

PRECALCULUS MATHEMATICS 12

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- **121 PROBLEMS**

LOGARITHMIC and EXPONENTIAL FUNCTIONS

JAN 1997

1. Change $r^s = t$ to logarithmic form.
A. $r = \log_s t$ B. $s = \log_t r$ C. $t = \log_r s$ D. $s = \log_r t$
2. Evaluate: $\log_4 135$ (Accurate to 2 decimal places)
A. 0.53 B. 1.53 C. 2.13 D. 3.54
3. Determine the range of the function $y = 3^x + 2$.
A. $y > -2$ B. $y > 2$ C. $y > 0$ D. all real numbers
4. Which expression is equivalent to $3 \log a + \log b - \frac{1}{2} \log c$?
A. $\log\left(\frac{6ab}{c}\right)$ B. $\log\left(\frac{a^3b}{\sqrt{c}}\right)$ C. $\log\left(3a + b - \frac{1}{2}c\right)$ D. $\log\left(\frac{a^3 + b}{\sqrt{c}}\right)$
5. Solve: $\log_2 x - \log_2 4 = 3$
A. 2 B. 12 C. 32 D. 36
6. If $\log_9 5 = x$ and $\log_{27} 2 = y$, express $\log_3 100$ in terms of x and y .
A. $2(9^x + 27^y)$ B. $9x + 27y$ C. $24xy$ D. $4x + 6y$
7. Given the function $f(x) = 7^{\frac{x}{2}} - 3$, determine its inverse, $f^{-1}(x)$.
A. $f^{-1}(x) = 2 \log_7 x - 6$ B. $f^{-1}(x) = 2 \log_7(x - 6)$
C. $f^{-1}(x) = 2 \log_7(x + 3)$ D. $f^{-1}(x) = 2 \log_7 x + 3$
8. The population of a city is increasing at a rate of 6.5% each year. If the present population is 12 000, how long will it take for the population to reach 32 000?

JUN 1997

9. Change $\log_a b = c$ to exponential form.

A. $b = a^c$

B. $b = c^a$

C. $c = a^b$

D. $c = b^a$

10. Evaluate: $\log_8 16$

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. $\frac{4}{3}$

D. 2

11. Solve: $2^{\log x} = \frac{1}{4}$

A. -2

B. $\frac{1}{100}$

C. 2

D. 100

12. If $\log_a b = 0.5$, evaluate $\log_a \sqrt{b} + \log_a \left(\frac{a}{b}\right)$.

A. 0.5

B. 0.75

C. 1

D. 1.5

13. Simplify: $\frac{1}{\log_a x} + \frac{1}{\log_b x}$

A. $-\log_{ab} x$

B. $-\log_x ab$

C. $\log_{ab} x$

D. $\log_x ab$

14. To the nearest power of 10, determine the difference between 10^{600} and 10^{200} .

A. 10^3

B. 10^{200}

C. 10^{400}

D. 10^{600}

15. Given $\log 6 = x$ and $\log 8 = y$, determine an expression for $\log 3$ in terms of x and y .

A. $\frac{x}{3} - y$

B. $x - \frac{y}{3}$

C. $\frac{y}{3} - x$

D. $y - \frac{x}{3}$

16. Solve for x : $2 \log(4 - x) - \log 3 = \log(10 - x)$

JAN 1998

17. Evaluate: $\log_3 3^{-15}$

A. -15

B. -5

C. $\frac{1}{15}$

D. $\frac{1}{5}$

18. Which expression is equivalent to $2 \log \frac{3}{x}$?

A. $\log 9 - 2 \log x$

B. $\log 9 - \log x$

C. $\log 6 - \log 2x$

D. $(\log 3 - \log x)^2$

19. Determine the domain of the function $y = \log_2(x - 4)$.

A. $x > -4$

B. $x > 0$

C. $x > 2$

D. $x > 4$

20. Solve: $\log_6 x + \log_6(x - 5) = 2$

A. 3.5

B. 8.5

C. 9

D. 20.5

21. Given $f(x) = \log_3 x$, determine its inverse, $f^{-1}(x)$.

A. $f^{-1}(x) = x^3$

B. $f^{-1}(x) = 3^x$

C. $f^{-1}(x) = \log_x 3$

D. $f^{-1}(x) = \log_3 \frac{1}{x}$

22. Solve for x : $(\log_2 3)(\log_x 5)(\log_5 2) = 9$

A. $3^{\frac{1}{9}}$

B. $9^{\frac{1}{3}}$

C. 2

D. 3

23. Consider the graph of $y = -\log_2 x$. Which value for x in the interval $a \leq x \leq b$, $a > 0$, will give the largest value for y ?
- A. a B. b C. $\frac{(a+b)}{2}$ D. \sqrt{ab}
24. A river system has a current population of 4 000 000 fish. In each year, an enhancement program produces a 7% growth in population followed by a 200 000 fish harvest (i.e. 200 000 fish are taken from the population). Determine the number of fish after the second harvest.

JUN 1998

25. Evaluate: $\log_2 8$
- A. 2 B. 3 C. 4 D. 16
26. Which of the following is equivalent to $\log\left(\frac{a^3}{\sqrt{b}}\right)$
- A. $3 \log a - \frac{1}{2} \log b$ B. $\frac{3 \log a}{\frac{1}{2} \log b}$ C. $\log 3a - \log \frac{1}{2}b$ D. $\frac{\log 3a}{\log \frac{1}{2}b}$
27. Which of the following equations can be used to determine the number of years, t , that are needed for a \$300 deposit to increase to \$1500 if it is invested at 9% compounded annually?
- A. $1500 = 300(1.09)^t$ B. $300 = 1500(1.09)^t$ C. $1500 = 300(1.9)^t$ D. $300 = 1500(1.9)^t$
28. Determine the domain of the function $y = \log_3(x+1) - 2$.
- A. $x > -1$ B. $x > 1$ C. $y > -2$ D. $y > 2$
29. Solve: $\frac{1}{\log_2 x} + \frac{1}{\log_8 x} = 2$
- A. 2 B. 4 C. 8 D. 16

30. If $\log_3 7 = x$ and $\log_2 9 = y$, determine an expression for $\log_9 7 + \log_2 3$ in terms of x and y .

- A. $\frac{1}{2}x + \frac{1}{2}y$ B. $2x + \frac{1}{2}y$ C. $\frac{1}{2}x + \sqrt{y}$ D. $2x + \sqrt{y}$

31. Solve: $\log(3x - 5) + \log(2x - 1) = 1$

JAN 1999

32. Evaluate: $\log_{12} 8000$

- A. 0.33 B. 2.82 C. 3.62 D. 3.90

33. Express $3 \log a + \log b - \log c$ as a single logarithm.

- A. $\log\left(\frac{a^3b}{c}\right)$ B. $\log(a^3 + b - c)$ C. $3\log\left(\frac{ab}{c}\right)$ D. $\log\left(\frac{3ab}{c}\right)$

34. Determine the range of the function $y = 7^{x+2} - 4$.

- A. $y > -4$ B. $y > -2$ C. $y > 2$ D. $y > 4$

35. Solve: $\log_3(2x + 3) - \log_3(x + 1) = 2$

- A. $-\frac{6}{7}$ B. $-\frac{3}{4}$ C. 0 D. no real roots

36. Simplify: $8^{\log_2 6x}$

- A. $18x$ B. $6x^3$ C. $216x^3$ D. $1296x^4$

37. Solve for x : $(\log_x 7)(\log_7 5) = 2$

A. $\frac{5}{2}$

B. $\sqrt{5}$

C. $\frac{2\sqrt{3}}{7}$

D. $\frac{\sqrt{35}}{7}$

38. A population of frogs doubles every 20 weeks. If the present population is 400 frogs, how long will it take for the population to reach 10 000?

JUN 1999

39. Change $y = \log_5 x$ to exponential form.

A. $x = 5y$

B. $x = 5^y$

C. $x = y^5$

D. $x = \left(\frac{1}{5}\right)^y$

40. Express $\log a - \log b + 2 \log c$ as a single logarithm.

A. $\log \frac{ac^2}{b}$

B. $\log \frac{a}{bc^2}$

C. $\log \frac{ab}{2c}$

D. $\log \frac{a}{2bc}$

41. Solve for x : $3^x = 18$

A. $x = \log 6$

B. $x = 3 \log 18$

C. $x = \frac{\log 18}{\log 3}$

D. $x = \log 18 - \log 3$

42. Determine an equation of the asymptote of the graph of $y = 4 \log_5(x - 4) + 5$

A. $x = 4$

B. $x = 5$

C. $y = 4$

D. $y = 5$

43. Solve: $\log_2(4 - x) - \log_2 x = 1$

A. 1

B. $\frac{4}{3}$

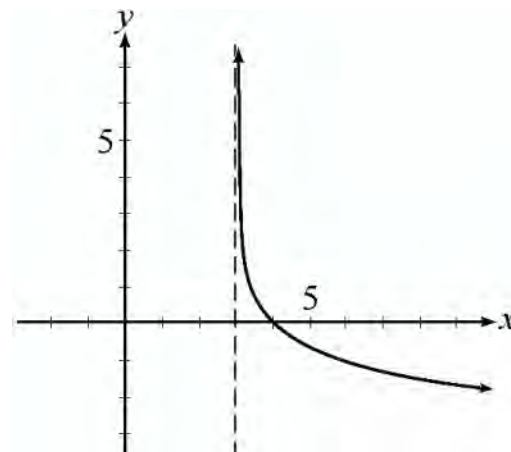
C. $\frac{3}{2}$

D. 2

44. Determine an expression equivalent to $\frac{1}{\log_a x} - \frac{1}{\log_b x}$.
- A. $\log_a x - \log_b x$ B. $\log_x a - \log_x b$ C. $\log_b x - \log_a x$ D. $\log_x b - \log_x a$
45. The point (m, n) is on the graph of $f(x) = a^x$. Which of the following must be a point on the graph of $g(x) = \log_a x$, ($a > 0$)?
- A. $(-m, -n)$ B. $(-n, -m)$ C. (m, n) D. (n, m)

JAN 2000

46. Change $a = b^c$ to logarithmic form.
- A. $\log_a b = c$ B. $\log_b c = a$ C. $\log_c a = b$ D. $\log_b a = c$
47. Solve for y : $x = \log c^y$
- A. $\frac{x}{\log c}$ B. $\log c^x$ C. $\frac{x}{c}$ D. $x - \log c$
48. A population of insects doubles every 5 days. If there are currently 1 200 insects, determine an equation for the population P , of insects, t days from now.
- A. $P = 1\,200\left(\frac{1}{2}\right)^{\frac{t}{5}}$ B. $P = 1\,200\left(\frac{1}{2}\right)^{5t}$ C. $P = 1\,200(2)^{5t}$ D. $P = 1\,200(2)^{\frac{t}{5}}$
49. Determine the equation of the logarithmic function shown:
- A. $y = \log_3(x - 3)$
 B. $y = \log_3(x + 3)$
 C. $y = -\log_3(x + 3)$
 D. $y = -\log_3(x - 3)$



50. Solve for x : $(\log_2 8)^x - (\log_9 3)^{x+1} = 0$

- A. -0.50 B. -0.39 C. -0.33 D. 1.71

51. If $\log 5 = m$ and $\log 7 = n$, determine $\log \frac{35}{10}$ in terms of m and n .

- A. $\frac{mn}{10}$ B. $\frac{m+n}{10}$ C. $m+n-1$ D. $m+n-10$

52. A population of wolves decreases by 2% each year. At the present time, there are 8 000 wolves. How long will it take for the population to become 500 wolves? (Answer to the nearest year.)

