# PRECALCULUS MATHEMATICS 12

# **TABLE OF CONTENTS**

#### **COMBINING FUNCTIONS**

Review of basic functions;

Addition, subtraction, multiplication and division of functions; composition of functions.

• 48 PROBLEMS

#### **TRANSFORMATIONS**

Transformations of graphs and equations (vertical and horizontal translations, stretches, and reflections) of parent functions and relations; inverses.

• 118 PROBLEMS

#### **POLYNOMIALS**

Factoring (Factor Theorem and the Remainder Theorem), the Rational Zero Theorem and polynomial division;

Graphing and polynomial characteristics;

Solving equations and inequalities algebraically and graphically.

• 183 PROBLEMS

#### RADICAL & RATIONAL FUNCTIONS

Characteristics and behaviour of graphs, including asymptotes (horizontal, vertical and oblique), intercepts and point discontinuities.

• 51 PROBLEMS

#### LOGARITHMIC and EXPONENTIAL FUNCTIONS

Graphing and characteristics of exponential and logarithmic functions including transformations;

Applying laws of logarithms;

Solving equations with the same and with different bases, including base e; Applications.

- 235 PROBLEMS
- LAWS OF LOGARITHMS WORKSHEET
- REVIEW PACKAGE

#### TRIGONOMETRY I

Examining angles in standard position in both radians and degrees;

Reference and coterminal angles; arc length;

Reciprocal trigonometric functions;

Solving first-degree equations;

Graphing and characteristics of primary trigonometric functions, including transformations.

- 165 PROBLEMS
- REVIEW PACKAGE

# TRIGONOMETRY II (EQUATIONS and IDENTITIES)

Trigonometric identities (Pythagorean, quotient, double angle, reciprocal, and sum and difference identities) and two-column proofs;

Solving second-degree equations (over restricted domains and all real numbers).

- 135 PROBLEMS
- TRIGONOMETRIC IDENTITIES PACKAGE
- REVIEW OF SUM and DIFFERENCE & DOUBLE ANGLE IDENTITIES WORKSHEET

# **CONIC SECTIONS**

The parabola, circle, ellipse and hyperbola: equations and graphs; General to Standard Form;

Transformations and applications.

• 194 PROBLEMS

#### **GEOMETRIC SEQUENCES and SERIES**

Common ratio, first term, general term;

Infinite geometric series and summation notation.

• 121 PROBLEMS

# GEOMETRIC SEQUENCES and SERIES

- 1. Determine the sum of the infinite geometric series: -45 + 30 20 + ...
  - A. -135

B. −35

C. -27

- D. there is no finite sum
- 2. In a geometric sequence  $t_3 = -36$  and  $t_6 = 972$ . Find the value of  $t_1$ .
  - A. -4

- B. -3
- C. 3

- D. 4
- 3. Given that  $2^x$ ,  $8^y$ , k is a geometric sequence, determine k.
  - A.  $2^{6y-x}$
- B.  $2^{3y-x}$
- C.  $2^{10y-5x}$
- D.  $2^{2y-2x}$

# JUN 1997

- 4. Determine a single geometric mean between 1 and 4.
  - A. 1.5

**B.** 2

C. 2.5

- D. 3
- 5. Determine the 20th term of the geometric sequence 2, 2.5, ... (Accurate to 2 decimal places.)
  - A. 11.50
- B. 111.02
- C. 138.78
- D. 173 47
- 6. Find the sum of the infinite geometric series:  $2-1+\frac{1}{2}-\frac{1}{4}+\dots$ 
  - A. 0

- B.  $\frac{5}{4}$
- C.  $\frac{4}{3}$
- D. 4

- Which of the following is a geometric sequence? 7.
  - A. 10, 20, 30, 40
- B. 2, 8, 12, 16
- C. 12,36,72,144 D. 8,16,32,64
- If the sum of an infinite geometric series is 90 and the common ratio is  $-\frac{1}{5}$ , determine the 8. value of the first term.
  - A. 72

B. 75

C. 108

- D. 112.5
- Find the sum of the first 30 terms of the geometric series 10+12+14.4+... (Accurate to at 9. least 2 decimal places.)

