# PRECALCULUS MATHEMATICS 12

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## **COMBINING FUNCTIONS**

Review of basic functions;

Addition, subtraction, multiplication and division of functions; composition of functions.

• 48 PROBLEMS

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• 118 PROBLEMS

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## **TRIGONOMETRY I**

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- 165 PROBLEMS
- REVIEW PACKAGE

## **TRIGONOMETRY II (EQUATIONS and IDENTITIES)**

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Solving second-degree equations (over restricted domains and all real numbers).

- 135 PROBLEMS
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Transformations and applications.

• 194 PROBLEMS

## **GEOMETRIC SEQUENCES and SERIES**

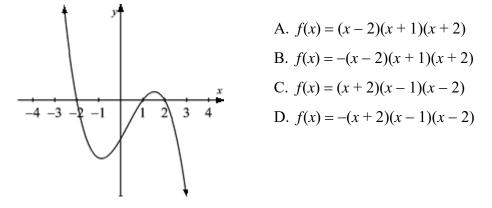
Common ratio, first term, general term;

Infinite geometric series and summation notation.

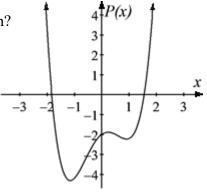
• 121 PROBLEMS

## JAN 1991

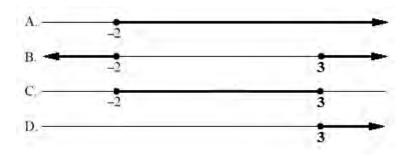
1. Which one of the following functions could describe the graph illustrated?



- 2. If 2x + 1 is a factor of a polynomial P(x), which of the following must have a value of zero? A. P(1) B. P(-1) C.  $P\left(\frac{1}{2}\right)$  D.  $P\left(-\frac{1}{2}\right)$
- 3. Which of the following approximates the zeros of the function shown?
  - A. -2.2, 1.6 B. -1.8, 1.6 C. -2.2, -2, 1.6 D. -1.8, -2, 1.6



4. Which graph below illustrates the solution set for the inequality  $(x+2)(x-3)^2 \ge 0$ ?



Page 2

- 5. Given the graph of the function y = P(x), how many positive zeros does the function y = P(x-2) 1 have?
  - A. 0 B. 2 C. 3 D. 4
- 6. Solve:  $3x^3 2x^2 7x 2 = 0$

#### **JUNE 1991**

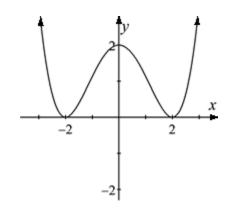
- 7. Which of the following is a possible root of the equation  $4x^4 + 2x^3 + kx + 7 = 0$ , where k is an integer ?
  - A. 2 B. 4 C.  $\frac{7}{2}$  D.  $\frac{2}{7}$
- 8. Given a polynomial P(x), what condition must be true for x 2 to be factor of P(x)?

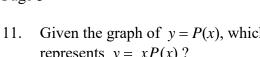
A. P(2) = 0 B. P(-2) = 0 C. P(x) = 2 D. P(x) = -2

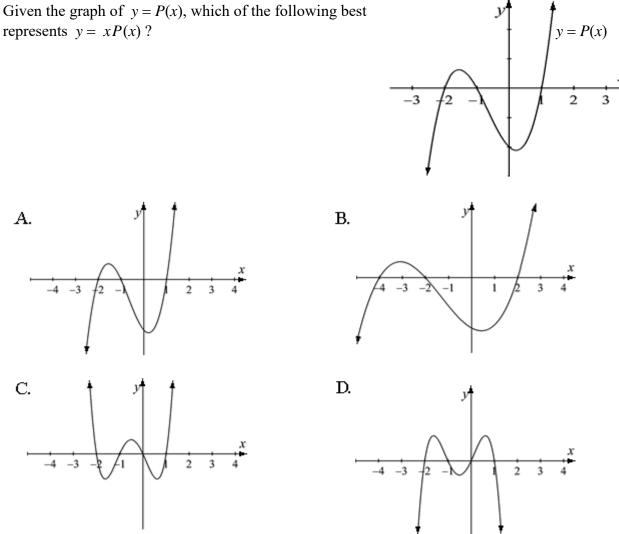
9. What is the quotient when  $5x^3 - 6x^2 + 64$  is divided by x + 2?

A.  $5x^2 + 4x + 8$ B.  $5x^2 - 16x + 32$ C.  $5x^2 + 4x + 72$ D.  $5x^2 - 16x + 96$ 

10. Select a cubic equation with roots -1, 1 and  $\frac{2}{3}$ : A.  $2x^3 + 3x^2 - 2x - 3$ B.  $2x^3 - 3x^2 - 2x + 3$ C.  $3x^3 + 2x^2 - 3x - 2$ D.  $3x^3 - 2x^2 - 3x + 2$ 







## JAN 1992

If x + 7 is a factor of a polynomial p(x), which of the following must be true? 12.

C. p(-7) = 0B. p(7) = 0D. p(x) = -7A. p(x) = 0

Using the Rational Zero Theorem, determine all possible rational roots of  $2x^3 + x^2 - 5x + 3 = 0$ . 13.

A. 
$$\pm 1, \pm 2$$
 B.  $\pm 1, \pm 2, \pm 3$  C.  $\pm 1, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm 3$  D.  $\pm 1, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm 2$ 

- What is the remainder when  $x^{21} 1$  is divided by x + 1? 14.
  - B. –2 A. -22 C. –1 D. 0

#### Page 3

- 15. What is the minimum degree of the polynomial function shown?
  - A. 1 B. 2 C. 3 D. 4
- 16. Which of the following is a factor of  $x^3 + 5x^2 + 2x 8$ ?
  - A.  $x^2 + 6x + 8$  B.  $x^2 + 3x + 2$  C. x 2 D. x 4
- 17. Determine a polynomial inequality whose solution is graphed below:

4	-3	-2	-1	0	1	2	3	
A. $(x-1)(x+2)^2 =$	≤0			В. (	(x+1)(	(x-2)	$^2 \leq 0$	
C. $(x+1)(x-2)^2 \ge 1$	≥0		Γ	<b>D</b> . ( <i>x</i> -	-1)( <i>x</i> -	$(-2)^2 \ge$	: 0	

18. Determine all real roots of the equation  $x^3 + x^2 - 5x - 5 = 0$ .

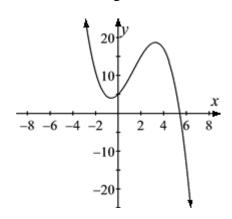
#### **JUNE 1992**

19. Let p(x) be a polynomial such that p(-3) = 0. Which of the following must be a factor of p(x)?

A. x B. x-3 C. x+3 D.  $x^2-9$ 

20. Determine all possible rational roots of  $2x^3 - 5x^2 + 3x - 5 = 0$ .

A. 
$$\pm 1, \pm 2$$
 B.  $\pm 1, \pm 5$  C.  $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$  D.  $\pm 1, \pm 2, \pm \frac{1}{5}, \pm \frac{2}{5}$ 

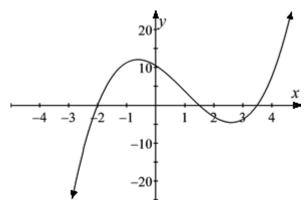


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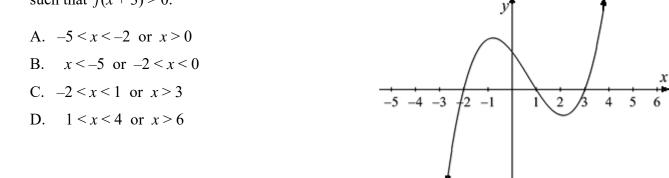
21. Estimate the real zeros of the function shown:

A. 10

- B. -2, 1.5, 3.5 C. 2, -1.5, -3.5
- D. 10, -2, 1.5, 3.5



- 22. Determine the remainder when  $p(x) = x^{28} 2x^5 + 3$  is divided by x + 1.
  - A. 2 B. 3 C. 4 D. 6
- 23. Using the graph of the polynomial function f(x) shown, determine all values of x such that f(x+3) > 0.