53. Given $\frac{1}{\log_y 4} = \log_{\frac{1}{4}} \left(\frac{1}{8x} \right)$, express y as a polynomial function of x . State the restrictions on x and y .

JUN 2000

54. Change $\log_a b = c$ to exponential form.

- A. $b = a^c$ B. $b = c^a$ C. $a = b^c$ D. $a = c^b$

55. Evaluate: $\log 2^{2000}$

- A. 0 B. 10.97 C. 602.06 D. infinitely large

63. Determine the domain of the function $y = \log_4(x + 7) - 3$.

- A. $x > -7$ B. $x > -3$ C. $x > 3$ D. $x > 7$

64. If $f(x) = 3 \log x$, determine $f^{-1}(x)$, the inverse of $f(x)$.

- A. $f^{-1}(x) = \frac{1}{3 \log x}$ B. $f^{-1}(x) = \frac{1}{3} \log \frac{1}{x}$ C. $f^{-1}(x) = 10^{\frac{x}{3}}$ D. $f^{-1}(x) = 10^{x-3}$

65. A culture has 300 bacteria. The number of bacteria doubles every 4 hours. How long will it take for the number of bacteria to reach 72 000?

- A. 31.63 hours B. 33.27 hours C. 36.88 hours D. 60 hours

66. Determine the number of solutions for the following system:



$$y = -4 \log_{12} x$$

$$y = 4 \sin x$$

- A. 2 B. 3 C. 4 D. 5

67. Simplify: $2^{\log_8 x^{27}}$

- A. $3x$ B. $9x$ C. x^3 D. x^9

JUN 2001

68. Change $b^a = c$ to logarithmic form.

- A. $\log_b a = c$ B. $\log_c b = a$ C. $\log_b c = a$ D. $\log_c a = b$

69. Express $2 \log k + \log 5 - \log p$ as a single logarithm.

- A. $\log \frac{5k^2}{p}$ B. $\log \frac{10k}{p}$ C. $\log (k^2 + 5 - p)$ D. $\log (2k + 5 - p)$

70. Solve: $\log_2 x + \log_2(x - 2) = 3$

- A. 2.5 B. 3 C. 4 D. $1 + \sqrt{7}$

71. The population of a type of bacteria triples every 20 hours. In how many hours will a population of 30 become a population of 1000?

- A. 63.84 B. 101.18 C. 106.83 D. 169.32

72. Determine the domain of the function $y = \log_{(2x-3)}(x)$.

- A. $x > 0, x \neq 1$ B. $x > 0, x \neq 2$ C. $x > \frac{3}{2}, x \neq 1$ D. $x > \frac{3}{2}, x \neq 2$

73. Solve for x : $\log_a a^{2x} = \log_{b^2} b^{3x-3}$

- A. -3 B. $-\frac{3}{2}$ C. $\frac{3}{2}$ D. 3

74. Simplify: $\log_{\frac{1}{x}} \frac{1}{y} - \log_{\frac{1}{x}} y - \log_x \frac{1}{y}$

- A. $\log_x y^3$ B. $\log_x y$ C. $-\log_x y$ D. $\log_x y - 2$

SPECS 2001

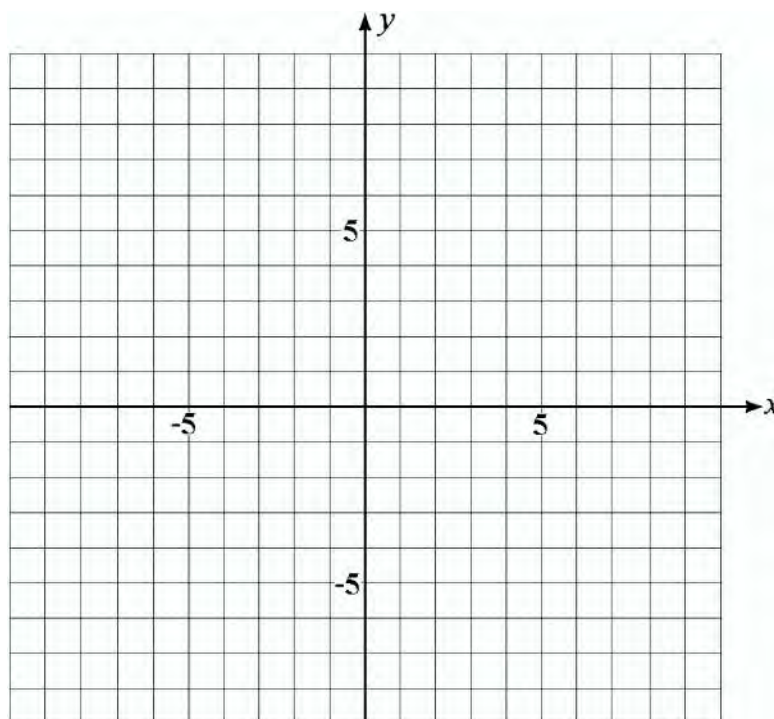
75. The population of ABC high school is currently 1 250 students and is decreasing at an annual rate of 3%. Which expression represents the population, P , of the school 5 years from now?

- A. $P = 1\,250(1.03)^5$ B. $P = 1\,250(1.03)^{-5}$ C. $P = 1\,250(0.97)^5$ D. $P = 1\,250(0.97)^{-5}$

76. The half-life of plutonium-239 is about 25 000 years. How many years does it take until only 36% of the plutonium still remains?

SAMPLE 2001

77. Graph $\log_5(y + 2) = x + 1$ on the grid below. State any asymptotes and give exact values for the x - and y -intercepts.



78. A recent earthquake in Washington measured 6.3 on the Richter scale. In 1964, the Alaskan earthquake measured 8.5. How many times as intense was the 1964 Alaskan earthquake compared to the recent Washington earthquake?
- A. 1.35 B. 2.2 C. $10^{1.35}$ D. $10^{2.2}$
79. Solve for x : $\log_3(x - 6) + \log_3 x = 3$
- A. 4.5 B. 9 C. 16.5 D. -3, 9
80. A bank account earns interest at a rate of 7% per year compounded continuously. Which equation will determine the effective annual growth rate, r ? (P_0 is the initial amount invested; t is time, in years, over which the amount is invested.)
- A. $P_0(1 + r)^t = P_0e^{0.07t}$ B. $P_0(1.07)^t = P_0e^{rt}$
C. $P_0(1 + r)^t = P_0e^{1.07t}$ D. $P_0(1.07)^t = P_0e^{(1+r)t}$
81. If 3 150 mg of a radioactive substance decays to 450 mg in 73 weeks, determine the half-life of the substance to the nearest week. (Solve algebraically using logarithms.)

JAN 2002

82. Change to logarithmic form: $p = q^r$

- A. $\log_p q = r$ B. $\log_q p = r$ C. $\log_r p = q$ D. $\log_q r = p$

83. Evaluate: $\log_2 7.5$

- A. 0.44 B. 0.57 C. 2.26 D. 2.91

84. Determine the range of the function $y = 2^{x-3} + 4$

- A. $y > 4$ B. $y > -4$ C. $x > 3$ D. $x > -3$

85. Solve: $16^{x+1} = 8^{1-x}$

- A. $-\frac{1}{3}$ B. $-\frac{1}{7}$ C. $\frac{2}{7}$ D. $\frac{2}{5}$

86. Which expression is equivalent to $\log\left(\frac{100x^3}{y}\right)$?

- A. $2 + \log(3x - y)$ B. $300 \log x - \log y$ C. $2 + 3 \log x - \log y$ D. $\log(100 + x^3 - y)$

87. Solve: $\log_3(x + 4) + \log_3(6 - x) = 2$

- A. 3 B. 5 C. 3, -5 D. 5, -3

88. Simplify: $a^{\log_a 8 + \log_a 2}$

- A. 10 B. 16 C. a^{10} D. a^{16}

89. The population of a city grows continuously according to the formula $P = P_0 e^{kt}$. Determine the value of the growth rate, k , if the population increases from 30 000 to 45 000 in 8 years.

- A. 0.02 B. 0.05 C. 0.41 D. 1.05

97. Determine an expression for: $\sum_{n=1}^5 \log_a n$
- A. $\log_a 5$ B. $\log_a 6$ C. $\log_a 15$ D. $\log_a 120$
98. At which of the following points is the relation $\log(y - x) + \log(y + x) = \log 9$ not defined?
- A. (0, 3) B. (-4, 5) C. (4, -5) D. (4, 5)
99. Solve algebraically: $\log_2(2 - 2x) + \log_2(1 - x) = 5$
100. A biologist determines that a particular type of bacteria grows continuously according to the formula $P = P_0 e^{kt}$. Determine the value of the continuous growth rate if the population of the bacteria increases from 500 to 1500 in 8 days.

JUN 2002

101. Determine the logarithmic form of $a = b^c$.
- A. $\log_a b = c$ B. $\log_a c = b$ C. $\log_c a = b$ D. $\log_b a = c$
102. Solve: $\left(\frac{1}{4}\right)^{1-2x} = 8^{x-3}$
- A. -7 B. $\frac{11}{7}$ C. $\frac{7}{4}$ D. no solution

103. A recent earthquake in Turkey measured 7.2 on the Richter scale. In 1960, the earthquake in Morocco measured 5.8. How many times as intense was the recent Turkey earthquake compared to the Moroccan earthquake?

