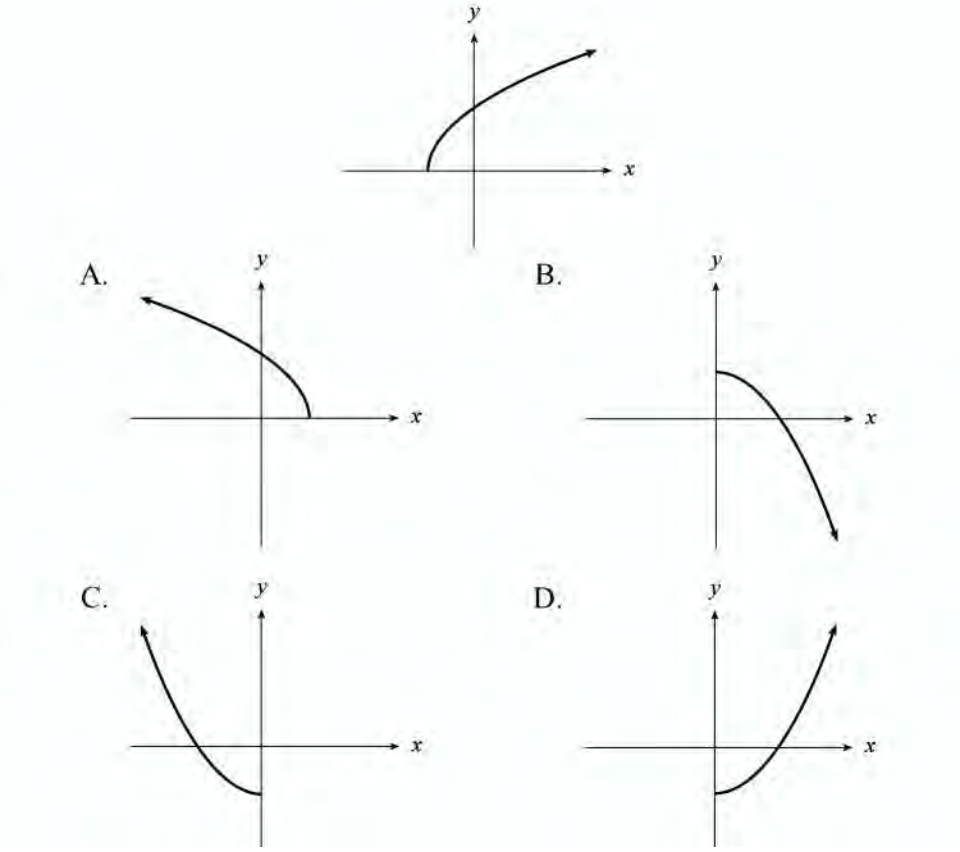
102. If the graph of $x^2 + y^2 = 4$ is compressed vertically by a factor of 2, which of the following equations represents this transformation?

A. $4x^2 + y^2 = 4$ B. $x^2 + 4y^2 = 4$ C. $x^2 + 2y^2 = 4$ D. $2x^2 + y^2 = 4$

103. The point $(-2, 6)$ is on the graph of $y = f(x)$. Which of the following points must be on the graph of $y = \frac{1}{3} f(2(x-1))$?

A. (0, 2) B. (−6, 2) C. (−3, 18) D. (−5, 18)

104. Which graph best represents the inverse relation of the graph shown?



105. Determine the inverse of $f(x) = x - 2$.

A. $f^{-1}(x) = x + 2$ B. $f^{-1}(x) = \frac{1}{x} - \frac{1}{2}$ C. $f^{-1}(x) = -\frac{x}{2}$ D. $f^{-1}(x) = \frac{1}{x - 2}$

106. Determine $Q^{-1}(t)$ if $Q(t) = \frac{C}{4t - 1}$, and C is a non-zero constant.

A. $Q^{-1} = \frac{C}{4}t + C$ B. $Q^{-1} = \frac{4t - 1}{C}$

C. $Q^{-1} = \frac{C + t}{4t}$, $t \neq 0$ D. $Q^{-1} = \frac{C - 4t}{t}$, $t \neq 0$

107. Suppose $g(4) = 30$ means the volume of water in a container is 30 mL when the depth of the water is 4 cm. What is the meaning of $g^{-1}(50) = 10$?