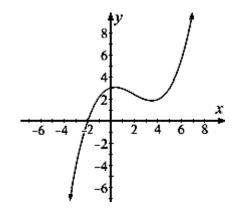


24. A polynomial function p(x), of degree 3, has the real zeros -2, 1 and 4, and a *y*-intercept of 24. Determine the value of p(6).

## JAN 1993

25. Determine a real zero of the function shown:

A. -2 B. 2 C. 3 D. 4



26. Determine all possible rational roots of  $6x^3 - 5x^2 - 7x - 3 = 0$ .

A. 
$$\pm 1, \pm 3$$
B.  $\pm 1, \pm 2, \pm 3, \pm 6$ C.  $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm 1, \pm 2, \pm 3, \pm 6$ D.  $\pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 3$ 

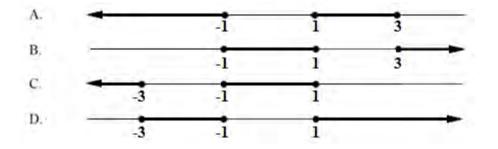
- 27. Determine the remainder if  $x^3 2x^2 + 3x 7$  is divided by x + 1.
  - A. -13 B. -9 C. -5 D. 5
- 28. Solve:  $x^3 + 5x^2 + 6x = 0$

A. -2, -3 B. 1, -6 C. 0, -2, -3 D. 0, 1, -6

29. Determine a polynomial equation that has roots  $\pm 3$  and 2.

A.  $x^3 - 2x^2 - 9x + 18 = 0$ B.  $x^3 + 2x^2 - 9x - 18 = 0$ C.  $x^3 + 2x^2 - 9x + 18 = 0$ D.  $x^3 - 2x^2 + 9x - 18 = 0$ 

- 30. How many real roots are there for the polynomial equation  $x(x^2 4)(x + 3)(x^2 + 5) = 0$ ?
  - A. 3 B. 4 C. 5 D. 6
- 31. Graph the solution of the inequality for  $(x-3)(x+1)(x-1) \ge 0$



- 32. If the graph of the polynomial function shown is of the form  $y = ax^3 + bx^2 + cx + d$  (where *a*, *b*, *c*, and *d* are constants), what are the conditions on *a* and *d*?
  - A.  $a > 0, d \neq 0$ B. a > 0, d = 0C.  $a < 0, d \neq 0$ D. a < 0, d = 0

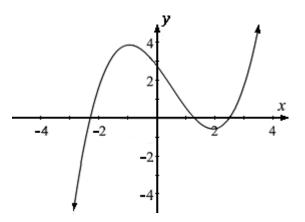
#### **JUNE 1993**

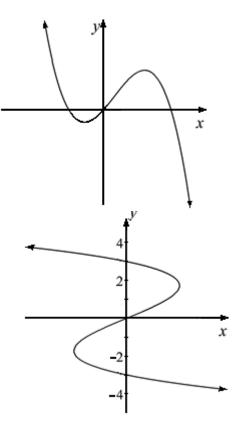
- 33. Determine a possible equation of the inverse of the relation shown.
  - A.  $y = x^3 3x$ B.  $y = 3x - x^3$ C.  $y = x^3 - 9x$ D.  $y = 9x - x^3$
- 34. If a polynomial p(x) is divided by x + 5, what is the remainder?
  - A. p(-5) B. p(5) C. p(x-5) D. p(x+5)
- 35. Estimate the real zeros of the function shown:
  - A. -2.3, 1.3, 2.5
    B. 2.3, -1.3, -2.5
    C. -2.7, 1.3, 2.5
  - D. 2.7, -1.3, -2.5
- 36. Determine a polynomial equation that has roots  $\sqrt{2}$ ,  $-\sqrt{2}$  and 1.

A. $x^3 - x^2 - 4x + 4 = 0$	B. $x^3 + x^2 - 4x - 4 = 0$
C. $x^3 + x^2 - 2x - 2 = 0$	D. $x^3 - x^2 - 2x + 2 = 0$

37. Determine the remainder when  $6x^3 - 11x^2 + 14x - 5$  is divided by  $2x^2 - 7x + 3$ .

A. -107x - 53 B. -107x + 43 C. 40x - 20 D. 20







38. What is the minimum degree of a polynomial inequality whose solution is shown below?

	- <u>3</u>	1	0 9	-
A. 3	B. 4	C. 5	D. 6	

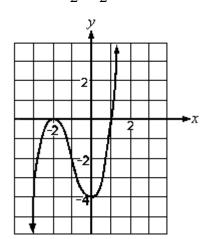
39. Solve:  $2x^3 + 3x^2 - 11x - 6 = 0$ 

#### **JAN 1994**

40. According to the Rational Root Theorem, what are the possible rational roots of  $2x^4 + 3x^2 - 7x + 3 = 0$ ?

A.  $\pm 1, \pm 3$  B.  $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$  C.  $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$  D.  $\pm 1, \pm 2, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$ 

- 41. Which equation could represent the following graph?
  - A.  $y = (x + 2)^2 (x 1)$ B.  $y = (x + 2)^2 (x + 1)$ C.  $y = (x - 2)^2 (x - 1)$ D.  $y = (x - 2)^2 (x + 1)$



42. Determine the quotient when  $x^3 - 2x^2 - 9$  is divided by x - 3.

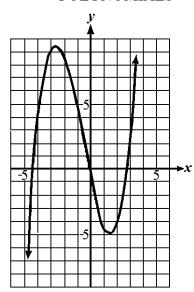
A.  $x^2 + 5x + 15$  B.  $x^2 + x - 6$  C.  $x^2 - 5x + 6$  D.  $x^2 + x + 3$ 

43. What value of k would make x + 2 a factor of  $2x^3 - 5x^2 - 2kx + 8$ ?

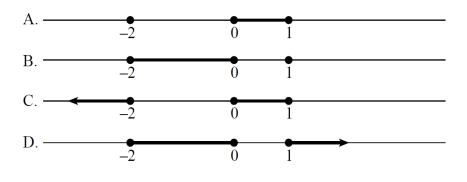
A. -7 B. -1 C. 1 D. 7

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- 44. From the graph of y = f(x) shown, find the approximate solutions to f(x) = 2.
  - A. -4.2
  - B. 8.6
  - C. -4.5, 0.3, 2.5
  - D. -4.2, -0.5, 3.0



45. Determine the graph of the solution set of the inequality  $x(x-1)^n (x+2)^m \le 0$ , if *n* is an even positive integer and *m* is an odd positive integer.