- A. 1.24 B. 1.4 C. 17.43 D. 25.12

104. If the graph of $y = \log_a x$ goes through the point (1024, 5), determine a .

- A. 4 B. 4.31 C. 10 D. 204.8

105. A sample of water contains 200 g of pollutants. Each time the sample is passed through a filter, 20% of its pollutants are removed. Determine an example that gives the number of grams of pollutants still in the water after it passes through five filters.

- A. $200(0.8)^4$ B. $200(1.2)^4$ C. $200(0.8)^5$ D. $200(1.2)^5$

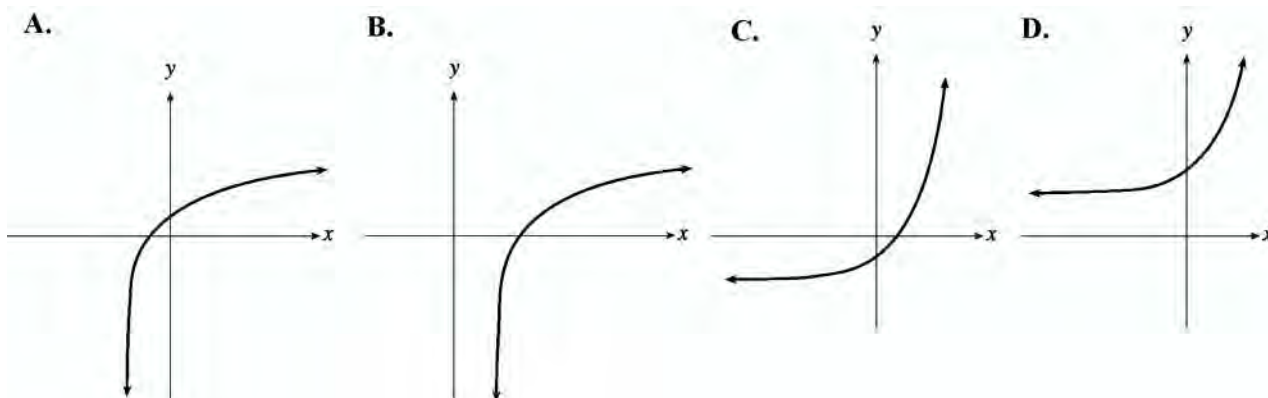
106. If $\log_a x = 3$ and $\log_a y = 4$, evaluate $\left(\log_a \frac{1}{xy}\right)^2$.

- A. $\frac{1}{49}$ B. 1 C. 14 D. 49

107. Simplify: $e^{\ln a}$

- A. a B. e^a C. $\ln a$ D. ae

108. Which graph best represents the function $y = \log_2(x - 2)$?



109. Solve algebraically: $\log_2 x + \log_2(x - 7) = 3$

AUG 2002

110. Change $\log_4 c = x$ to exponential form.

- A. $x^4 = c$ B. $4^x = c$ C. $4^c = x$ D. $c^x = 4$

111. Determine the domain of $y = 2 \log_4(x - 1) + 5$.

- A. $x > 1$ B. $x > 4$ C. $x > 5$ D. all real numbers

112. Solve: $25^{x+3} = 125^{2x-1}$

- A. $-\frac{16}{3}$ B. 1 C. $\frac{11}{8}$ D. $\frac{9}{4}$

113. Solve: $\log_4(x^2 + 1) - \log_4 6 = \log_4 5$

- A. $\sqrt{10}$ B. $\pm\sqrt{10}$ C. $\sqrt{29}$ D. $\pm\sqrt{29}$

114. Determine the x -intercept of $y = \log_2(x + 4) + 1$.

- A. -3 B. -3.5 C. -3.9 D. -4

115. Max invests \$5 000 at an interest rate of 6% per annum, compounded monthly. Which expression represents the amount of Max's investment after t years?

- A. $5000(1.06)^{12t}$ B. $5000(1.005)^{12t}$ C. $5000(1.06)^t$ D. $5000(1.005)^{\frac{t}{12}}$

116. Which expression is equivalent to $\log(m^2n)^3$?

- A. $6 \log m + 3 \log n$ B. $6 \log m + \log n$ C. $(2 \log m + \log n)^3$ D. $\log 3m^2 + \log 3n$

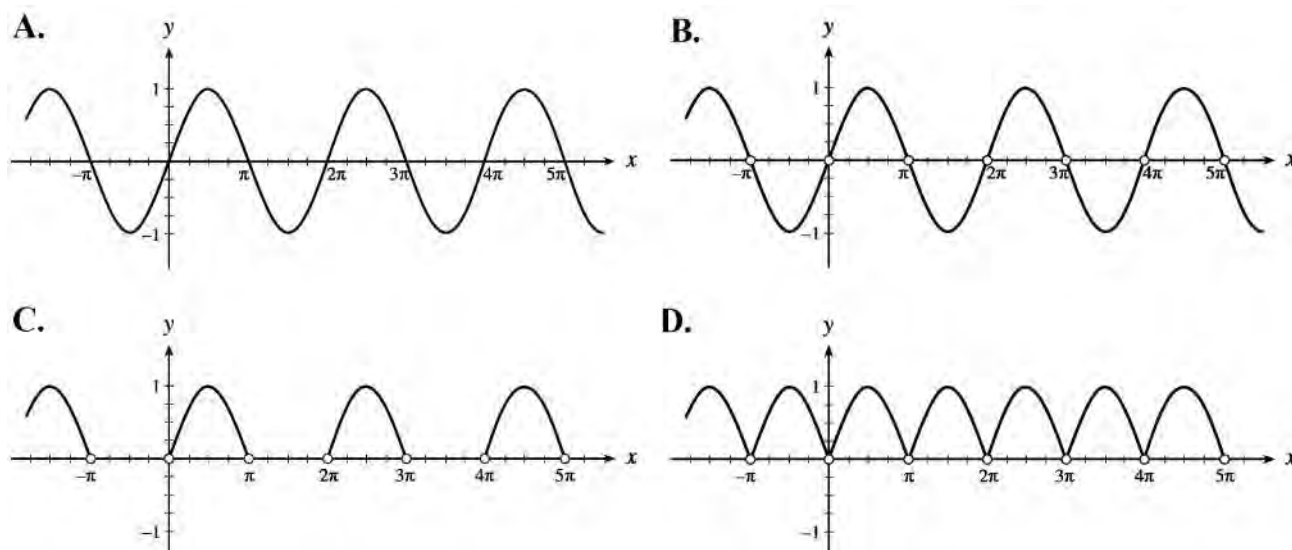
123. If $\log_3(m+n) = 2$, $(m+n) > 0$, express m in terms of n .

- A. $m = 9 - n$ B. $m = 6 - n$ C. $m = \frac{9}{n}$ D. $m = \frac{6}{n}$

124. A radioactive substance decays continuously according to the formula $N = Ce^{kt}$, where N is the final amount, C is the initial amount, k is the time in years. If 50 grams of the substance decays to 20 grams in 10 years, determine the value of k .

- A. -0.0916 B. -0.0398 C. 0.0610 D. 0.0916

125. Which graph best represents the function $\log y = \log \sin x$?



126. Solve algebraically: $2 \log(3 - x) = \log 4 + \log(6 - x)$

JUN 2003

127. Determine the domain of the function $y = \log(3x - 5)$.

- A. $x > -\frac{5}{3}$ B. $x > -\frac{3}{5}$ C. $x > \frac{3}{5}$ D. $x > \frac{5}{3}$

128. Express as a single logarithm: $\log a - 2 \log b - \log c$
- A. $\log \frac{ac}{2b}$ B. $\log \frac{ac}{b^2}$ C. $\log \frac{a}{2bc}$ D. $\log \frac{a}{b^2c}$
129. Solve for x : $8^{x-1} = \left(\frac{1}{16}\right)^{5-x}$
- A. $-\frac{19}{4}$ B. -3 C. $\frac{23}{7}$ D. 17
130. An earthquake off the coast of Alaska measured 6.4 on the Richter scale. Another earthquake near Japan was 50 times as intense. What was the Richter scale reading for the earthquake near Japan?
- A. 7.1 B. 7.9 C. 8.1 D. 10.9
131. Which expression gives the amount that an investment of P dollars will grow to after 4 years if it is compounded semi-annually at a rate of 5% per annum?
- A. $P(1.05)^4$ B. $P(1.025)^4$ C. $P(1.05)^8$ D. $P(1.025)^8$
132. Given that $y_1 = \log_a 0.4$ and $y_2 = \log_a 4$, where $0 < a < 1$, which of the following must be true?
- A. $y_1 < y_2$ B. $y_1 > y_2$ C. $0.4 < y_1 < 4$ D. $0.4 < y_2 < 4$
133. If 200g of a substance decays to 17g in 28 days, determine the half-life of this substance. (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

JAN 2004

134. Evaluate: $\log_3 59.2$
- A. 0.27 B. 1.30 C. 3.71 D. 19.73

135. Determine the domain of $y = \log_a(-x)$.

- A. $x < 0$ B. $x > 0$ C. $x \leq 0$ D. $x \geq 0$

136. Express as a single logarithm: $\log A - 3 \log B + \log C$

- A. $\log \frac{AC}{3B}$ B. $\log \frac{AC}{B^3}$ C. $\log \frac{A}{B^3C}$ D. $\log(A - 3B + C)$

137. If the point $(2, 9)$ is on the graph of $y = a^x$, what point must be on the graph of $y = \log_a x$?

- A. $\left(2, \frac{1}{9}\right)$ B. $(2, 9)$ C. $(9, -2)$ D. $(9, 2)$

138. Solve: $\log_2(3 - 2x) - \log_2(2 - x) = \log_2 3$

- A. -2 B. $\frac{1}{2}$ C. 3 D. no solution

139. The number of insects in a colony can triple in 7 weeks. After 50 weeks, how many times greater will the number of insects be than after 20 weeks?

- A. 81 B. 110.87 C. 243 D. 2.06×10^{14}

140. A radioactive substance decays from 600 g to 105 g in twelve days. Determine the half-life, in days, for this substance.