A. The volume of the water is 10 mL when the depth of the water is 50 cm.

B. The depth of the water is 10 cm when the volume of the water is 50 mL.

C. The depth of the water is 0.2 cm when the volume of the water is 50 mL.

D. The volume of the water is 5 mL when the depth of the water is 10 cm.

108. The function $y = f(x)$ has a domain of $[-3, 15]$ and a range of $[-5, 12]$. Determine the range for each of the following:

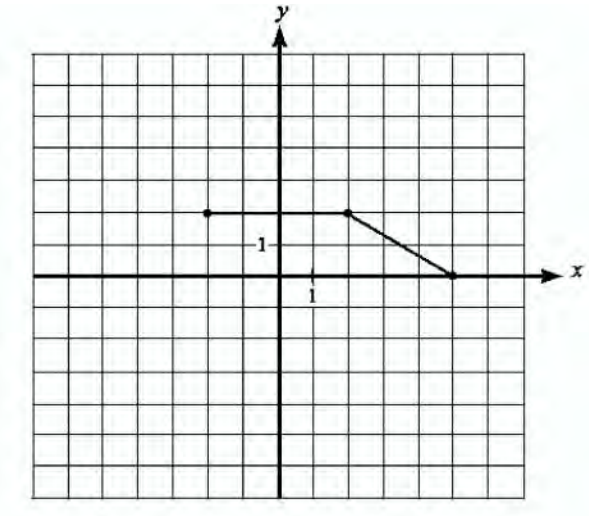
a) $y = |f(x)|$

b) $y = \sqrt{f(x)}$

c) $y = f^{-1}(x)$

d) $y = \dfrac{1}{f(x)}$

109. The graph of $y = 2f(x - 1)$ is sketched below.

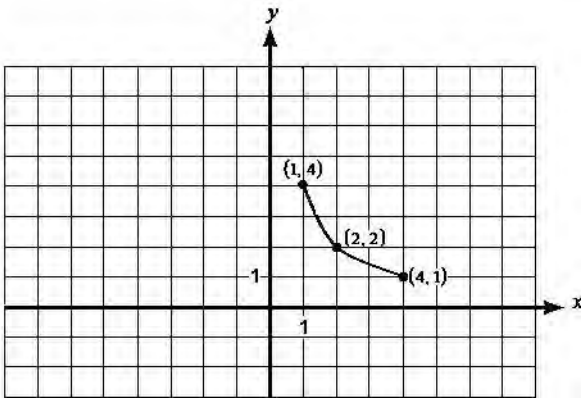


On the same grid, sketch a clearly labelled graph of $y = f(x)$.

110. For which of the following functions is $f(x) = f^{-1}(x)$, where $f^{-1}(x)$ is the inverse function of $f(x)$?

A. $f(x) = x^2$ B. $f(x) = \dfrac{1}{x}$ C. $f(x) = |x|$ D. $f(x) = \log x$

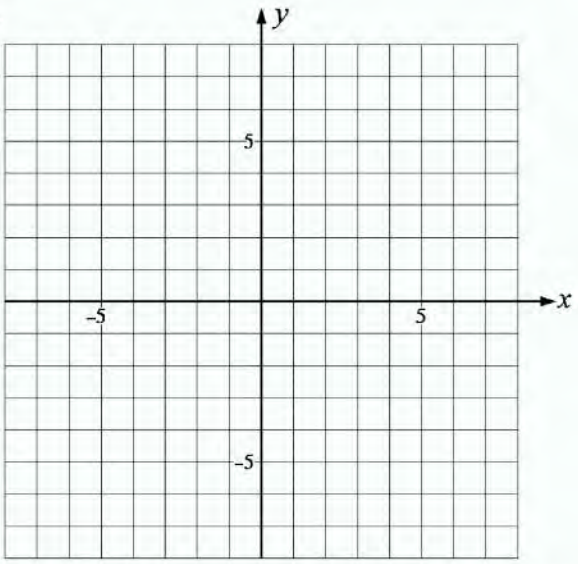
111. The graph of $y = f(x)$ is shown below.