# **JUN 1998**

- 10. Determine the common ratio of the geometric sequence: -2,  $6\sqrt{2}$ , -36, ...
  - A.  $-3\sqrt{2}$
- B.  $-\frac{1}{3\sqrt{2}}$  C.  $\frac{1}{3\sqrt{2}}$
- D.  $3\sqrt{2}$
- 11. Determine the second term of the series defined by  $\sum_{i=0}^{5} 3\left(\frac{1}{2}\right)^{j-1}$ .
  - A.  $-\frac{3}{2}$
- B.  $\frac{3}{2}$

C. 3

- D. 6
- $20 4 + \frac{4}{5} \frac{4}{25} + \dots$ 12. Find the sum of the infinite geometric series:
  - A.  $\frac{10}{3}$

B.  $\frac{50}{3}$ 

C. 24

- 13. Find the sum of the infinite geometric series:  $8 6 + \frac{9}{2} \frac{27}{8} + \dots$ 
  - A. -32
- B.  $-\frac{32}{7}$  C.  $\frac{32}{7}$

D. 32

- 14. Which of the following is equivalent to  $\sum_{k=1}^{20} 3^{k+2}$ ?
  - A.  $\frac{27(1-3^{20})}{-2}$  B.  $\frac{27(1-3^{19})}{-2}$  C.  $\frac{3(1-3^{20})}{-2}$  D.  $\frac{3(1-3^{19})}{-2}$

# **JUN 1999**

- 15. Find the sum of the infinite geometric series:  $-45 + 30 20 + \dots$ 
  - A. -135

B. -35

C. -27

- D. there is no finite sum
- 16. Given a geometric series with a first term of 14 and a common ratio of 1.8, determine the sum of the first 10 terms.
  - A. -4986.45
- B. 2777.03
- C. 3453.79
- D. 6230.82
- 17. Determine the number of terms in the following geometric sequence:

$$\frac{a^2}{b}$$
, a, b, ...,  $\frac{b^{15}}{a^{14}}$ 

A. 14

B. 15

C. 16

D. 17

# **JAN 2000**

- 18. Determine a formula for  $t_n$  in terms of n for the geometric sequence 2, 6, 18, 54, ...
  - A.  $t_n = 2(3)^n$
- B.  $t_n = 2(3)^{n-1}$  C.  $t_n = -1 + 3n$  D.  $t_n = 2 + 3n$
- 19. Determine two positive geometric means between 64 and 1.

  - A. 16, 4 B. 32, 16 C. 48, 16 D. 32, 8

- 20. When two geometric means are inserted between 9 and 243, which of the following is one of them?
  - A. 78

B. 81

C. 87

- D. 165
- 21. Find the sum of the first 20 terms of the geometric sequence 125, 100, 80, 64, ...
  - A. -2250
- B. 37489
- C. 617.79
- D. 42868.09
- 22. For what values of x,  $x \neq \frac{1}{2}$ , will the following infinite geometric series have a finite sum?  $1 + (2x 1) + (2x 1)^2 + (2x 1)^3 + \dots$
- A. -1 < x < 0 B. -1 < x < 1 C.  $-\frac{1}{2} < x < \frac{3}{2}$  D. 0 < x < 1

- 23. Evaluate:  $\sum_{k=2}^{4} \log_6 k$ 
  - A. 0.60
- B. 1.23
- C. 177
- D. 4.00

# **JAN 2001**

- 24. Determine the sum of the infinite geometric series:  $100 - 60 + 36 - \dots$ 
  - A. 62.5
- B. 76

C. 160

- D. 250
- 25. Determine two positive geometric means between 16 and 800.
  - A. 42 55, 113 14
- B. 58.94, 217.15
- C. 212, 408
- D. 277.33, 538.67
- 26. Which expression represents the sum of the series  $\sum_{k=1}^{n} 2(3)^k$ ?