- A. 4.77 B. 5.27 C. 7.43 D. 30.17

141. Solve algebraically using logarithms: $2^x = 5^{x+1}$ (Answer accurate to at least 2 decimal places.)

JUN 2004

142. Determine the domain of $f(x) = \log_7(x+6) + 12$.

- A. $x > 6$ B. $x > -6$ C. $x > 12$ D. $x > -12$

143. Express $\log_5 30$ using logarithms in base 4.

- A. $\log_4 30 - \log_4 5$ B. $\frac{\log_4 5}{\log_4 30}$ C. $\frac{\log_4 30}{\log_4 5}$ D. $\frac{\log_{30} 4}{\log_5 4}$

144. Solve: $\left(\frac{1}{9}\right)^x = 27^{2-x}$

- A. -6 B. $\frac{6}{5}$ C. 2 D. 6

145. Which expression is equivalent to $\log \frac{x}{2y^3}$?

- A. $\log x - \log 2 + 3 \log y$ B. $\log x - 3 \log 2 + 3 \log y$
C. $\log x - \log 2 - 3 \log y$ D. $\log x - 3 \log 2 - 3 \log y$

146. Solve: $\log_2 x + \log_2(x-1) = 3$

- A. 2.37 B. 3 C. 3.37 D. 3.5

147. The formula $A = P(1.09)^t$ is an example of exponential growth with base 1.09. Determine an equivalent continuous growth formula using base e , $A = Pe^{kt}$.

- A. $A = Pe^{0.086t}$ B. $A = Pe^{1.086t}$ C. $A = Pe^{0.86t}$ D. $A = Pe^{1.86t}$

148. Determine an exponential function in the form $y = 3^{x-h} + k$ with y -intercept 5 and asymptote $y = -4$

- A. $y = 3^{x-4} + 5$ B. $y = 3^{x-2} - 4$ C. $y = 3^{x-5} - 4$ D. $y = 3^{x+2} - 4$

AUG 2005

149. Express as a single logarithm: $\log m - \log n - 3 \log k$
- A. $\log \frac{m}{nk^3}$ B. $\log \frac{m}{3nk}$ C. $\log \frac{mk^3}{n}$ D. $\log \frac{3mk}{n}$
150. Determine the domain of the function $y = \log(x - 5)$.
- A. $x \geq 5$ B. $x > 5$ C. $x \leq 5$ D. $x < 5$
151. Simplify: $9 \log_{27} x - 4 \log_9 x$
- A. $\log_3 x$ B. $\log_9 x$ C. $\log_{27} x$ D. $\frac{3}{4} \log_3 x$
152. A particular type of bacteria multiplies 5-fold every 30 minutes. Initially there are 100 bacteria. Determine an expression for the number of bacteria after k minutes.
- A. $\frac{100(5)^k}{30}$ B. $100(5)^{30k}$ C. $100(5)^{\frac{30}{k}}$ D. $100(5)^{\frac{k}{30}}$
153. Given $f(x) = 2^x + 5$, determine $f^{-1}(x)$, the inverse of $f(x)$.
- A. $f^{-1}(x) = 5 + \log_2 x$ B. $f^{-1}(x) = -5 + \log_2 x$
C. $f^{-1}(x) = \log_2(x + 5)$ D. $f^{-1}(x) = \log_2(x - 5)$
154. Solve algebraically: $2 \log_3(x + 4) - \log_3(-x) = 2$

AUG 2006

155. Change $\log_a p = t$ to exponential form.

- A. $p^t = a$ B. $a^t = p$ C. $a^p = t$ D. $t^p = a$

156. Solve: $\log_5(3x) - \log_5(x - 3) = 2$

- A. -6 B. $-\frac{1}{2}$ C. $\frac{75}{22}$ D. 11

157. A population grows continuously according to the formula $P = P_0 e^{kt}$, where P is the final population in t years, P_0 is the initial population and k is the continuous growth rate. What will be the population in 7 years if the initial population is 25 000 and the continuous growth rate is 1.2%?

- A. 27191 B. 57909 C. 177113 D. 197312

158. A radioactive substance has a half-life of 17 days. How many days will it take for 300 g of this substance to decay to 95 g? (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

SAMPLE 2008

159. Determine the domain of $y = \log(x + 1)$.

- A. $x < 1$ B. $x > 1$ C. $x < -1$ D. $x > -1$

160. Determine an equivalent expression for $\log \frac{100a^2}{\sqrt{b}}$.

- A. $2 \log 100a - \frac{1}{2} \log b$ B. $2 + 2 \log a - \frac{1}{2} \log b$
C. $4 \log a - \frac{1}{2} \log b$ D. $100 + 2 \log a - \frac{1}{2} \log b$

161. Evaluate: $\log_{\sqrt{7}} 7^3$

- A. $\frac{2}{3}$ B. $\frac{3}{2}$ C. 6 D. 9

162. As an iceberg melts during the summer, it loses 3% of its mass every 5 days. This iceberg reduces to 40% of its original mass after t days. Which equation could be used to determine the value of t ?

- A. $40 = 100(0.97)^{\frac{t}{5}}$ B. $40 = 100(0.97)^{\frac{5}{t}}$ C. $40 = 100(1.03)^{\frac{t}{5}}$ D. $40 = 100(1.03)^{\frac{5}{t}}$

163. Solve: $\log_2 (\log_9 x) = -1$

- A. $\frac{1}{81}$ B. $\frac{1}{3}$ C. 3 D. 81

164. Solve: $5^{x+1} = 2(3^{2x})$

- A. $x = \frac{-\log 5}{1 - 2\log 6}$ B. $x = \frac{-\log 5}{\log 5 - 2\log 6}$ C. $x = \frac{\log 2 - \log 5}{1 - 2\log 3}$ D. $x = \frac{\log 2 - \log 5}{\log 5 - 2\log 3}$

165. Change to logarithmic form: $a^3 = b$

- A. $3 = \log_a b$ B. $3 = \log_b a$ C. $b = \log_a 3$ D. $a = \log_b 3$

166. A population grows continuously according to the formula $P = P_0 e^{kt}$, where P is the final population at the end of t years, P_0 is the initial population and k is the annual growth rate. Determine the population (millions) at the end of 8 years if the initial population is 15 million and the annual growth rate is 4%.

- A. 20.66 B. 124.90 C. 179.02 D. 367.99

167. Determine the magnitude of an earthquake that is half as intense as an earthquake of magnitude 8.0 on the Richter scale.

- A. 4.0 B. 5.0 C. 7.7 D. 8.3

168. Solve algebraically: $\log 2 - \log(x - 1) = \log(x + 1) - \log(x + 17)$

JAN 2008

169. Determine an equation for the asymptote of the graph of $y = 2^{x+3} + 4$.

- A. $y = 4$ B. $x = 3$ C. $x = -3$ D. $y = -4$

170. Solve: $9^x = 27^{x-3}$

- A. -9 B. 3 C. $\frac{9}{2}$ D. 9

171. Solve: $\log_5(3x) - \log_5(x - 3) = 2$

- A. -6 B. $-\frac{1}{2}$ C. $\frac{75}{22}$ D. 11

172. Evaluate: $\log_5 \sqrt{5^3}$

- A. $\frac{1}{6}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 6

173. If $\log_2 5 = x$ and $\log_2 3 = y$, determine an expression for $\log_2 \left(\frac{15}{2} \right)$, in terms of x and y .

- A. xy B. $x + y$ C. $xy - 1$ D. $x + y - 1$

174. Solve: $\log_2(\log_x(x + 6)) = 1$

- A. 2 B. 3 C. $2, 3$ D. $-2, 3$

175. Change $\log_2(3x) = 5$ to exponential form.

- A. $3x = 2^5$ B. $3x = 5^2$ C. $2 = (3x)^5$ D. $2 = 3x^5$

176. In 1872, Washington State experienced an earthquake of magnitude 6.8 on the Richter scale. Determine the magnitude on the Richter scale of an earthquake that is half as intense as the Washington State earthquake.

- A. 3.4 B. 6.0 C. 6.5 D. 7.1

177. A population grows continuously according to the formula $P = P_0 e^{kt}$, where P is the final population at the end of t years, P_0 is the initial population, and k is the annual growth rate. What will the population be at the end of 10 years if the initial population is 5000 and the annual growth rate is 3%?

- A. 6 720 B. 6 749 C. 51 523 D. 100 428

178. In a population of moths, 78 moths increase to 1000 moths in 40 weeks. What is the doubling time for this population of moths?

SAMPLE 2009

179. Solve for x : $81^{x-1} = \left(\frac{1}{27}\right)^{x-4}$

- A. -8 B. -3 C. $-\frac{3}{7}$ D. $\frac{16}{7}$

180. Solve for x : $9^{x+2} = (3^{4x-3})(3^5)$

- A. 0 B. 1 C. $\frac{17}{19}$ D. $\frac{19}{18}$

181. Solve for x : $5 = 3^x$

A. $x = \log_5 3$

B. $x = \log_3 5$

C. $x = 3^5$

D. $x = 5^3$

182. Solve for x : $ab^x = c$

A. $x = \frac{\log c}{\log a + \log b}$

B. $x = \frac{\log c + \log a}{\log b}$

C. $x = \frac{\log c - \log a}{\log b}$

D. $x = \frac{\log c}{\log b} - \log a$

183. Solve: $2^x = 3(5^{x+1})$

184. Solve for x : $\log(3 - x) + \log(3 + x) = \log 5$

A. $x = -2$

B. $x = 2$

C. $x = \pm 2$

D. no solution

185. Solve: $\log_2 8 + \log_3 \frac{1}{3} = \log_4 x$

A. $\frac{1}{64}$

B. $\frac{1}{16}$

C. 16

D. 64

186. Solve: $\log_2(\log_4(\log_5 x)) = -1$

A. $\frac{1}{25}$

B. 5

C. 25

D. 125

187. Solve: $2 \log_4 x - \log_4(x+3) = 1$

188. Simplify: $\log_2 4^x$

A. x

B. $2x$

C. 2^x

D. x^2

189. Write as a single logarithm: $3 + \frac{1}{2} \log_2 x - 3 \log_2 y$

A. $\log_2 \left(\frac{1000\sqrt{x}}{y^3} \right)$

B. $\log_2 \frac{8\sqrt{x}}{y^3}$

C. $\log_2(1000 + \sqrt{x} - y^3)$

D. $\log_2(8 + \sqrt{x} - y^3)$

190. If $\log_4 x = a$, determine $\log_{16} x$ in terms of a .

A. $\frac{a}{4}$

B. $\frac{a}{2}$

C. $2a$

D. $4a$

191. If $\log 2 = a$, $\log 3 = b$, determine an expression for $\log 2400$.

A. $2a^3b$

B. $3a + b + 2$

C. $3a + b + 100$

D. $a^3 + b + 2$

192. Simplify: $a^{\log_a 8 + \log_a 2}$

A. 10

B. 16

C. a^{10}

D. a^{16}

193. Determine the value of $\log_n ab^2$ if $\log_n a = 5$ and $\log_n b = 3$.

A. 11

B. 14

C. 16

D. 45

194. Given $\log_a 2 = x$ and $(\log_a 8)(a^{\log_a x}) = 12$, solve for a .

- A. 2 B. ± 2 C. $\sqrt{2}$ D. $\pm\sqrt{2}$

195. Change to exponential form: $\log_k l = m$

- A. $l = m^k$ B. $l = k^m$ C. $k = m^l$ D. $k = l^m$

196. If (a, b) is on the graph of $y = 3^x$, which point must be on the graph of $y = \log_3 x$?

- A. (a, b) B. (b, a) C. $(3a, b)$ D. $(a, 3b)$

197. Determine the inverse of $f(x) = 3^{x-1} - 2$.

- A. $f^{-1}(x) = \log_3(x+2) + 1$ B. $f^{-1}(x) = \log_3(x+2) - 1$
C. $f^{-1}(x) = \log_3(x-1) + 2$ D. $f^{-1}(x) = \log_3(x-1) - 2$

198. If \$5000 is invested at 7.2% per annum compounded monthly, which equation can be used to determine the number of years, t , for the investment to increase to \$8000?