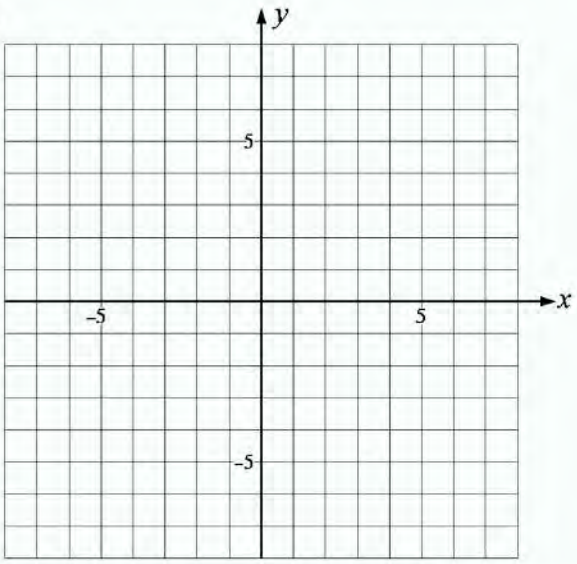
On the grids provided, sketch the graphs of

A. $y = -f(x+1)$ B. $y = \dfrac{1}{f(x)}$ C. $y = 2f(2x)$

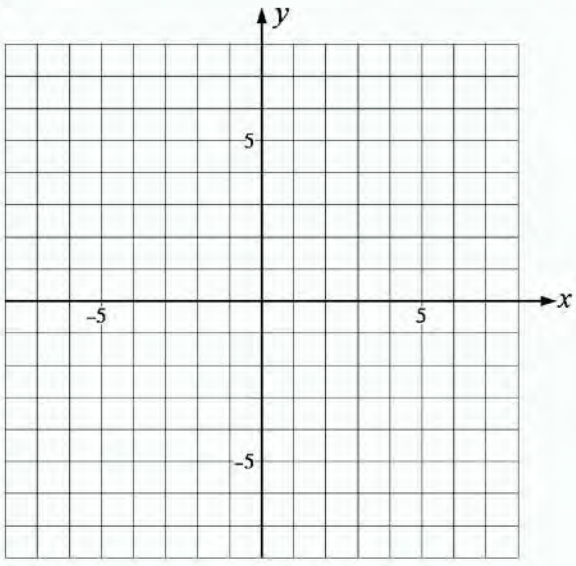
A.



B.



C.

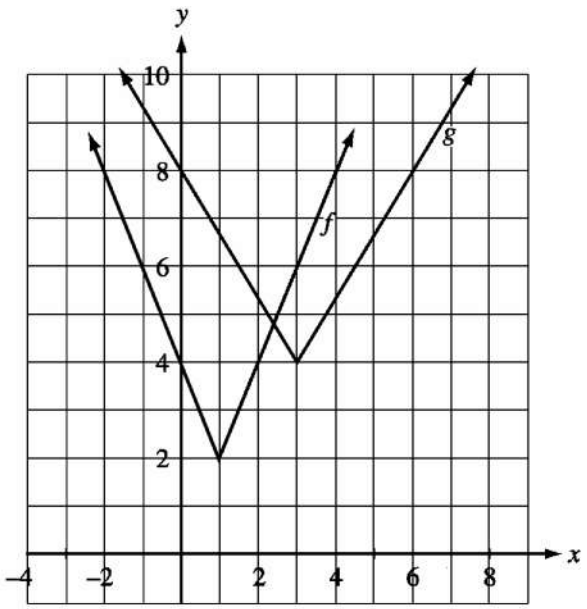


112. Given the functions $f(x) = |x - 2| + 3$ and $g(x) = |x + 2| + 1$, determine the correct set of translations that will transform $y = f(x)$ into $y = g(x)$.