- A.  $-2(1-3^n)$  B.  $-3(1-3^n)$  C.  $-6(1-3^n)$  D.  $-2(1-3^{n-1})$

- 27. What is the single positive geometric mean between 1 and 4?
  - A. 1.5

**B.** 2

C. 2.5

D. 3

- 28. Which of the following sequences is arithmetic?
  - A.  $\sqrt{2}$ , 2,  $2\sqrt{2}$ , 4

B.  $\sqrt{2}$ ,  $2\sqrt{2}$ ,  $4\sqrt{2}$ ,  $6\sqrt{2}$ 

C.  $\sqrt{2}$ ,  $2\sqrt{2}$ ,  $3\sqrt{2}$ ,  $4\sqrt{2}$ 

- D.  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{4}$ ,  $\sqrt{5}$
- 29. Determine the sum of the infinite geometric series: 24 12 + 6 3 + ...
  - A. 8

B. 16

C. 48

- D. there is no finite sum
- 30. Determine the sum of the first 15 terms of the geometric series: 3+9+27+...
  - A. 7174452
- B. 14348907
- C. 21523359
- D. 43046721

#### **SAMPLE 2001**

- 31. Determine the common ratio of the geometric sequence: 3,  $\frac{2}{5}$ ,  $\frac{4}{75}$ , ...
  - A.  $\frac{2}{15}$
- B.  $\frac{3}{10}$
- C.  $\frac{2}{5}$
- D.  $\frac{6}{5}$
- 32. A geometric sequence has a common ratio of 2 and the 12th term in the sequence is 16 384. Determine the first term.
  - A. 2

B. 4

C. 8

- 33. Evaluate:  $\sum_{k=2}^{6} 2(3)^{k-2}$ 
  - A. 93

B. 186

- C. 242
- D. 728
- 34. In a geometric sequence  $t_3 = 45$  and  $t_6 = 1215$ . Determine the first term.
  - A.  $\frac{3}{5}$

C. 3

- D. 5
- 35. In a sequence,  $t_n = \sum_{k=1}^n \left(\frac{1}{x}\right)^{k-1}$ . If  $t_\infty = 3$ , determine the value of x.
  - A.  $\frac{2}{3}$  B.  $\frac{3}{4}$  C.  $\frac{4}{3}$

D.  $\frac{3}{2}$ 

# **JAN 2002**

- 36. Determine the common ratio of the geometric sequence 8, 12, 18, 27, ...
  - A.  $\frac{2}{3}$

- B.  $\frac{4}{3}$  C.  $\frac{3}{2}$

- D. 4
- 37. An aquarium originally containing 30 litres of water loses 6% of its water to evaporation every day. Determine a geometric sequence which shows the number of litres of water in the aquarium on consecutive days.
  - A. 30, 30(0.94),  $30(0.94)^2$ ,  $30(0.94)^3$ , ...
  - B. 30, 30(0.06),  $30(0.06)^2$ ,  $30(0.06)^3$ , ...
  - C. 30,  $\frac{30}{1.06}$ ,  $\frac{30}{(1.06)^2}$ ,  $\frac{30}{(1.06)^3}$ , ...
  - D. 30,  $\frac{30}{0.94}$ ,  $\frac{30}{(0.94)^2}$ ,  $\frac{30}{(0.94)^3}$ , ...
- 38. Which term of the geometric sequence 5, 15, 45, ... is 885 735?
  - A. 10th
- B. 11th
- C. 12th
- D. 13th

- 39. Determine an expression which represents  $\sum_{k=1}^{n} 4(5)^{k-1}$ 
  - A.  $4(5)^n$
- B.  $4(5)^{n-1}$
- C.  $1-(5)^n$
- D.  $(5)^n 1$
- 40. A ball is dropped from a height of 5 m. After each bounce, it rises to 60% of its previous height. What is the total vertical distance the ball travels before it comes to rest?
  - A. 12.5 m
- B. 15 m
- C. 20 m
- D. 25 m

# **APR 2002**

- 41. Determine the common ratio of the geometric series:  $3-1+\frac{1}{3}-\frac{1}{9}+\ldots+\frac{1}{243}$ 
  - A. -3
- B.  $-\frac{2}{3}$  C.  $-\frac{1}{3}$
- D.  $\frac{1}{3}$
- 42. Calculate the 9th term of the geometric sequence: 8 000, 4 000, 2 000, ...
  - A. 8