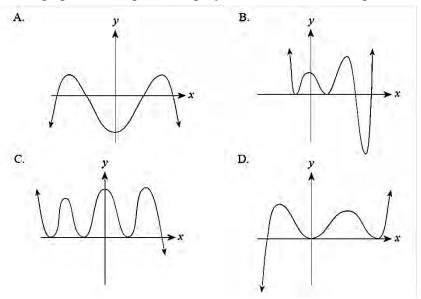
46. Solve:  $2x^3 - x^2 - 8x + 4 = 0$ 

#### **JUNE 1994**

47. According to the Rational Root Theorem, which one of the following is a possible root of the equation  $8x^4 + 19x^3 - 13x^2 + 7x - 3 = 0$ ?

A. 2 B. 3 C. 4 D. 8

48. Which graph could represent a polynomial function of degree 5?



49. When  $4x^2 + 2kx - 5$  is divided by x + 2 the remainder is 3. What is the value of k?

- A. -6 B. -2 C. 2 D.  $\frac{11}{4}$
- 50. Solve:  $x^3 2x^2 5x + 6 = 0$ A. 1, 2, -3 B. 1, -2, 3 C. -1, 2, -3 D. -1, -2, 3
- 51. Determine the remainder when  $p(x) = 4x^3 6x^2 + 4x 3$  is divided by 2x 1.
  - A. -7 B. -4 C. -3 D. -2
- 52. Determine a polynomial equation that has roots of  $\sqrt{3}$ ,  $-\sqrt{3}$  and 2.

A. $x^3 - 2x^2 - 3x + 6 = 0$	B. $x^3 + 2x^2 - 3x - 6 = 0$
C. $x^3 - 2x^2 - 9x + 18 = 0$	D. $x^3 + 2x^2 - 9x - 18 = 0$

53. Which polynomial inequality has the solution -3 < x < -2 or x > 1?

A. (x+3)(x+2)(x-1) < 0B. (x+3)(x+2)(x-1) > 0C. (x-3)(x-2)(x+1) < 0D. (x-3)(x-2)(x+1) > 0

 $\rightarrow x$ 

(-2,4)

B. 
$$y = (x + 2)^{2}(x - 1)$$
  
C.  $y = 2(x + 2)^{2}(x + 1)$   
D.  $y = 2(x + 2)^{2}(x - 1)$   
JAN 1995  
5. Determine the minimum degree of the polynomial function shown.  
A. 3 B. 4 C. 5 D. 6

55

The graph of a cubic polynomial function, y = f(x), is shown.

Determine the equation of y = f(x) - 4.

A.  $y = (x+2)^2(x+1)$ 

56. If  $p(x) = x^3 - 3x^2 + kx + 1$ , determine k if p(3) = -5. C. 4

B. –2

A. –12

57.

Determine all real roots of the equation  $(x^2 - 4)(x^2 + 9)(x - 5)^2 = 0$ .

A. 2, 3, 5 B. ±2, 5 C.  $\pm 2, \pm 3, 5$ D.  $\pm 2, \pm 3, \pm 5$ 

The polynomial equation  $x^3 - ax^2 + bx - c = 0$ , where a, b and c are integers, has 6 as one of its 58. roots. According to the Rational Root Theorem, which of the following could be a value of c?

D. 16

A. 2 B. 3 C. 9 D. 18

Determine the quotient and remainder:  $(t^4 + 3t^3 + 5t^2 + 21t - 14) \div (t^2 + 3t - 2)$ 59.

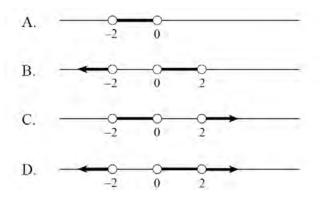
B. quotient:  $t^2 + 7$ , remainder: -28A. quotient:  $t^2 + 7$ , remainder: 0 C. quotient:  $t^2 + 3$ , remainder: 12t - 8 D. quotient:  $t^2 + 3$ , remainder: 30t - 20

Determine the remainder when  $x^{39} - 3x^{15} - 2x + 1$  is divided by x - 1. 60.

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54.

61. Solve the inequality:  $x(x-2)(x^2-4) < 0$ 



62a. A sheet of paper 12 cm long and 8 cm wide is used to make a box with no lid. Equal squares of side length x cm are cut from each of the corners and the sides are folded up to make the box. Which of the following expresses the volume of the box?

A. $V(x) = x(12 + x)(8 + x)$	B. $V(x) = x(12 - x)(8 - x)$
C. $V(x) = x(12 + 2x)(8 + 2x)$	D. $V(x) = x(12 - 2x)(8 - 2x)$

62b. A square piece of cardboard 10 cm by 10 cm will have equal squares with sides of length x cm cut from each corner. The sides will then be folded up to create a box with no top. Determine the value of x that will give the box a maximum volume.

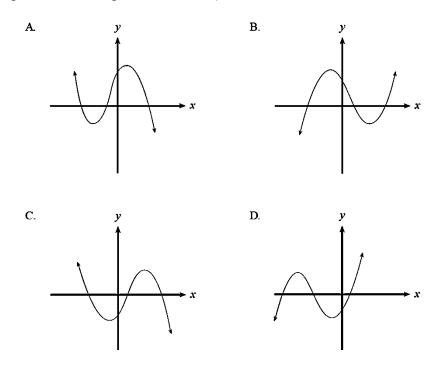
#### **JUNE 1995**

- 63. If x + 8 is a factor of the polynomial P(x), which of the following must be true?
  - A. P(-8) = 0 B. P(8) = 0 C. P(x) = 8 D. P(x) = -8
- 64. What is the maximum number of real roots that a polynomial equation can have if its degree is 6?
  - A. 3 B. 5 C. 6 D. 7

- 65. According to the Rational Zero Theorem, which number is a possible zero of the function  $f(x) = 6x^3 + 7x^2 3x + 4$ ?
  - A.  $-\frac{3}{2}$  B.  $\frac{1}{4}$  C.  $\frac{1}{3}$  D. 3
- 66. Determine the remainder when  $2x^4 + 4x^3 5x^2 + 8$  is divided by x 2.

A. -12 B. 18 C. 30 D. 52

67. Which graph is the best representation of  $y = ax^3 + bx^2 + cx - 24$  where a > 0?



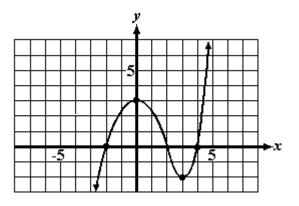
68. Determine all the real zeros of the function  $P(x) = 2x(x^2+9)(x^2-2)$ .