A. 4 units left and 2 units down
B. 4 units right and 2 units up
C. 1 unit left and 3 units up
D. 2 units left and 4 units down

113. The graph of the function $y = f(x)$ is transformed to produce the graph of the function $y = g(x)$ as shown. Determine an equation for $y = g(x)$ in terms of $y = f(x)$.

A. $g(x) = \frac{1}{2}f(3x)$
B. $g(x) = 2f(3x)$
C. $g(x) = \frac{1}{2}f\left(\frac{x}{3}\right)$
D. $g(x) = 2f\left(\frac{x}{3}\right)$



114. The graph of $y = f(x)$ is transformed into the graph of $g(x) + 4 = 2(f(x - 3))$.
For $y = f(x)$, the domain is $[-1, 3]$ and the range is $[2, 6]$.
For $y = g(x)$, the domain is $[a, b]$ and the range is $[c, d]$.
For the graph of $y = g(x)$, the values of a, b, c, and d are, respectively

A. $-4, 0, 0, 8$
B. $2, 6, 0, 8$
C. $-4, 0, -4, 4$
D. $2, 6, -4, 4$

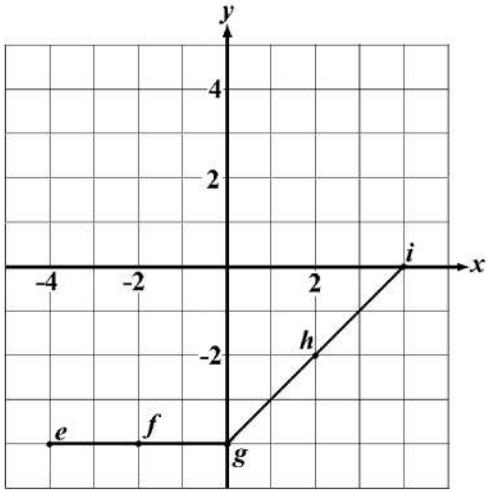
115. Consider the following transformations on the graph of $y = f(x)$.

I. $y = f(x + 2)$
II. $y = 2f(x)$
III. $y = f(-x)$
IV. $y = -f(x)$

Which transformations will have no effect on the zeros of the original graph of $y = f(x)$?

A. I and II only
B. II and III only
C. II and IV only
D. III and IV only

116. The graph of $y = f(x)$ is shown below.



Determine the location of invariant points under each of the following transformations on $y = f(x)$:

A. $y = -f(x)$ B. $y = f(-x)$ C. $x = f(y)$

117. The ordered pairs below represent possible transformations of the point $P(a,\ b)$ on the graph of the function $y = f(x)$.

Point 1: $(4a,\ b)$ Point 2: $(-a,\ b)$

Point 3: $(a,\ -b)$ Point 4: $\left(a,\ \frac{b}{4}\right)$

Point 5: $\left(\frac{a}{4},\ b\right)$ Point 6: $(a,\ 4b)$

Match each of the following single transformations with the correct ordered pairs of the corresponding point P on the new graph.