- B. 15.625
- C. 31.25
- D. 2 048 000
- 43. If the sum of the first 5 terms of a geometric series is -328 and the common ratio is -4, determine the first term.
  - A. -3.86
- B. -1.6
- C. 0.96
- D. 643

- 44. Evaluate:  $\sum_{k=1}^{\infty} 50 \left(\frac{1}{4}\right)^k$ 
  - A.  $\frac{75}{8}$  B.  $\frac{50}{4}$

- C.  $\frac{50}{3}$
- D.  $\frac{200}{3}$
- 45. For a geometric sequence,  $t_7 = 5x + 2$  and  $t_{10} = x 23$ . If the common ratio, r, is 2, find the value of  $t_{10}$ 
  - A. -26
- B. −24
- C. -12
- D. -3

- 46. The general term of a geometric sequence is  $t_n = 5(-2)^{n-1}$ . Determine the common ratio.
  - A. -5
- B. −2

D. 5

- 47. Determine the first term in the expansion of  $\sum_{k=2}^{8} 3(2^k)$ 
  - A. 3

B. 6

C. 12

- D. 36
- 48. Determine the number of terms in the geometric sequence:  $\frac{1}{128}$ ,  $\frac{1}{32}$ ,  $\frac{1}{8}$ , ..., 2 048
  - A. 8

B. 9

C. 10

- D. 11
- 49. Find the sum of the infinite geometric series:  $3 1 + \frac{1}{3} \frac{1}{9} + \dots$ 
  - A.  $\frac{20}{9}$

B.  $\frac{9}{4}$ 

c.  $\frac{9}{2}$ 

- D. there is no finite sum
- 50. A doctor prescribes medication to be taken for 9 days. The amount taken on the first day is 270 mg. On each successive day, the amount taken is one half the amount taken on the previous day. What is the total amount of medication taken? (Accurate to the nearest mg.)
  - A. 538

B. 539

- C. 540
- D. 541

# **AUG 2002**

- 51. Determine the common ratio of the geometric series: -4, -1,  $-\frac{1}{4}$ , ...
  - A.  $\frac{1}{4}$
- B.  $-\frac{1}{4}$  C. 4

- D. -4
- 52. Bob worked for a company for 5 years. His starting annual salary was \$38 000. Each year his salary was increased by 2% over the previous year's salary. What is the total amount of money Bob earned with this company?
  - A. \$156 621 10
- B. \$193 200.00
- C. \$197 753.53 D. \$239 7081.60

- 53. Evaluate:  $\sum_{k=3}^{7} 5(2)^k$ 
  - A. 600
- B. 635

- C. 1240
- D. 1270
- 54. The 3rd term of a geometric sequence is 48 and the 6th term is  $\frac{81}{4}$ . Find the first term of the sequence.
  - A. 3

- B. 27
- C.  $\frac{256}{3}$
- D. 768

- 55. Determine the common ratio of the geometric sequence: -64, 48, -36
  - A.  $-\frac{4}{3}$
- B.  $-\frac{3}{4}$  C.  $\frac{3}{4}$

- D.  $\frac{4}{2}$
- 56. If the sum of *n* terms of the geometric sequence 4, 8, 16, ... is  $S_n = \frac{4(1-2^5)}{1-2}$ , determine the value of *n*. the value of n.
  - A. 2

B. 4

C. 5

- D. 6
- 57. In the geometric sequence  $\frac{1}{8}$ ,  $\frac{1}{2}$ , 2, ..., which term is 524 288?
  - A. 11

B. 12

C. 13

- D. 14
- $\sum_{k=1}^{\infty} 200(0.6)^{k-1}$ 58. Determine the sum of the infinite geometric series:
  - A. 125

B. 133

C. 500

- D. no finite sum
- 59. If x, 4, 8x are three consecutive terms in a geometric sequence, determine the values of
  - A.  $\pm 1$

- B.  $\pm \sqrt{2}$
- C. ±2

D.  $\pm 2\sqrt{2}$ 

- 60. Determine the common ratio of the geometric sequence:  $24, -36, 54, -81, \dots$ 
  - A.  $-\frac{3}{2}$
- B.  $-\frac{2}{3}$  C.  $\frac{2}{3}$