A.  $0, \pm \sqrt{2}$  B.  $0, \pm 3$  C.  $0, \sqrt{2}, 3$  D.  $0, \pm \sqrt{2}, \pm 3$ 

69. Solve the inequality:  $(x + 2)^2(x - 2)(x - 4) < 0$ A. x < -2 B. -2 < x < 4 C. 2 < x < 4 D. x < 2 or x > 4

70. The graph of the function f(x) is shown. If g(x) = 3 f(x), determine the zeros of g(x).

A2, 2, 4	B6, 6, 12
С6, 9	D2, 3



#### **JAN 1996**

71. According to the Rational Root Theorem, determine all possible rational roots of  $3x^3 - 8x^2 + 16x - 4 = 0$ .

A.	$\pm 1, \pm 3$	B. $\pm 1, \pm 2, \pm 4$
C.	$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$	D. $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}$

72. If p(x) is a polynomial function where p(-2) = 5, then which of the following could not be a zero of this function?

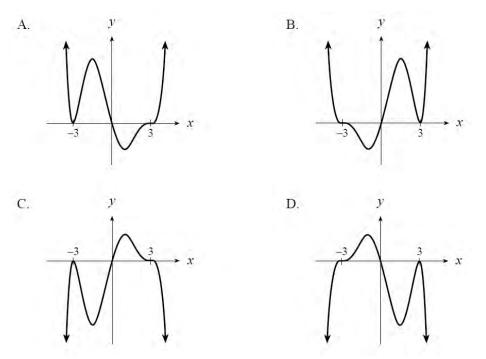
A. -5 B. -2 C. 2 D. 5

73. Find the remainder when  $x^3 - 2x^2 + 5$  is divided by  $x^2 + x - 1$ .

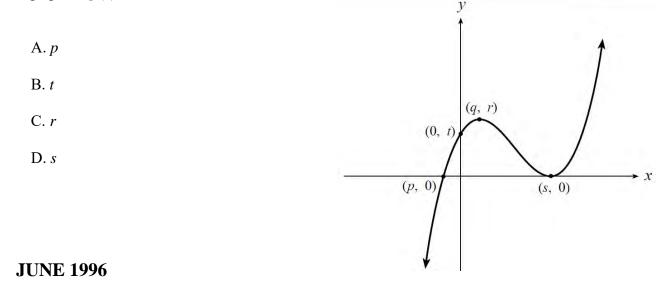
A. 4 B. 2x + 2 C. 2x + 4 D. 4x + 2

- 74. Determine the value of k such that x + 2 is a factor of the polynomial  $2x^3 + 5x^2 + kx 12$ .
  - A. -12 B. -4 C. 4 D. 12

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- 75. Which graph best represents  $y = -x(x+3)^2 (x-3)^3$ ?



76. Given that p(x) and f(x) are polynomial functions such that p(x) = x f(x) + c, determine c if the graph of p(x) is shown.



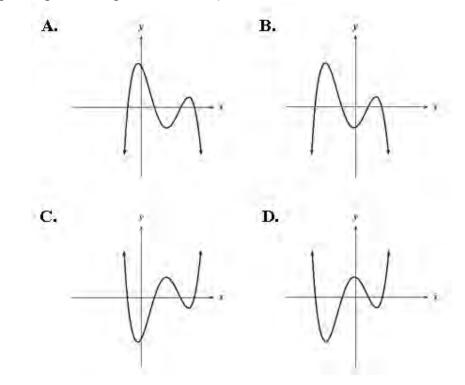
- 77. If the polynomial p(x) is divided by x 6, which of the following represents the remainder?
  - A. p(6) B. p(-6) C. p(x) + 6 D. p(0)
- 78. Determine the value of k if 2 is a zero of the function  $p(x) = x^3 7x^2 + kx + 12$ .

A. k = -16 B. k = 4 C. k = 5 D. k = 16

79. Determine the quotient when  $x^3 - 12x^2 + 9x - 5$  is divided by x - 3.

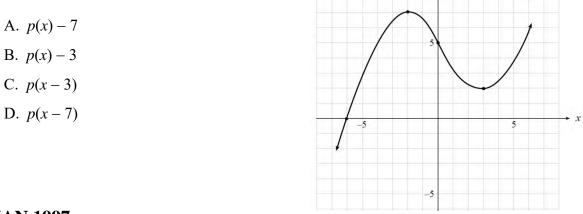
A. 
$$x^2 - 9x - 16$$
 B.  $x^2 - 9x - 18$  C.  $x^2 - 15x + 54$  D.  $x^2 + 9x + 36$ 

- 80. If x + 4 is a factor of the polynomial  $mx^3 11x^2 10x + n$ , where *m* and *n* are integers, according to the Rational Root Theorem, which of the following could be a value for *n*?
  - A. 2 B. 6 C. 8 D. 10
- 81. Solve:  $x^3 4x^2 > 12x$ 
  - A. -2 < x < 6B. x < -2 or x > 6C. -2 < x < 0 or x > 6D. -6 < x < 0 or x > 2
- 82. Which graph is a possible representation of  $y = ax^4 + bx^3 + cx 6$ , where *a* is a negative integer?



- 83. Determine a polynomial equation that has the following roots: 2,  $\pm\sqrt{5}$ 
  - A.  $x^3 + 2x^2 5x 10 = 0$ B.  $x^3 - 2x^2 + 5x - 10 = 0$ C.  $x^3 - 2x^2 - 5x + 10 = 0$ D.  $x^3 + 2x^2 + 5x + 10 = 0$

84. The graph of the cubic polynomial function p(x) is given below. Which of the following functions must have 3 unequal real zeros?

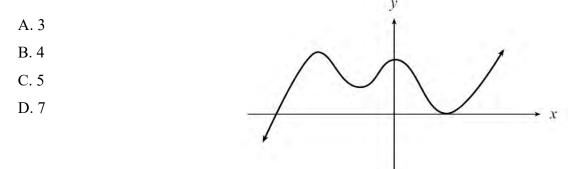


#### **JAN 1997**

85. If x + 4 is a factor of the polynomial p(x), then which of the following must be true?

A. p(-4) = 0 B. p(4) = 0 C. p(0) = -4 D. p(0) = 4

- 86. According to the Rational Root Theorem, which number could not be a root of the equation  $4x^3 + kx^2 + 3x 3 = 0$ , where k is an integer?
  - A. -3 B. -1 C.  $\frac{4}{3}$  D.  $\frac{3}{2}$
- 87. What is the minimum degree of the polynomial function graphed below?