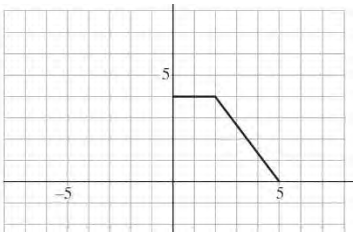
a) $y = -f(x)$ b) $y = f\left(\frac{1}{4}x\right)$ c) $y = \frac{1}{4}f(x)$ d) $y = f(-x)$

118. Given $f(x) = -3x + 7$, evaluate $y = f^{-1}(-2)$.

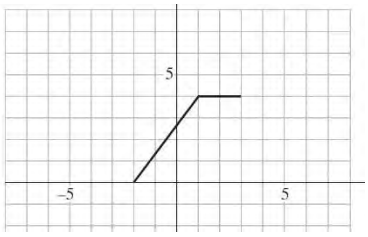
TRANSFORMATIONS ANSWER KEY

1 D
2 C
3 A
4 B
5 D
6 A
7 C
8 B

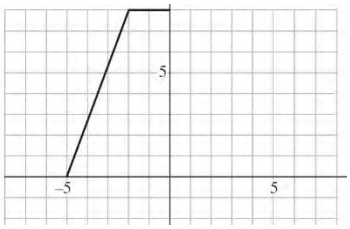
9a)



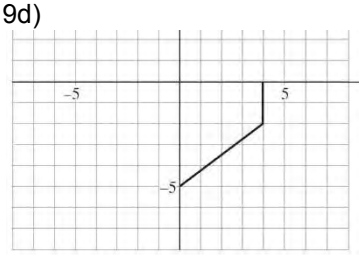
9b)



9c)

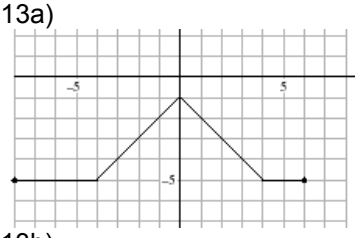


9d)

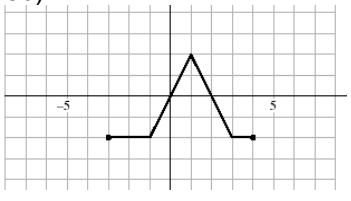


10 B
11 C
12 B

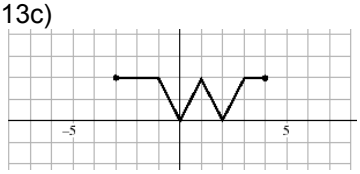
13a)



13b)

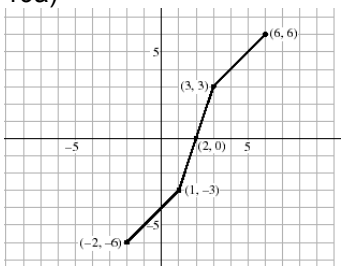


13c)

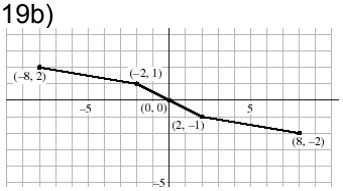


14 A
15 A
16 A
17 A
18 B

19a)

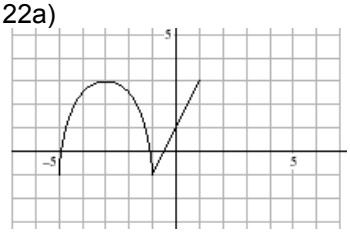


19b)

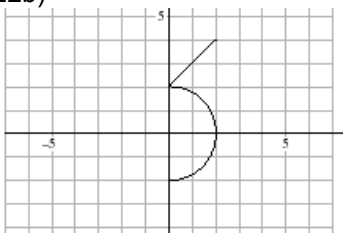


20 D
21 A

22a)

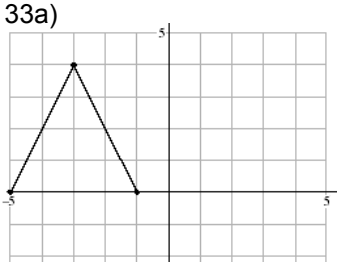


22b)