- A. $8000 = 5000(1.072)^t$ B. $8000 = 5000(1.006)^t$
C. $8000 = 5000(1.072)^{12t}$ D. $8000 = 5000(1.006)^{12t}$

199. The population of a particular country is 25 million. Assuming the population is growing continuously, the population, P , in millions, t years from now can be determined by the formula $P = 25e^{0.022t}$. Determine the population, in millions, 20 years from now.

- A. 29.90 B. 37.97 C. 38.63 D. 38.82

200. The population of a nest of ants can multiply threefold (triple) in 8 weeks. If the population is now 12 000, how many weeks will it take for the population to reach 300 000 ants?

201. The radioactivity of a certain substance decays by 20% in 30 hours. What is the half-life of the substance?
202. The intensity of light reduces by 7% for every 3 metres below the surface of water. At what depth will the light intensity be reduced to 60% of its original amount?
203. The population of Canada is 30 million people and is growing at an annual rate of 1.4%. The population of Germany is 80 million people and is decreasing at an annual rate of 1.7%. In how many years will the population of Canada be equal to the population of Germany?
204. Determine the domain of the function $y = \log(2x + 3)$.
- A. $x > -\frac{3}{2}$ B. $x > -\frac{2}{3}$ C. $x > \frac{2}{3}$ D. $x > \frac{3}{2}$
205. In 1976, an earthquake in Guatemala had a magnitude of 7.5 on the Richter scale and in 1960, an earthquake in Morocco had a magnitude of 5.8. How many times as intense was the 1976 Guatemalan earthquake compared to the 1960 Moroccan earthquake?
- A. 1.29 B. 1.7 C. $10^{1.29}$ D. $10^{1.7}$

211. Which of the following is equivalent to $\log 3x^2$?

- A. $2(\log 3 + \log x)$ B. $\log 9 - 2 \log x$ C. $2 \log 3 + \log x$ D. $\log 3 + 2 \log x$

212. If $a = b^{c \log_b d}$, then which of the following must be true?

- A. $a = cd$ B. $a = b^c$ C. $a = d^c$ D. $a = dc$

213. If $a = 2 \log_4 Q$ and $b = \log_4 P$, determine an expression for $\frac{Q}{P}$.

- A. $\frac{a}{2b}$ B. $\frac{2a}{b}$ C. 2^{a-2b} D. 2^{2a-b}

214. Given that $y_1 = \log_a 5$ and $y_2 = \log_a 3$ where $0 < a < 1$, which of the following must be true?

- A. $y_1 > 5$ B. $y_1 < y_2$ C. $y_1 > y_2$ D. $3 < y_2 < 5$

215. The inverse relation of $y = \log 2x$ is given by which one of the following?

- A. $y = \frac{10^x}{2}$ B. $y = 5^x$ C. $y = 10^{2x}$ D. $y = \frac{1}{\log 2x}$

216. Solve for x : $\log_2[\log_x(\log_3 9)] = -1$

- A. 2 B. 3 C. 4 D. 5

217. Evaluate: $\sum_{k=3}^5 \log_k k^2$

- A. 1 B. 2 C. 6 D. 8

218. Evaluate: $\sum_{k=2}^4 \log_6 k$

- A. 0.60 B. 1.23 C. 1.77 D. 4.00

219. Determine the sum of the first 12 terms of the series $\log_b 1 + \log_b 10 + \log_b 100 + \dots$

A. $\frac{66}{\log b}$

B. $\frac{72}{\log b}$

C. 66

D. 72

For the next 6 problems, solve for x :

220. $2 \log(3 - x) = \log 2 + \log(22 - 2x)$

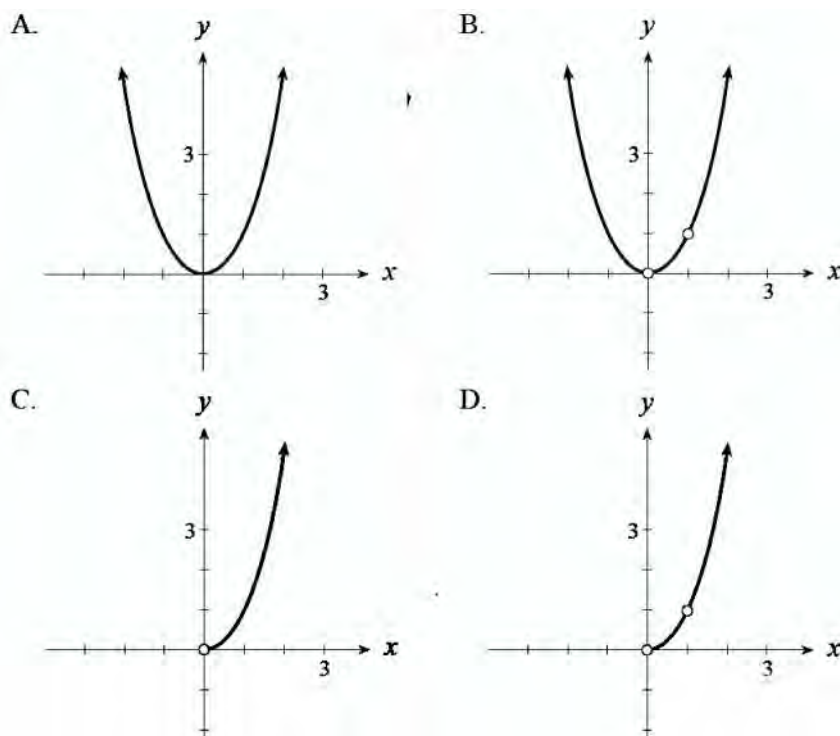
221. $\log_{12}(3 - x) + \log_{12}(2 - x) = 1$

222. $\log_5(2x + 1) = 1 - \log_5(x + 2)$

223. $\log(10 - 3x) - 2 \log x = 0$

224. $\log_4(7 - 3x) + \log_4(x + 4) = 2$

225. $\log_2(x + 7) + \log_2(x + 5) = 3$

226. Which of the following is a graph of $\log_x y = 2$?

227. Determine how many monthly investments of \$50 would have to be deposited into a savings account that pays 3% annual interest, compounded monthly, for the account's future value to be \$50,000. Express your answer as a whole number.

Use the formula: $FV = \frac{R((1+i)^n - 1)}{i}$, where

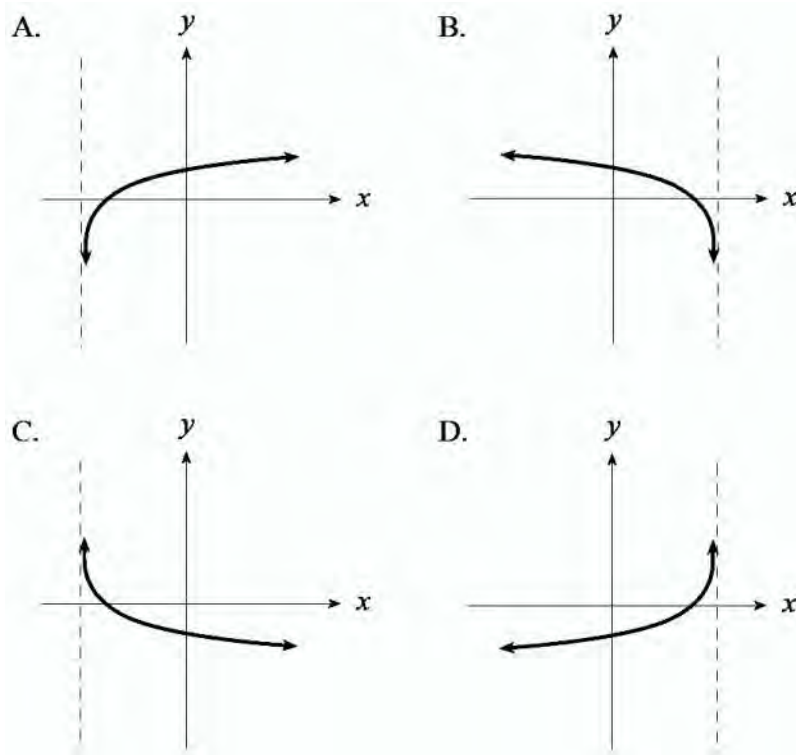
FV = the future value

R = the investment amount

i = (the annual interest rate) ÷ (the number of compounding periods per year)