88. When  $x^3 + x^2 - kx - 5$  is divided by x - 2, the remainder is 1. Find the value of k.

A. 3 B. 3.5 C. 4.5 D. 5

89. Determine a polynomial equation that has the roots  $\pm 2, \pm \sqrt{7}$ .

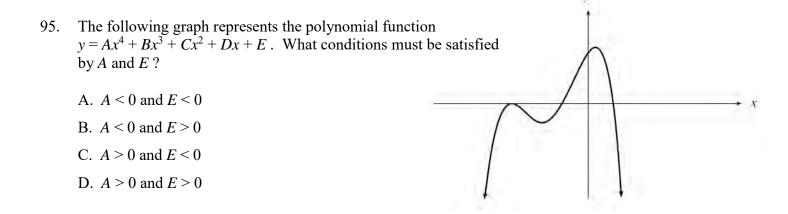
A. $x^4 - 11x^2 + 28 = 0$	B. $x^4 + 11x^2 + 28 = 0$
C. $x^4 - 9x^2 + 14 = 0$	D. $x^4 + 9x^2 + 14 = 0$

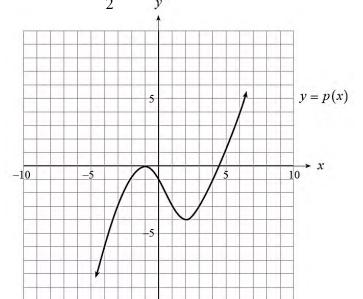
- 90. Find the remainder for the following division.  $x^2 + 2x 4\sqrt{x^4 + 2x^3 3x^2 + 2x + 6}$ 
  - A. 2 B. 10 C. 4x + 2 D. 16x 22
- 91. Determine the real root(s):  $2x^3 3x^2 + 6x 9 = 0$ 
  - A.  $-\frac{3}{2}$  B.  $\frac{3}{2}$  C.  $-\frac{3}{2}, \pm\sqrt{3}$  D.  $\frac{3}{2}, \pm\sqrt{3}_y$
- 92. Use the graph of the function y = p(x) shown to solve the equation p(x-5) + 6 = 0.
  - A. -4
  - B. -1
  - C. 1
  - D. 4

#### **JUNE 1997**

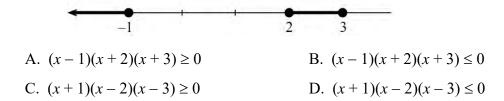
- 93. According to the Rational Root Theorem, list all possible rational roots of  $2x^8 5x^3 + 6x^2 4 = 0$ .
  - A.  $\pm 1$ ,  $\pm 2$ ,  $\pm 4$  B.  $\pm \frac{1}{2}$ ,  $\pm 1$ ,  $\pm 2$ ,  $\pm 4$  C.  $\pm \frac{1}{4}$ ,  $\pm \frac{1}{2}$ ,  $\pm 1$ ,  $\pm 2$  D.  $\pm \frac{1}{2}$ ,  $\pm 1$ ,  $\pm 2$ ,  $\pm 4$ ,  $\pm 8$
- 94. Determine the remainder when  $6x^3 11x^2 + 14x 5$  is divided by  $2x^2 7x + 3$ .

A. -107x - 53 B. -107x + 43 C. 40x - 20 D. 20





96. Which polynomial inequality describes the solution shown?



97. For the polynomial function  $p(x) = ax^3 + bx - 3$ , p(-1) = 4. Determine the value of p(1).

A. -10 B. -4 C. 4 D. 10

98. If -2 is a root of  $2x^3 + kx^2 - 11x + 6 = 0$ , determine the other two roots.

#### **JAN 1998**

- 99. Given a polynomial p(x), what condition must be true for x 2 to be a factor of p(x)?
  - A. p(2) = 0 B. p(-2) = 0 C. p(x) = 2 D. p(x) = -2

100. According to the Rational Root Theorem, determine all possible rational roots of  $2x^3 - 5x^2 + 12x - 6 = 0$ .

- A.  $\pm 1$ ,  $\pm 2$ ,  $\pm 3$ ,  $\pm 6$ B.  $\pm \frac{1}{2}$ ,  $\pm \frac{1}{3}$ ,  $\pm \frac{2}{3}$ ,  $\pm \frac{1}{6}$ C.  $\pm 1$ ,  $\pm 2$ ,  $\pm 3$ ,  $\pm 6$ ,  $\pm \frac{1}{2}$ ,  $\pm \frac{3}{2}$ D.  $\pm 1$ ,  $\pm 2$ ,  $\pm \frac{1}{2}$ ,  $\pm \frac{1}{3}$ ,  $\pm \frac{2}{3}$ ,  $\pm \frac{1}{6}$
- 101. What is the quotient when  $5x^3 6x^2 + 64$  is divided by x + 2?
  - A.  $5x^2 + 4x + 8$  B.  $5x^2 16x + 32$  C.  $5x^2 + 4x + 72$  D.  $5x^2 16x + 96$

102. Determine the remainder when  $3x^{45} + 4x^8 - 5x^3 + 2$  is divided by x + 1.

A. -10 B. -2 C. 4 D. 8

103. What is the least number of real zeros that a polynomial function can have if its degree is 5?

- A. 0 B. 1 C. 3 D. 5
- 104. Determine the real roots:  $x^3 + 3x^2 6x 8 = 0$

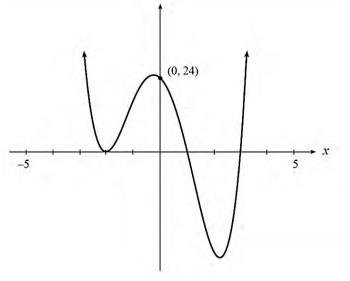
POLYNOMIALS

A. -4, -1, 2 B. -4, 1, 2 C. -2, 1, 4 D. -1, 2, 4

105. Solve the inequality: (x + 5)(x - 2)(6 - x) > 0

```
A. x < -5 or x > 6B. x < -5 or x > 2C. -5 < x < 2 or x > 6D. x < -5 or 2 < x < 6
```

- 106. Determine a possible equation of the polynomial function y = f(x) shown:
  - A.  $f(x) = 2(x+2)^2(x-1)(x-3)$
  - B.  $f(x) = 4(x+2)^2(x-1)(x-3)$
  - C.  $f(x) = 2(x-2)^2(x+1)(x+3)$
  - D. f(x) = 4(x+2)(x-1)(x-3)



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#### **JUNE 1998**

107. According to the Rational Root Theorem, determine all possible rational roots of  $5x^3 - 4x^2 + 15 = 0$ .

A. 
$$\pm 1$$
,  $\pm 5$ B.  $\pm 1$ ,  $\pm 3$ ,  $\pm 5$ ,  $\pm 15$ C.  $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{5}, \pm \frac{3}{5}$ D.  $\pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{1}{5}, \pm \frac{1}{15}$ 

108. If 3x-1 is a factor of p(x), which of the following must have a value of 0?

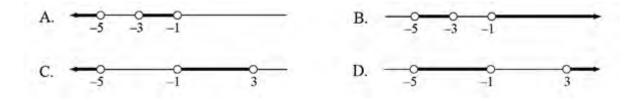
A.  $p\left(\frac{1}{3}\right)$  B.  $p\left(-\frac{1}{3}\right)$  C. p(-1) D. p(1)

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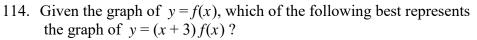
- 109. How many real roots are there for the polynomial equation  $x(x^2-4)(x^2+9) = 0$ ?
  - A. 1 B. 2 C. 3 D. 5
- 110. Factor:  $x^3 2x^2 5x + 6$ A. (x + 1)(x - 2)(x + 3)
  - A. (x + 1)(x 2)(x + 3)B. (x + 1)(x + 2)(x - 3)C. (x - 1)(x - 2)(x + 3)D. (x - 1)(x + 2)(x - 3)
- 111. Determine the quotient when  $2x^3 5x^2 + 7x + 3$  is divided by 2x + 1.