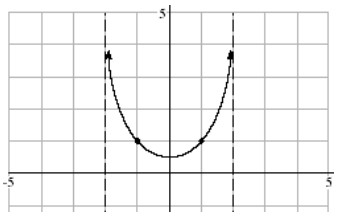


23 A
24 B
25 A
26 D
27 A
28 A
29 A
30 B
31 C
32 B

33a)

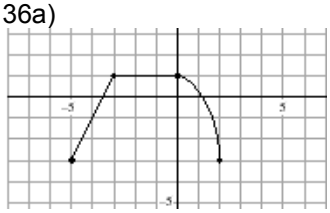


33b)

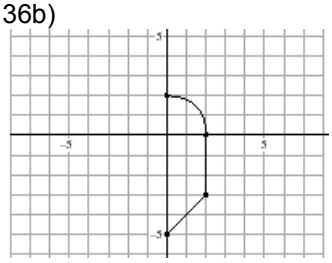


34 B
35 B

36a)

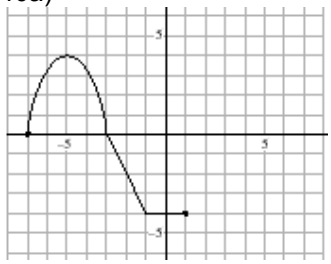


36b)

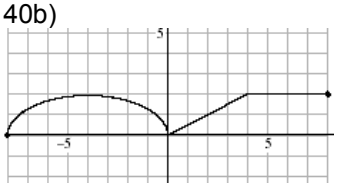


37 A
38 A
39 C

40a)

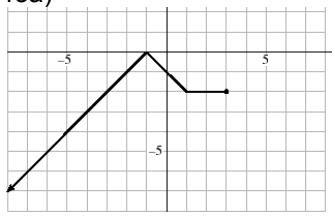


40b)

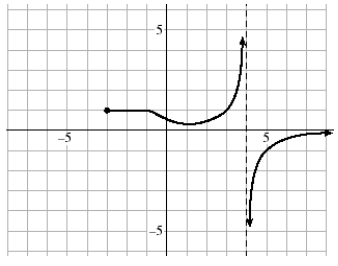


41 A
42 D

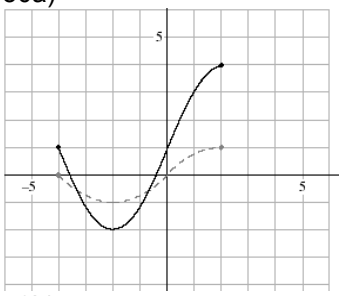
43a)



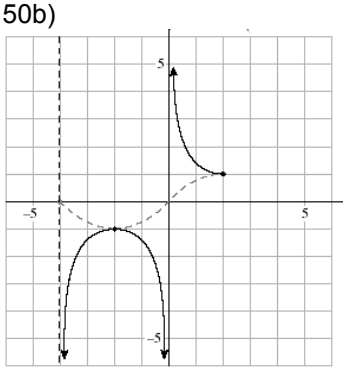
43b)



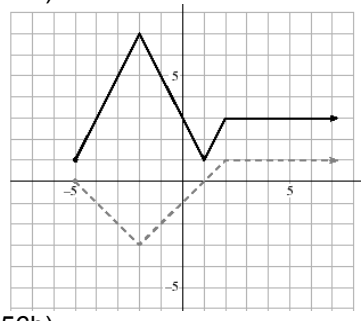
44 C
45 D
46 A
47 B
48 D
49 D
50a)



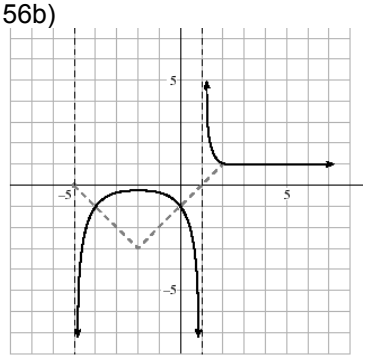
50b)