- D.  $\frac{3}{2}$
- 61. Calculate the 15th term of the geometric sequence: 1.1, 1.32, 1.584, 1.9008, ...
  - A. 14.12
- B. 16.95
- C. 48.76
- D. 64.36
- 62. Determine the sum of the first 10 terms of the geometric series: 100 + 250 + 625 + ...
  - A. 254 246.48
- B. 381 469 73
- C. 635 716.21
- D. 953 674.32

- 63. Evaluate:  $\sum_{k=3}^{5} \log k$ 
  - A. log 12
- B. log 15
- C. log 60
- D. log 120
- 64. A ball is dropped from a height of 10 m. After each bounce, it rises to 80% of its previous height. What is the total vertical distance the ball travels before it comes to rest?
  - A. 22.5 m
- B. 50 m
- C. 90 m
- D. 100 m

# **JAN 2004**

- 65. Determine the third term of the geometric series: x, 3x, , where x = 0.
  - A. 5x

B. 6x

C. 9x

- D.  $3x^{2}$
- 66. How many terms are there in the series defined by  $\sum_{k=4}^{31} 2(3)^{k-1}$ ?
  - A. 27

B. 28

C. 30

- 67. Determine the sum of the first 10 terms of the geometric series defined by  $\frac{2}{3} 2 + 6 18 + \dots$ 
  - A. -984133
- B. 3 280 67
- C. 9 841 67
- D. 19 682 67
- 68. In a geometric sequence,  $t_2 = 480$  and  $t_7 = -15$ . Determine the common ratio r.
  - A. -3

- B. −2
- C.  $-\frac{1}{3}$
- D.  $-\frac{1}{2}$
- 69. Consider the geometric sequence 1, (a+b),  $(a+b)^2$ , ... Which term of this geometric sequence, when expanded, contains the expression  $35a^4b^3$ ?
  - A. 5th term
- B. 6th term
- C. 7th term
- D. 8th term

- $\frac{1}{20}$ ,  $\frac{1}{5}$ ,  $\frac{4}{5}$ , ... 70. Determine the 9th term of the geometric series:
  - A. 3 276.80
- B. 13 107 20
- C. 19 531.25 D. 97 656.25

- 71. Evaluate:  $\sum_{k=1}^{8} 5(3)^k$ 
  - A. 5 465
- B. 16 395
- C. 16 400
- D. 49 200

- 72. Which infinite geometric series has a finite sum?
  - A.  $\frac{1}{2} 1 + 2 4 + \dots$

B.  $64 + 48 + 36 + 27 + \dots$ 

C.  $\frac{1}{24} + \frac{1}{12} + \frac{1}{6} + \frac{1}{3} + \dots$ 

D.  $16 - 20 + 25 - 31.25 + \dots$ 

- 73. Determine a value for k for the geometric series: 8, k, 20
  - A.  $\frac{\sqrt{10}}{2}$
- B.  $2\sqrt{10}$
- C.  $4\sqrt{5}$  D.  $4\sqrt{10}$
- 74. A new well produced 48 000 L of water in the first month. If the volume of water pumped decreases by 6% each month, determine the total volume of water, in litres, that will be pumped from the well before it runs dry.
  - A. 51 063 83
- B. 93 120
- C. 752 000
- D. 800 000

# **AUG 2005**

- 75. Determine the common ratio of the geometric series:  $3\sqrt{3}$ , 3,  $\sqrt{3}$ , ...
  - A.  $\sqrt{3}$
- B.  $3\sqrt{3}$
- C.  $\frac{1}{\sqrt{3}}$
- D.  $\frac{3}{\sqrt{3}}$
- 76. In the geometric sequence 162, 108, 72, ..., determine the 10th term.
  - A. -4.21
- B. -1.87
- C. 2.81
- D. 6.32
- 77. The sum of the first 8 terms of a geometric series is 16 400. If the common ratio of this senes is 3, determine the first term.
  - A. -128 13
- B. 5

- C. 7.5 D. 15
- 78. An infinite geometric series has all positive terms. If  $t_1 = 64$  and  $t_3 = 1$ , determine the sum of this series.
  - A.  $\frac{256}{5}$
- B.  $\frac{512}{9}$
- C.  $\frac{512}{7}$
- D.  $\frac{256}{3}$