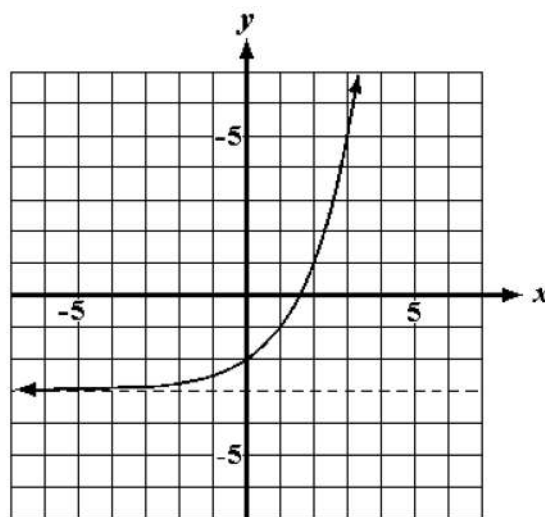
n = the number of investments

228. Which graph best represents the function $y = -\log_3(x + 5)$?

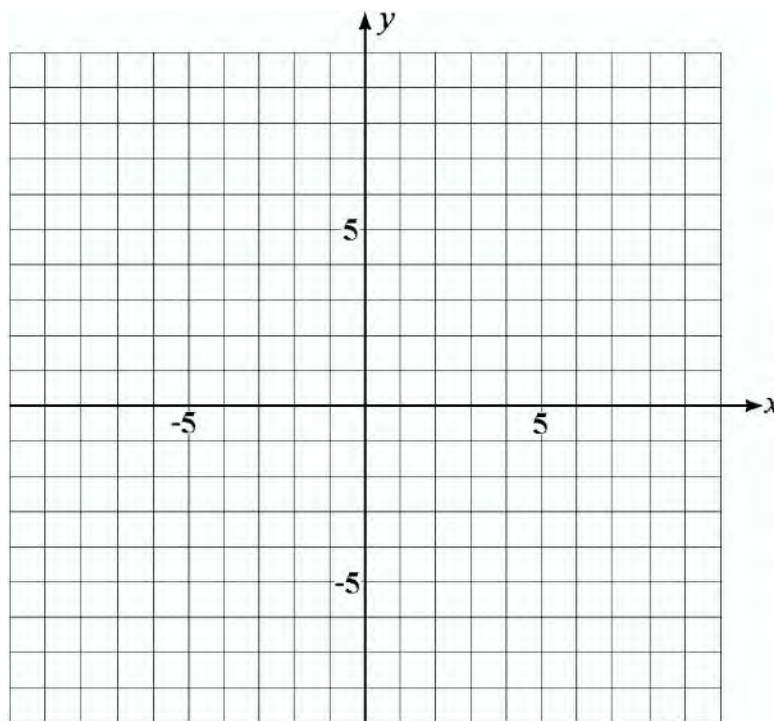


229. The graph of the function $f(x)$ shown is best described by the equation:

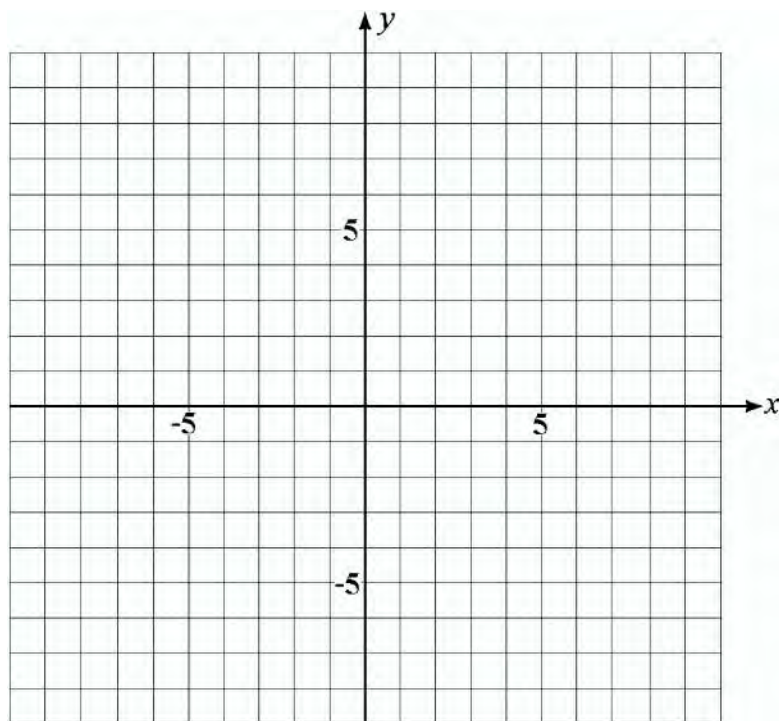
- A. $f(x) = 2^{x+3}$
- B. $f(x) = 2^x + 3$
- C. $f(x) = 2^{x-3}$
- D. $f(x) = 2^x - 3$



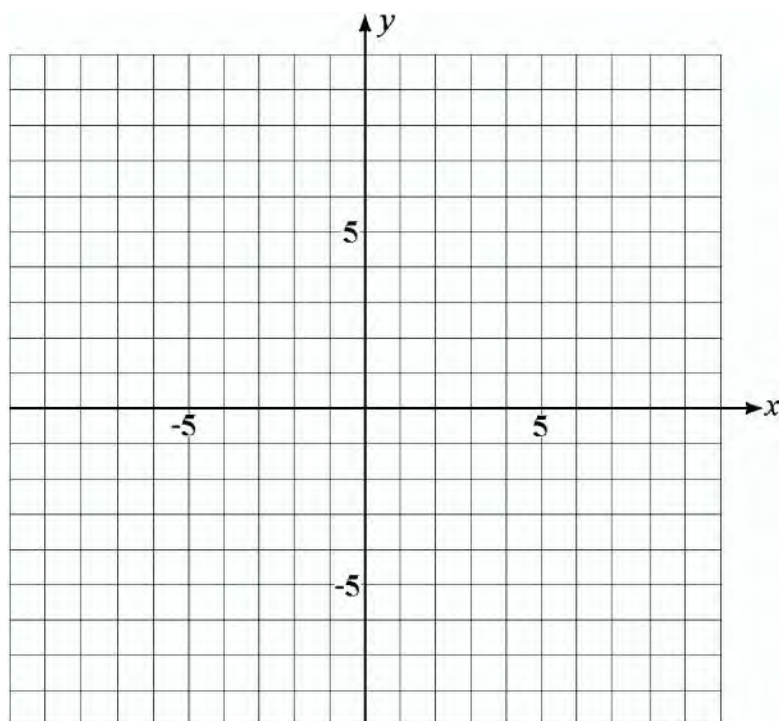
230. Sketch the graphs of: a) $y = 3^x$ and b) $y = 2(3^x) + 1$



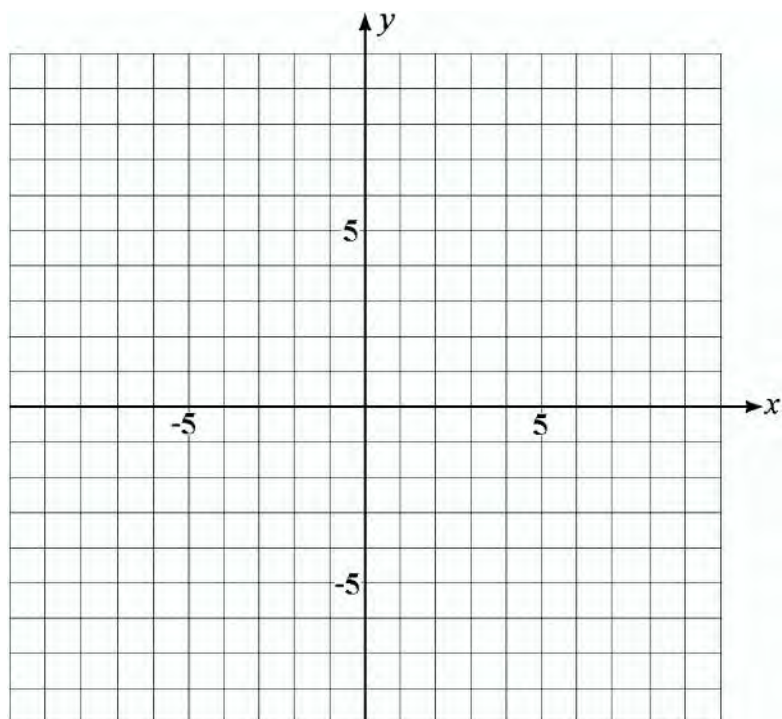
231. Sketch the graphs of: a) $y = \left(\frac{1}{4}\right)^x$ and b) $y = 2\left(\frac{1}{4}\right)^x$



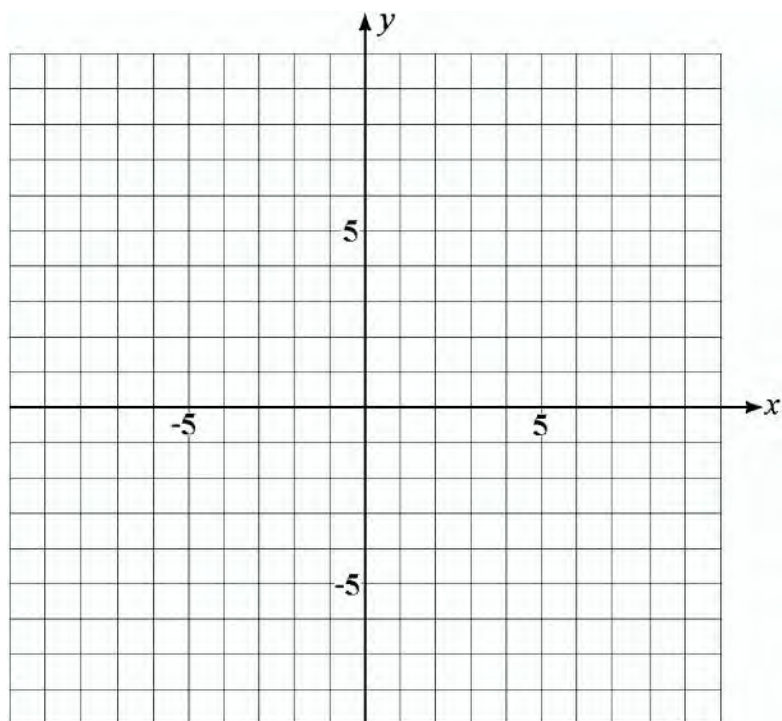
232. Sketch the graphs of: a) $y = 3^x$ and b) $y = \log_3 x$



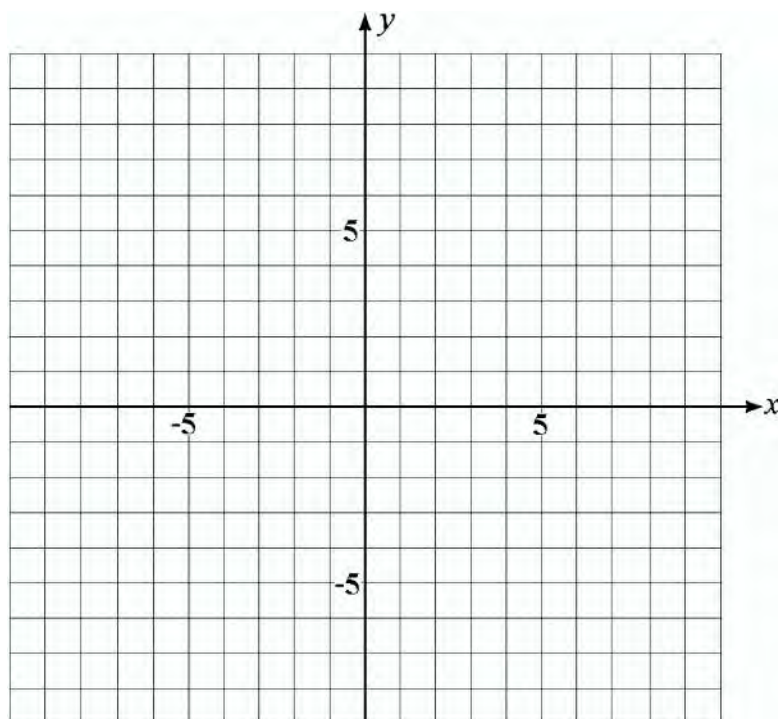
233. Sketch the graph of: $y = \log_5(x+2)$



234. Given the function $f(x) = 3^{x-2} + 1$, sketch the graphs of a) $y = f(x)$ and b) $y = f^{-1}(x)$, and determine the equation of $f^{-1}(x)$.