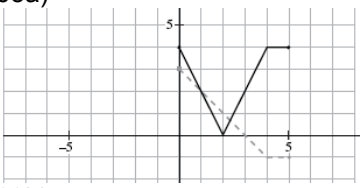
51 A
52 B
53 B
54 A
55 B
56a)



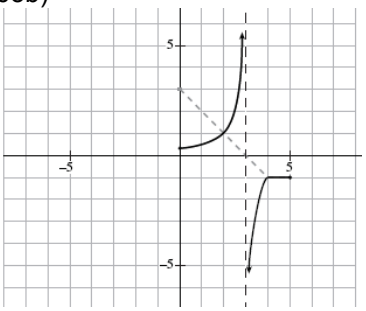
56b)



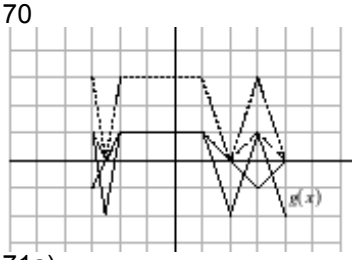
57 C
58 A
59 A
60 B
61 A
62 C
63a)



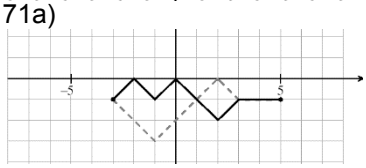
63b)



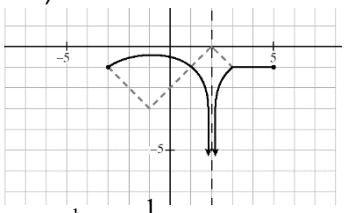
64 C
65 B
66 A
67 A
68 C
69 B
70



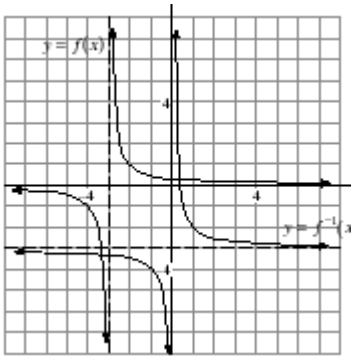
71a)



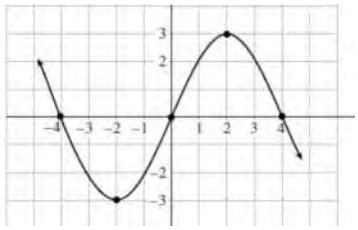
71b)



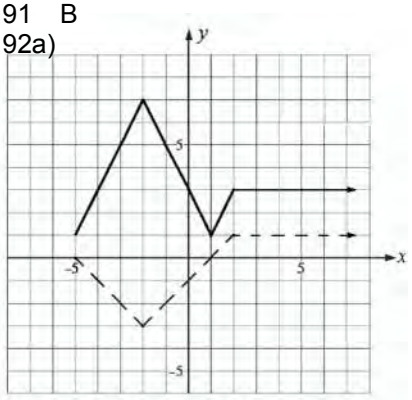
72 $f^{-1}(x) = \frac{1}{x} - 3$



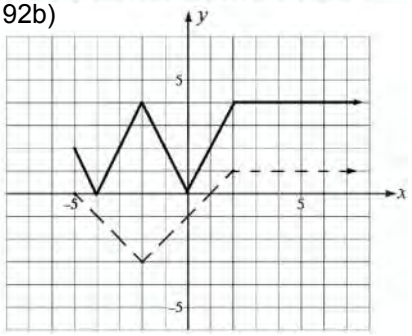
73 D
74 C
75 A
76 D
77 C
78 B
79 B
80 A
81 A
82 B
83 B
84 C
85 B
86 C
87 C
88 D
89