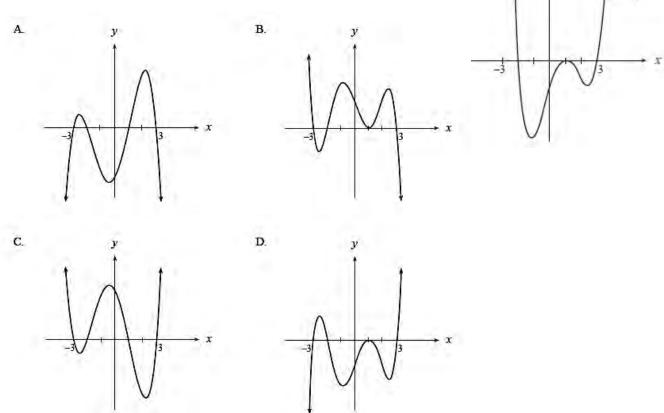
A.  $x^2 - 3x + 4$  B.  $x^2 - 3x + 5$  C.  $x^2 - 2x - 2$  D.  $x^2 - 2x + 2$ 

- 112. If the cubic polynomial function f(x) = k(x-1)(x+2)(x-3) passes through the point (2, 6), determine the value of k.
  - A.  $-\frac{3}{2}$  B.  $-\frac{2}{3}$  C.  $\frac{2}{3}$  D.  $\frac{3}{2}$
- 113. Solve: (x+5)(x+1)(3-x) < 0



y = f(x)





#### **JAN 1999**

115. If 5 is a zero of the polynomial P(x), then which of the following must be true?

A. P(x) = 5 B. P(5) = 0 C. P(0) = 5 D. P(-5) = 0

116. According to the Rational Root Theorem, determine all possible rational roots of  $4x^5 - 3x^3 + 6x - 2 = 0$ .

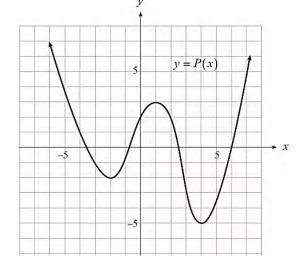
A. 
$$\pm 1, \pm 2$$
B.  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$ C.  $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$ D.  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{4}$ 

- 117. Determine the remainder when  $x^{12} 2x^7 + 6x^2 4$  is divided by x + 1.
  - A. 0 B. 1 C. 4 D. 5

118. Which of the following is a real zero of the polynomial function  $f(x) = x^3 - 3x + 3$ ?

(D) A. -2.10 B. -2.00 C. 0.82 D. 3.00

- 119. The graph of a polynomial function y = P(x) is shown below. If f(x) = P(x) + k, determine all values of k such that f(x) will have two unequal real zeros and no other real zeros.
  - A. k < -3 or k > 2B. 2 < k < 5
  - C. -2 < k < 5
  - D.  $2 \le k \le 5$  or  $k \le -3$



120. Solve:  $x^3 - 8x^2 \ge -4x + 20$ 

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#### **JUNE 1999**

121. When the polynomial p(x) is divided by x - 4, the remainder is 6. Which of the following must be true?

A. p(4) = 6 B. p(-4) = 6 C. p(6) = 4 D. p(-6) = 4

122. Solve:  $x^3 - 7x - 6 = 0$ 

A. -1, -2, 3 B. -1, 2, -3 C. 1, -2, 3 D. 1, 2, -3

123. Determine the largest root of 
$$x^3 - 30x^2 + 235x - 430 = 0$$
.  
A. 2.64 B. 8.74 C. 18.62 D. 18.75

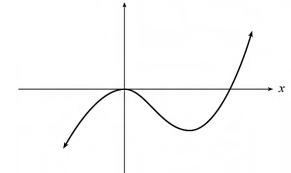
124. According to the Rational Root Theorem, which of the following equations has possible rational roots of  $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$ ?

A. 
$$3x^3 - 4x^2 + 5x + 1 = 0$$
  
B.  $6x^3 - 4x^2 + 5x + 1 = 0$   
C.  $2x^3 - 4x^2 + 5x + 3 = 0$   
D.  $3x^3 - 4x^2 + 5x + 2 = 0$ 

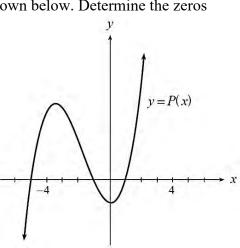
125. Which of the following is a polynomial function with zeros of  $-\sqrt{2}$ ,  $\sqrt{2}$  and -1?

A. 
$$P(x) = x^3 - x^2 - 2x + 2$$
  
B.  $P(x) = x^3 + x^2 - 2x - 2$   
C.  $P(x) = x^3 - x^2 - 4x + 4$   
D.  $P(x) = x^3 + x^2 - 4x - 4$ 

- 126. The graph of the polynomial function  $P(x) = ax^3 + bx^2 + cx + d$ , where *a*, *b*, *c* and *d* are constants, is shown. What are the conditions on *c* and *d*? *y* 
  - A. c = 0, d = 0B. c = 0, d > 0C. c > 0, d = 0D.  $c \neq 0, d = 0$



- 127. The graph of the cubic polynomial function y = P(x) is shown below. Determine the zeros of y = xP(-x).
  - A. -5, -1, 0, 1
    B. -5, -1, 1
    C. -1, 0, 1, 5
    D. -1, 1, 5



128. Solve:  $x^3 - 8x^2 > 18x - 20$ 

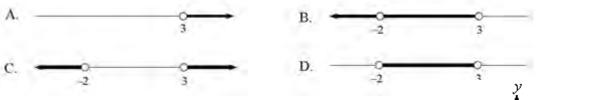
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#### **JAN 2000**

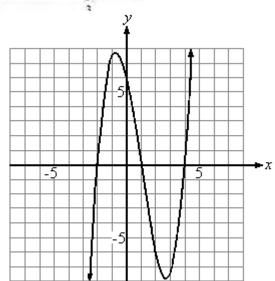
- 129. According to the Rational Root Theorem, determine all possible rational roots of  $5x^3 3x^2 + x 2 = 0$ .
  - A.  $\pm 1$ ,  $\pm 2$  B.  $\pm 1$ ,  $\pm 5$  C.  $\pm 1$ ,  $\pm 2$ ,  $\pm \frac{1}{2}$ ,  $\pm \frac{5}{2}$  D.  $\pm 1$ ,  $\pm 2$ ,  $\pm \frac{1}{5}$ ,  $\pm \frac{2}{5}$

130. How many different real roots are there for the polynomial equation  $x(x-3)(x^2+6) = 0$ ?

- A. 1 B. 2 C. 3 D. 4
- 131. Determine the remainder when  $3t^3 7t^2 11t + 20$  is divided by  $t^2 + 2t 4$ 
  - A. 3t-13 B. -25t+24 C. -25t+72 D. 27t-32
- 132. A cubic polynomial function that passes through the point (3, 24) has zeros at 5, -1 and -3. Determine an equation of this function.
  - A. y = -2 (x 5)(x + 1)(x + 3)B.  $y = -\frac{1}{2} (x - 5)(x + 1)(x + 3)$ C.  $y = \frac{1}{2} (x - 5)(x + 1)(x + 3)$ D. y = 2 (x - 5)(x + 1)(x + 3)
- 133. Solve the inequality:  $-(x-3)(x+2)^2 < 0$