235. Sketch the graphs of: a) $y = \log_2 x$ and b) $y = \log_2 (-(x - 2))$

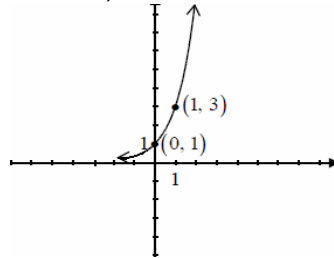


LOGARITHMS ANSWER KEY

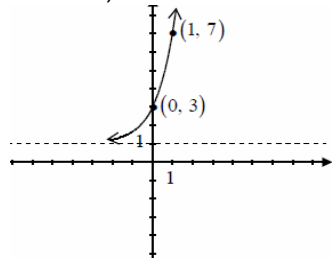
1 D	56 A	109 $x = 8$
2 D	57 C	110 B
3 B	58 A	111 A
4 B	59 D	112 D
5 C	60 $y = \frac{x^2}{27}, x > 0, y > 0, y \neq 1$	113 D
6 D	61 B	114 B
7 C	62 D	115 B
8 15.6 years	63 A	116 A
9 A	64 C	117 C
10 C	65 A	118 40.71 years
11 B	66 D	119 D
12 B	67 D	120 D
13 D	68 C	121 C
14 D	69 A	122 D
15 B	70 C	123 A
16 $x = -2$	71 A	124 A
17 A	72 D	125 C
18 A	73 A	126 $x = -3$
19 D	74 A	127 D
20 C	75 C	128 D
21 B	76 36 848 years	129 D
22 A	77 asymptote: $y = -2$, x -int: $\log_5 2 - 1$, y -int: 3	130 C
23 A	78 D	131 D
24 4 165 600	79 B	132 B
25 B	80 A	133 7.87 days
26 A	81 26 weeks	134 C
27 A	82 B	135 A
28 A	83 D	136 B
29 B	84 A	137 D
30 A	85 B	138 D
31 $x = \frac{5}{2}$	86 C	139 B
32 C	87 D	140 A
33 A	88 B	141 $x = -1.76$
34 A	89 B	142 B
35 A	90 130.03 days	143 C
36 C	91 D	144 D
37 B	92 C	145 C
38 92.88 weeks	93 A	146 C
39 B	94 D	147 A
40 A	95 D	148 D
41 C	96 C	149 A
42 A	97 D	150 B
43 B	98 C	151 A
44 B	99 $x = -3$	152 D
45 D	100 $k = 0.14$	153 D
46 D	101 D	154 $x = -1$
47 A	102 A	155 B
48 D	103 D	156 C
49 D	104 A	157 A
50 B	105 C	158 28.20 days
51 C	106 D	159 D
52 137 years	107 A	160 B
53 $y = 8x, x > 0, y > 0, y \neq 1$	108 B	161 C
54 A		162 A
55 C		163 C

164 D
 165 A
 166 A
 167 C
 168 $x = 7$
 169 A
 170 D
 171 C
 172 C
 173 D
 174 B
 175 A
 176 C
 177 B
 178 $n = 10.87$
 179 D
 180 B
 181 B
 182 C
 183 $x = -2.96$
 184 C
 185 C
 186 C
 187 $x = 6$
 188 B
 189 B
 190 B
 191 B
 192 B
 193 A
 194 C
 195 B
 196 B
 197 A
 198 D
 199 D
 200 23.44 weeks
 201 93.19 h
 202 21.12 m
 203 31.59 years
 204 A
 205 D
 206 A
 207 C
 208 B
 209 B
 210 D
 211 D
 212 C
 213 C
 214 B
 215 A
 216 C

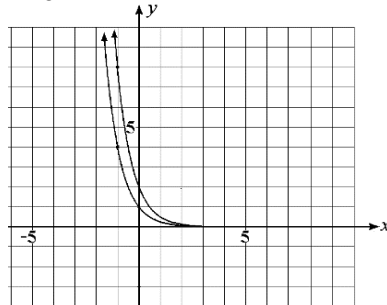
217 C
 218 C
 219 A
 220 $x = -5$
 221 $x = -1$
 222 $x = \frac{1}{2}$
 223 $x = 2$
 224 $x = \frac{4}{3}$ or -3
 225 $x = -3$
 226 D
 227 502
 228 C
 229 D
 230 a)



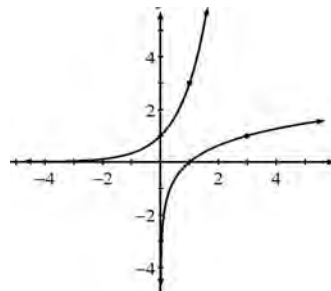
230 b)



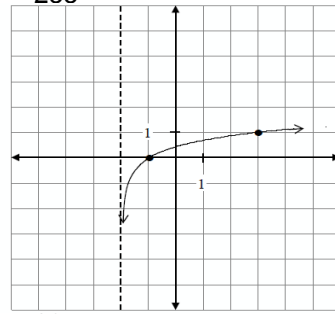
231



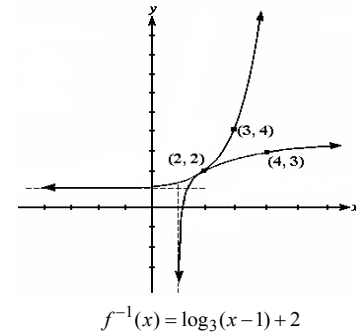
232



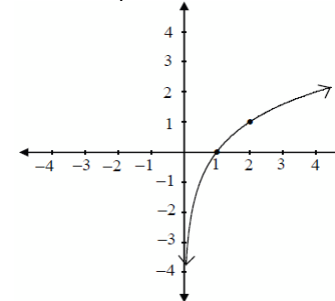
233



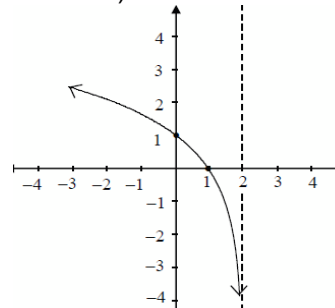
234



235 a)



b)



LAWS OF LOGS WORKSHEET

1. Simplify: $\log_5 5 + \log_3 3^4 + 2^{\log_2 3}$

2. Simplify: $\log_5 125 + \log_4 \sqrt{4}$

3. Solve for x :

a. $\log_3 x = 4$

b. $\log_{\sqrt{3}} 81 = x$

c. $\log_x 64 = -3$

d. $2\log x = \log 4 + \log 9$

e. $3\log x + 2\log x = 25$

f. $\log_4(x+3) + \log_4(x-3) = 2$

g. $\log_{\sqrt{2}} x = \log_{\sqrt{2}} 3 + \log_{\sqrt{2}} 6 - \log_{\sqrt{2}} 2$

h. $\log_8(x+1) - \log_8 x = \log_8 4$

i. $4\log \sqrt[3]{x} + \log \sqrt[3]{x^2} = \log \frac{1}{8}$

j. $\log_5(2x+1) = 1 - \log_5(x+2)$

k. $\log_2[\log_3 x] = 1$

4. Simplify :

a. $(\log_A B)(\log_B C)(\log_C A)$

b. $\log_{\frac{1}{4}} 4$

c. $\log_7(\sqrt{7})^4$

d. $10^{\log M} + \log 10^M$

5. Solve (exact answer and 3 decimal places): $5^{x-1} = 2^{3+x}$

6. Sketch the graphs of the following functions, stating the domain and range:

a. $y = 2^{x+4} + 1$

b. $y = \log_3(x-3) - 5$

7. Sketch the graph: $\log_{(x-2)} y = \log_{(x-2)} x^2$

(represent y as a function of x first, and state restrictions)

8. If $\log_3 x = 3$ and $\log_3 y = 5$, evaluate $\log_3(9x^{-2}y^3)$

9. If $\log_2 x = 4$, evaluate $\log_2(8x^3)$

10. If $\log_2 5 = x$ simplify $\log_2 10\sqrt{5} + \log_2 100$

LAWS OF LOGS WORKSHEET ANSWERS

1. 8

2. 3.5

3. a. 81 b. 8 c. 0.25 d. 6 e. 100 000

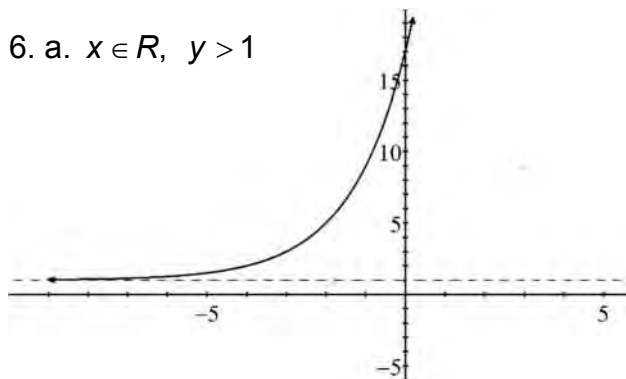
f. 5 g. 9 h. $\frac{1}{3}$ i. $\sqrt{\frac{1}{8}}$