- 79. Evaluate:  $\sum_{k=1}^{4} {}_{4}C_{k}$ 
  - A. 1

B. 4

C. 15

#### **AUG 2006**

- $3 + \frac{6}{5} + \frac{12}{25} + \dots$ 80. Determine the sum of the infinite geometric series:
  - A. 5

B. 7.5

- C. 12.5
- D. 15
- 81. Determine the sum of the first 10 terms of the geometric series:  $-36 + 24 16 + \dots$ 
  - A. -21 23
- B. -21.60
- C. -21 97
- D. -22.16
- 82. Determine the first term of the geometric series defined by  $\sum_{k=3}^{7} 5\left(-\frac{1}{2}\right)^{k-1}$ .
  - A.  $-\frac{5}{2}$
- B.  $-\frac{5}{8}$  C.  $\frac{5}{4}$

- D. 5
- 83. A ball is dropped from a height of 4 m. After each bounce, it rises to 70% of its previous height. What is the maximum height the ball will reach after it hits the ground for the 5th time?
  - A. 0.47 m
- B. 0.67 m
- C. 0.96 m
- D. 11.09 m
- 84. If x, 6, 3x are three consecutive terms in a geometric sequence, determine the values of x.
  - A.  $\pm 1$

- B.  $\pm \sqrt{3}$  C.  $\pm 2$

D.  $\pm 2\sqrt{3}$ 

# **SAMPLE 2008**

- 85. Determine the common ratio of the geometric sequence:  $\frac{a^2}{b^3}$ ,  $\frac{a}{b}$ , b.
  - A.  $\frac{a}{b}$

- B.  $\frac{b}{a}$
- C.  $\frac{a}{b^2}$

Sam gave his nephew, Norman, \$1 on his first birthday, \$2 on his second birthday, \$4 on his third birthday, and so on. That is, on each subsequent birthday, Sam gave Norman double the previous year's amount.

- 86. How much money did Sam give Norman on his 15th birthday?
  - A. \$16 333
- B. \$16 384
- C. \$32 767
- D. \$32 768
- 87. In total, how much money did Sam give Norman up to and including his 21st birthday'?
  - A. \$1 048 575
- B. \$1 048 576
- C. \$2 097 151
- D. \$2 097 152

- 88. Evaluate:  $\sum_{k=2}^{\infty} (-0.3)^k$ 
  - A. -0.23
- B. 0.07
- C. 0.13
- D. 0.77
- 89. In a geometric sequence,  $t_4 = 108$  and  $t_6 = 243$ . Determine a possible first term.
  - A.  $\frac{3}{2}$

- B.  $\frac{64}{3}$
- C. 32

- D. 48
- 90. If x-2, x+4, 5x+2 are three consecutive terms in a geometric sequence, determine the numerical value(s) of the common ratio(s).
  - A. -1

- B. -4, -1 C. -3, 3 D. 3, -1

# **JAN 2008**

- 91. Which of the following is a geometric sequence?

- A. 1, 4, 9, ... B. 2, 4, 10, ... C. 12, 7, 2, ... D. 18, 12, 8, ...
- 92. Determine the number of terms in the geometric sequence: 3, 6, 12, ..., 49 152

A. 13

B. 14

C. 15

- 93. Determine the sum of the infinite geometric series:  $16 12 + 9 \dots$ 
  - A.  $\frac{48}{7}$
- B.  $\frac{64}{7}$

- C. 64 D. no finite sum

- 94. Evaluate:  $\sum_{k=2}^{12} 32 \left(-\frac{1}{2}\right)^k$ 
  - A. -21.35
- B. -21.31
- C. -2.67 D. -2.66
- 95. The 4th term of a geometric sequence is 250 and the 7th term is -16. Determine the 10th
  - A.  $-\frac{2}{5}$
- B.  $\frac{2}{5}$  C.  $-\frac{128}{125}$  D.  $\frac{128}{125}$
- 96. For what values of x, (x = 0) will the following infinite geometric series have a finite sum?  $x + 3x^2 + 9x^3 + \dots$ 