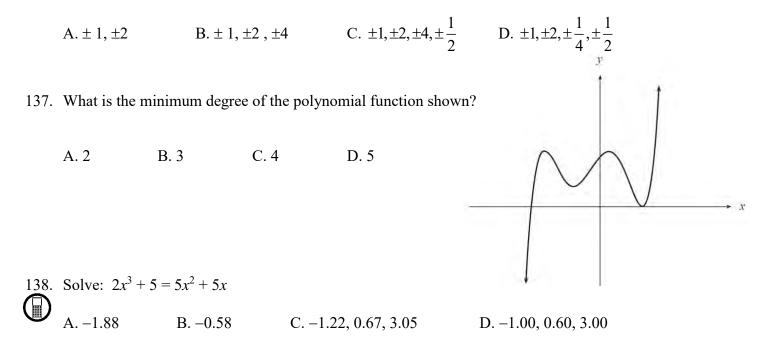
- 134. The graph of the polynomial function y = f(x) is shown. Determine the remainder when f(x) is divided by (x - 2).
  - A. -6
  - B. 0
  - C. 1
  - D. 6



135. Solve:  $x^3 + 10x^2 = 22 - 10x$ 

#### **JUNE 2000**

136. According to the Rational Root Theorem, determine all possible rational roots of  $4x^3 - 7x^2 + 3x - 2 = 0$ .



139. Solve the following inequality for x, given that a, b and c are constants such that a < b < c.  $(x - a)^3(x - b)^2 (x - c) > 0$ 

A. x > c B. x < a or x > c C. x < c,  $x \neq a$ ,  $x \neq b$  D. a < x < c,  $x \neq b$ 

140. Determine all values for k such that  $y = 2x^3 + 3x^2 - 12x + k$  has only one real zero. A. k < -20 B. k > 7 C. -20 < k < 7 D. k < -20 or k > 7

141. When  $2x^3 - 8x^2 + kx + 18$  is divided by x + 2, the remainder is -14. Find k, then find all real roots of  $2x^3 - 8x^2 + kx + 18 = 0$ .

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#### **JAN 2001**

- 142. Which expression represents the remainder when the polynomial P(x) is divided by x 9?
  - A. *P*(9) B. *P*(-9) C. *P*(0) D. *P*(*x* 9)
- 143. According to the Rational Root Theorem, which of the following is a possible root of the equation  $5x^3 + mx^2 + nx + 20 = 0$ , where *m* and *n* are integers?
  - A.  $\frac{1}{10}$  B.  $\frac{1}{5}$  C.  $\frac{1}{4}$  D.  $\frac{1}{2}$
- 144. Determine the quotient when  $x^4 8x^2 + 2x 7$  is divided by x + 3.

A.  $x^2 - 5x - 13$  B.  $x^2 - 11x + 35$  C.  $x^3 - 3x^2 + x - 1$  D.  $x^3 + 3x^2 + x + 5$ 

145. Determine the value of k if x - 2 is a factor of the polynomial  $x^3 - 4x^2 + kx + 6$ .

A. -9 B. -1 C. 1 D. 9

146. Solve  $(x + a)^2 (x + b)(x + c) < 0$ , where *a*, *b*, *c* are real number constants and 0 < a < b < c.

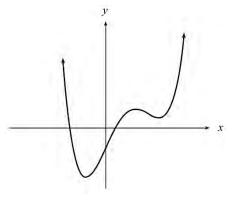
A. b < x < c B. -b < x < -c C. -c < x < -b D. -b < x < -a, x < -c

147. Solve:  $x^3 - 15x^2 = -10x - 30$ 

#### **JUNE 2001**

148. Determine the number of real zeros of the function shown.

A. 1 B. 2 C. 3 D. 4



A. 
$$x^2 - 2x$$
 B.  $x^2 - 4x + 6$  C.  $2x^2 - x + 1$  D.  $2x^2 - 5x + 7$ 

150. The polynomial equation  $mx^3 + 7x^2 - 3x + n = 0$ , where *m* and *n* are integers, has a root of  $\frac{4}{9}$ . According to the Rational Root Theorem, which of the following could be a value for *m*?

151. Solve:  $x^3 < x$ 

A. x < 0, x > 1 B. -1 < x < 1 C. -1 < x < 0, x > 1 D. x < -1, 0 < x < 1

152. Solve: 
$$x^3 + 2x^2 - 104x + 192 = 30$$
  
A. 1.65, 8.24 B. 2.37, 7.73 C. -12.11, 2.37, 7.73 D. -11.89, 1.65, 8.24

153. The points (-2, 0), (0, 5) and (2, -4) are on the graph of a third degree polynomial function, y = p(x). If p(x) is divided by x - 2, determine the remainder.

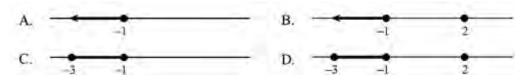
- A. -4 B. 0 C. 4 D. 5
- 154. A cubic polynomial function has a double zero at -2 and a single zero at 3. If this function passes through the point (4, -24), determine an equation of the function. Answer may be left in factored form.

## ADDITIONAL QUESTIONS

155. What numbers should replace p and q in the incomplete synthetic division shown below?

A. 
$$p = -5$$
,  $q = -3$  B.  $p = -5$ ,  $q = 3$  C.  $p = 5$ ,  $q = -3$  D.  $p = 5$ ,  $q = 3$ 

- 156. Determine the coefficient of x in the quotient when  $2x^4 7x^3 + 9x^2 + 2x 8$  is divided by  $x^2 3x + 4$ .
  - A. -13 B. -1 C. 1 D. 13
- 157. Graph the solution to  $(x+1)(x-2)^2(x+3)^3 \le 0$ .



158. Find a polynomial equation of lowest degree with integral coefficients such that one root of f(x) = 0 is  $\sqrt{2} + \sqrt{3}$ .

159.	Given the following table of values for the polynomial function $y = f(x)$ ,	x	У
	determine the minimum number of zeros for $f(x)$ .	-3	-15
		-2	-12
	A. 1	-1	2
	B. 2	0	3
	C. 3	1	5
		2	-7
	D. 4	3	-19

- 160. When  $x^4 + kx^2 5$  is divided by  $x^2 + 1$ , the remainder is -6. Find the value of k.
  - A. -2 B. 0 C. 1 D. 2
- 161. A polynomial function of degree 3 has zeros -2, 2, 4, and passes through the point (3, -25).Determine an equation of the function. (Answer may be left in factored form.)