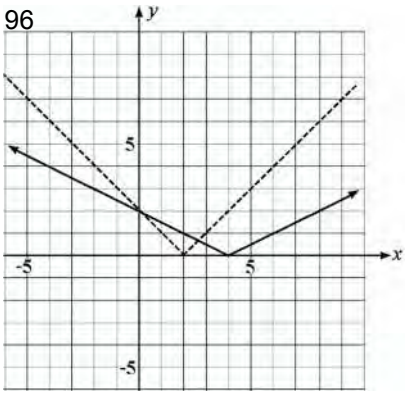
90 C
91 B
92a)



92b)



93 C
94 A
95 C
96

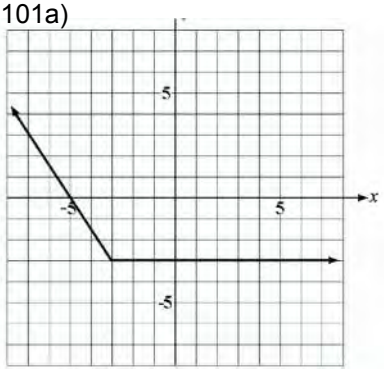


97 A
98 C

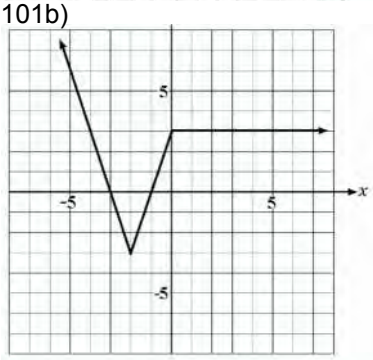
99 $f^{-1}(x) = \dfrac{x}{3-x}$

100 D

101a)



101b)



102 B
103 A
104 D
105 A
106 C
107 B

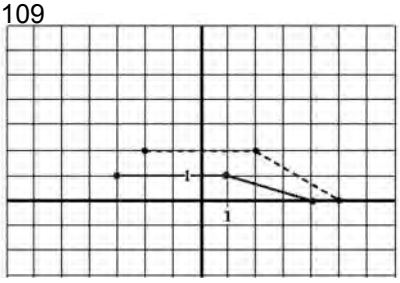
108 a) $0 \le y \le 12$

b) $0 \le y \le \sqrt{12}$

c) $-3 \le y \le 15$

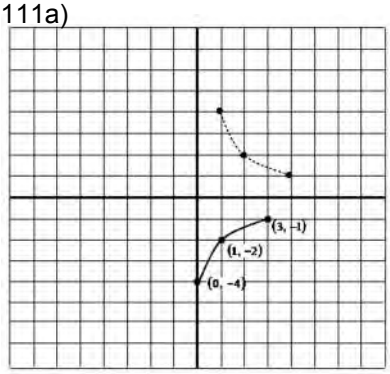
d) $y \le -\dfrac{1}{5}$ or $y \ge \dfrac{1}{12}$

109

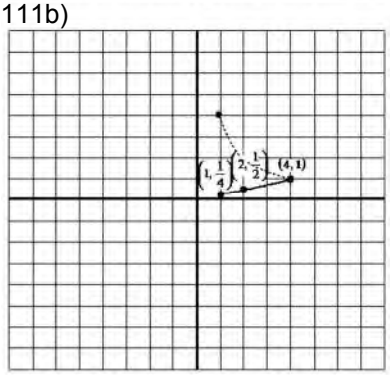


110 B

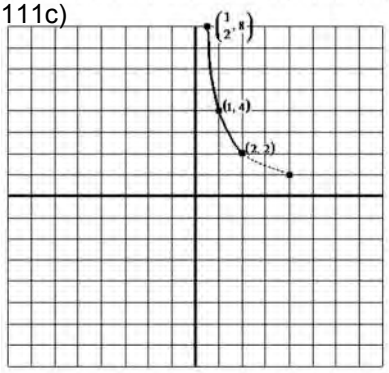
111a)



111b)



111c)



112 A
113 D
114 B
115 C
116 a) i b) g c) e
117 a) 3 b) 1 c) 4 d) 2
118 3