j. $\frac{1}{2}$ k. 9

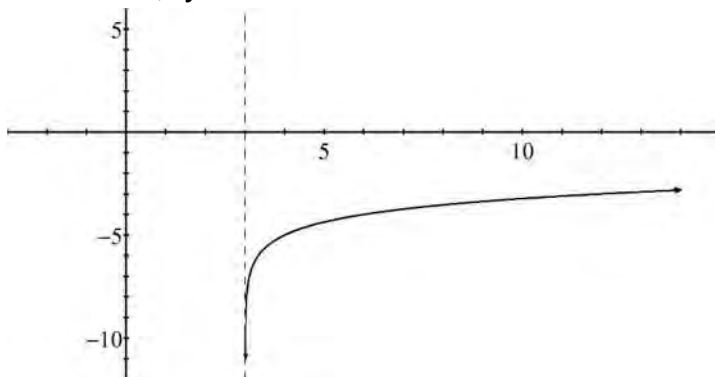
4. a. 1 b. -1 c. 2 d. 2M

5. $\frac{\log 8 + \log 5}{\log 5 - \log 2} \approx 4.026$

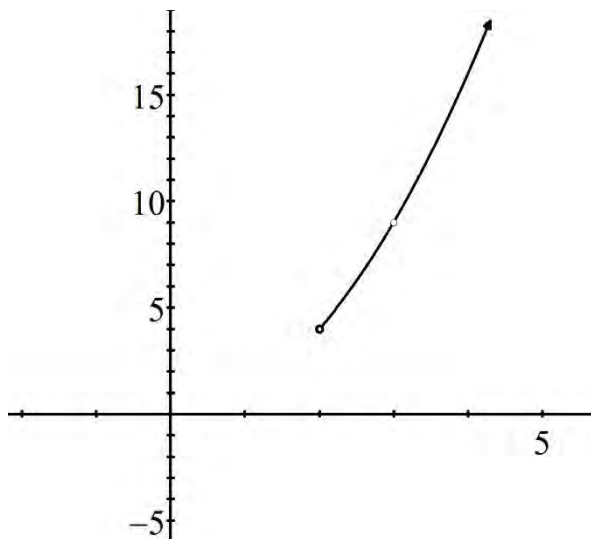
6. a. $x \in \mathbb{R}, y > 1$



b. $x > 3, y \in \mathbb{R}$



7. $y = x^2$, restrictions: $y > 0, x > 2, x \neq 3$



8. 11

9. 15

10. $\frac{7}{2}x + 3$

LOGARITHMS REVIEW

Evaluate/Simplify:

1. $\log_4 \sqrt{128}$

2. $\log_x \frac{1}{\sqrt{x^2}}$

3. $a^{\log_a x} + \log_a a^x$

4. $10^{\log_{100} c}$

5. $\log_9 \sqrt{3^5}$

6. $e^{6 \ln k}$

7. $\log_4 \frac{1}{\sqrt{32}}$

8. $\log_7 7000$

9. $(\log_x y)(\log_y x)$

10. Solve: $\left(\frac{1}{9}\right)^{2x+5} = 27^{(1-x)}$

11. Solve: $\log_x 9 = -\frac{2}{3}$

12. Solve: $5^x = 37$

13. Express $-2 \log_{\pi} a - \frac{1}{2} \log_{\pi} b + \log_{\pi} \left(\frac{a}{3}\right)$ as a single logarithm.

14. Which of the following statements is/are true?

- I. $\log(2 \cdot 9) = \log 18$
 - II. $\log(2 \cdot 9) = \log 2 + \log 9$
 - III. $\log(2 \cdot 9) = \log 2 + 2 \log 3$
 - IV. $\log(2 \cdot 9) = \log 1.8 + 1$
-

15. Which of the following statements is/are true?

- I. $\log(3 + 5) = \log 15$
 - II. $\log(3 \cdot 5) = \log 3 + \log 5$
 - III. $\log(3 + 5) = 3 \log 2$
 - IV. $\log(3 \cdot 5) = \log 10^{\log(15)}$
-

16. If $\log_{16} x = a$, express $\log_{64} x$ in terms of a .

17. If $\log_2 7 = x$, express $\log_2 \frac{49}{16}$ in terms of x

18. Given the function $f(x) = \log_b x$, how is $y = \log_{\frac{1}{b}} x$ related to $f(x)$?

19. A population of 20 million people is growing at 1.7% annually.

- a) What will the population be in 10 years?
 - b) When will the population reach 30 million?
 - c) Determine the doubling time.
-

20. A poison has a tenth-life (reduces to one-tenth) in the bloodstream of 720 minutes.

- a) Determine the half-life of the poison.
 - b) How much of a 1.20 g sample would remain after 300 minutes?
-

21. An iron pipe is rusting so that it loses 9% of its mass each year. What fraction of its original mass will remain after 10 years?

22. Consider an earthquake registering 7.7 on the Richter Scale:

- a) Determine the Richter Scale magnitude of an earthquake that is 7000 times more powerful.
 - b) Determine the Richter Scale magnitude of an earthquake that has half the intensity.
-

23. A population of insects can double in 30 days.

- a) By what factor will the population have grown in an 80 day period?
 - b) How long will it take the population to triple?
-

24. An amount is invested at an interest rate of 6.4% p.a. compounded twice every day.

- a) Determine the effective annual interest rate
 - b) Calculate the number of years for an investment of \$1800 to grow to \$5000.
-

25. Determine the half-life of a decaying radioactive material of which 0.4%, or $\frac{4}{1000}$, remains after 420 000 years.

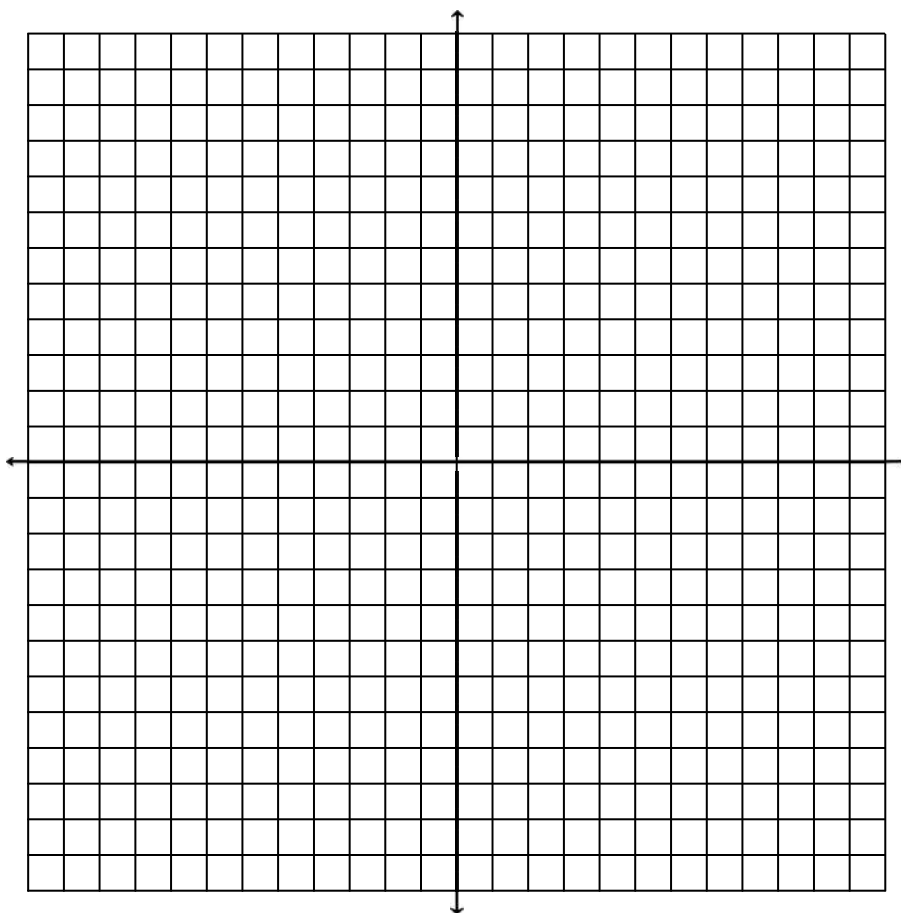
26. A population increases by a factor of 1.50 every 8 years.

- a) If the population now is 1.8 million, estimate the population 20 years earlier.
 - b) How long will it take the population to reach 4 million?
 - c) If this growth is expressed in the form $y = y_0 e^{kt}$, determine the growth constant "k".
-

27. What annual percent growth will increase an amount by a factor of 2 in seven years?

28. Consider the function $y = \log_4(x + 2) - 3$

a)



Sketch a graph of the function showing any asymptotes and their equations, and labelling EXACT coordinates of at least two points on the graph.

b) Find the x and y intercepts (exact) of the function by algebra.

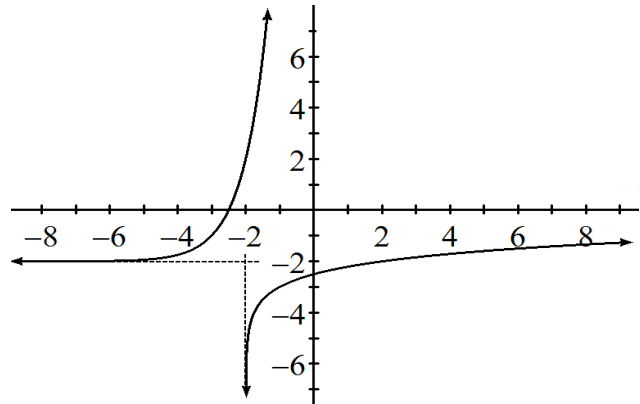
c) Graph the inverse of $f(x)$ on the same grid, and also determine the equation of $f^{-1}(x)$.

29. Solve algebraically, leaving answer in exact form: $3^{2x-5} = 7(5)^{x+1}$

30. Solve algebraically: $2\log_2(3 - x) - \log_2(x + 5) = 2$

LOGARITHMS REVIEW ANSWER KEY

1. $\frac{7}{4}$
2. $-\frac{2}{3}$
3. $2x$
4. \sqrt{c}
5. $\frac{5}{8}$
6. k^6
7. -5
8. ≈ 4.55
9. 1
10. -13
11. $\frac{1}{27}$
12. ≈ 2.24
13. $\log_{\pi}\left(\frac{1}{3a\sqrt{b}}\right)$
14. all true
15. all true except I
16. $\frac{2a}{3}$
17. $2x - 4$
18. $-f(x)$ (reflected in x -axis)
19. a) 23.67 million
b) 24.05 years
c) 41.11 years
20. a) 216.74 min.
b) 0.46 g
21. 0.389
22. a) 11.55
b) 7.399
23. a) 6.35
b) 47.55 days
24. a) 6.6089 %
b) 15.964 years
25. 52 726 years
26. a) 653 000
b) 15.75 years
c) $\frac{\ln 1.5}{8} \approx 0.050683$
27. 10.41%
28. a)



b) x -int: 62; y -int: -2.5

c) $f^{-1}(x) = 4^{x+3} - 2$

29. $x = \frac{5 \log 3 + \log 7 + \log 5}{2 \log 3 - \log 5}$

30. $x = -1$