162. Determine the cubic polynomial function which has zeros of -1, 2 and 3, and goes through the point (4, 6).

A. 
$$f(x) = (x+1)(x-2)(x-3)$$
  
B.  $f(x) = \frac{3}{5}(x+1)(x-2)(x-3)$   
C.  $f(x) = (x-1)(x+2)(x+3)$   
D.  $f(x) = \frac{1}{21}(x-1)(x+2)(x+3)$ 

163. If 
$$p(x) = (x - 2)q(x) + r$$
, determine  $p(2)$ .  
A.  $q(2)$  B.  $q(-2)$  C.  $-r$  D.

164. A polynomial function of degree 3 has zeros 5, 3, -1, and passes through the point (2, -6).Determine an equation of this function. (Answer may be left in factored form.)

r

- 165. Determine a factor of degree 2 of the polynomial p(x) if p(3) = 0 and p(-4) = 0.
  - A.  $x^2 + x 12$  B.  $x^2 x + 12$  C.  $x^2 x 12$  D.  $x^2 + x + 12$

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- 166. Determine the values of k for which  $\frac{1}{3}$  is a zero of  $p(x) = -9x^3 + 3x^2 3kx + k^3$ . A. -2, -1, 0 B. -2, 0, 1 C. -1, 0, 1 D. -1, 0, 2
- 167. Determine the polynomial function of degree 3, with zeros of -2, 0, and 3, that passes through the point (2, 5). Answer may be left in factored form.

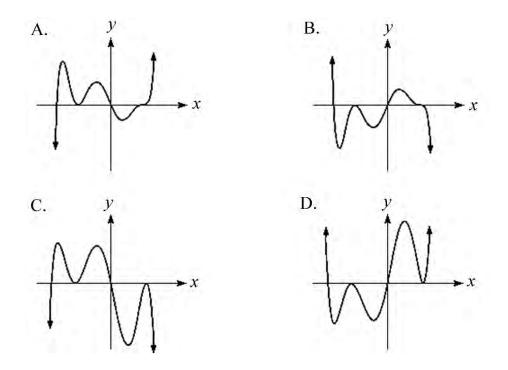
- 168. Determine the number of rational roots for the equation  $x^5 2x 1 = 0$ .
  - A. 1 B. 2 C. 3 D. 5
- 169. When a polynomial P(x) is divided by x + 4, the remainder is 5. Which point must be on the graph of the function y = P(x)?
  - A. (-4, 5) B. (5, -4) C. (-4, -5) D. (-5, -4)
- 170. A polynomial function of degree 3 has a zero of -1 and a double zero of 4. Determine this function if it passes through the point (1, 10). Answer may be left in factored form.

171. If x + 2 is a factor of the polynomial  $P(x) = 2x^3 + kx^2 - 32x - 4k^2$ , determine all possible values of k.

A. 
$$y \ge -18.81$$
 B.  $y \ge -16.54$  C.  $y \ge -8$  D. all real numbers

174. Determine the cubic polynomial function with zeros 1, 2, and -3 that passes through (3, -10). (Answer may be left in factored form.)

- 175. The function H(x) is the product of a 3rd degree polynomial function and a 2nd degree polynomial function. What is the maximum number of zeros of H(x)?
  - A. 2 B. 3 C. 5 D. 6
- 176. Which graph could represent  $f(x) = x(a x)(x b)^2 (x c)^3$ , where *a*, *b* and *c* are constants?



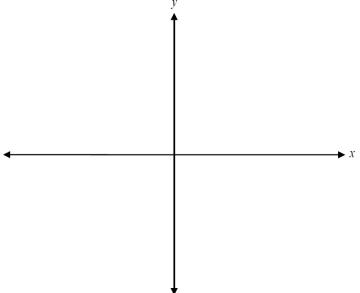
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177. Solve:  $x^4 - x^3 \ge 8x^2 + 2$ 

- 178. If the polynomial  $p(x) = ax^2 + bx 6$  is divided by (x 1), the remainder is -9. When p(x) is divided by (x + 2), the remainder is 12. Find the value of b.
  - A. -5 B. -2 C. 2 D. 5
- 179. If 2 is a root of the polynomial equation  $6x^3 + kx^2 + x + 2 = 0$ , determine the other roots.

180. Determine all of the zeros of the function  $p(x) = x^3 - 5x^2 - 2x + 24$ , given that one of the factors of p(x) is (x-3).

181. Sketch the graph of y = -2(x - 1)(x - 3)(x + 1)



4	D	47	Р	04	6	140	D
1	D	47	В	94 05	C	142	D
2	D	48	D	95	В	143	-1.66, 1.22, 4.44
3	В	49	С	96	С	144	A
4	A	50	В	97	В	145	В
5	С	51	D	98	D	146	C
6	$x = -1, -\frac{1}{3}, 2$	52	А	99	A	147	C
		53	В	100	$k = -3, x = \frac{1}{2}, 3$	148	С
7	С	54	D	101	A	149	1.09, 1.95, 14.14
8	А	55	D	102	С	150	В
9	В	56	В	103	В	151	D
10	D	57	В	104	D	152	D
11	С	58	D	105	В	153	D
12	С	59	А	106	A	154	D
13	С	60	А	107	D	155	Α
14	В	61	С	108	A	156	
15	C	62	A	109	C	100	$y = -\frac{2}{3}(x+2)^2(x-3)$
16	A	63a)	D	110	A	157	A
17	В	63b)		111	C	158	В
18		030)	$x = \frac{5}{3}cm$	112	D	159	D
	$-1, \pm \sqrt{5}$	64	-	112	B	160	
19	С	64 05	A				$x^4 - 10x^2 + 1 = 0$
20	С	65	С	114	A	161	В
21	В	66	С	115	D	162	
22	D	67	D	116	D	163	f(x) = 5(x-2)(x+2)(x-4)
23	A	68	D	117	В	164	В
24	240	69	A	118	С	165	D
25	A	70	С	119	D	166	$p(x) = -\frac{2}{3}(x-5)(x-3)(x+1)$
26	D	71	А	120	A		3
27	A	72	D	121	D	167	A
28	С	73	В	122	$x \ge 7.82$	168	С
29	А	74	D	123	A	169	$y = -\frac{5}{8}x(x+2)(x-3)$
30	В	75	С	124	A		8
31	В	76	В	125	С	170	A
32	D	77	С	126	D	171	A
33	D	78	В	127	В	172	$5(x+1)(x-4)^2$
34	А	79	А	128	A		$y = \frac{5}{9}(x+1)(x-4)^2$
35	А	80	В	129	С	173	4, -3
36	D	81	В	130	-2.48 < x < 0.83 or $x > 9.65$	174	-1.36 < x < 0.83 or $x > 3.53$
37	С	82	С	131	D	175	В
38	С	83	С	132	В	176	5
39		84	В	133	D		$y = -\frac{5}{6}(x-1)(x-2)(x+3)$
	$x = -3, -\frac{1}{2}, 2$	85	C	134	B	177	C
40	С	86	В	135	A	178	B
41	A	87	A	136	A	179	$x \le -2.43$ or $x \ge 3.40$
42	D	88	C	137	-8.52, -2.51, 1.03	180	A
43	D	89	C	137	–8.32, –2.31, 1.05 D	181	
43 44	D	90	A	130	D	101	$x = -\frac{1}{3}, \frac{1}{2}$
44 45	B	90 91	A	139	C	182	
45 46		91 92	B			102	3, 4, -2
40	$x = -2, \frac{1}{2}, 2$	92 93		141	В		
	2	93	В				