  - A. -3 < x < 0 B.  $-\frac{1}{3} < x < \frac{1}{3}$  C. -1 < x < 1 D. -3 < x < 3

# 2009 SAMPLE QUESTIONS

- 97. Determine the common ratio of the geometric sequence 1,  $-\frac{1}{3}$ ,  $\frac{1}{9}$ ,  $-\frac{1}{27}$ 
  - **A.** −3
- B.  $-\frac{1}{3}$  C.  $\frac{1}{3}$

- D. 3
- 98. Determine the 14th term of the geometric series:  $6 + 12 + 24 + \dots$ 
  - A. 12 288
- B. 24 576
- C. 49 152
- D. 98 304
- 99. The second term of a geometric series is -16 and the seventh term is 512. Determine the first term.
  - A. -2

B. 2

C. -8

- 100. If x-1, x+6, 3x+4 are the first three terms in a geometric sequence, determine the possible values of the first term.

  - A.  $-9, \frac{3}{2}$  B.  $-8, \frac{5}{2}$  C.  $-\frac{7}{2}, 7$  D.  $-\frac{5}{2}, 8$

- 101. In the World Dominoes tournament, 78 125 players are grouped 5 players at each table. One game is played by these 5 players and the winner at each table advances to the next round, and so on until the ?nal game of 5 players. How many rounds would the ultimate winner have played (including the final round)?
- 102. The first and second terms of a geometric sequence have a sum of 15, while the second and third terms have a sum of 60. Use an algebraic method to find the three terms.

- 103. Determine the number of terms in the series defined by  $\sum_{k=12}^{38} 3(2)^{k-1}$ .
  - A. 26

B. 27

C. 37

- D. 38
- 104. While training for a race, a runner increases her distance by 10% each day. If she runs 2 km on the first day, what will her total distance be for 26 days of training? (Accurate to 2 decimal places.)
  - A. 21.67 km
- B. 23.84 km
- C. 196.69 km
- D. 218.36 km
- 105. If the sum of *n* terms in a geometric series is given by the expression  $S_n = 4(3^n 1)$ , determine  $t_4$ .
  - A. 108
- B. 160

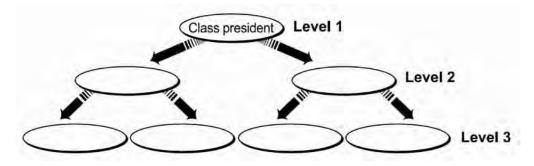
- C. 216
- D. 320

- 106. Evaluate:  $\sum_{k=3}^{5} \log_k k^2$ 
  - A. 1

B. 2

C. 6

- D. 8
- 107. A graduation class informs its members of changes in plans by telephone. The president of the class calls two members, each of whom in turn calls two members, and so on, as shown in the diagram. By the 9th level, all members of the graduation class have been contacted. Determine the maximum number of students in total in the graduation class.



- $800 + 300 + \frac{225}{2} + \dots$ 108. Determine the sum of the infinite geometric series:
  - A. 1280

B. 1212.5

C. 1254.69

D. there is no finite sum

- 109. Evaluate:  $\sum_{n=1}^{\infty} \frac{1}{3^n}$ 
  - A.  $\frac{1}{3}$

B.  $\frac{1}{2}$ 

C.  $\frac{2}{3}$ 

- D. 1
- 110. For what values of x, x = 1, will the following infinite geometric series have a finite sum?  $(x+1) + (x+1)^2 + (x+1)^3 + \dots$ 
  - A. -1 < x < 0

- B. -1 < x < 1 C. -3 < x < l D. -2 < x < 0

- 111. A ball is dropped from a height of 5 m. After each bounce, it rises to 60% of its previous height.
  - a) What is the maximum height the ball will reach after it hits the ground for the 4th time?
  - b) What is the total vertical distance the ball travels by the time the ball hits the ground for the 7th time?
  - c) What is the total vertical distance the ball travels before it comes to rest?
- 112. Find the sum of the infinite geometric series:  $\frac{1}{5} \frac{4}{15} + \frac{16}{45} \frac{64}{135} + \dots$ 
  - A.  $-\frac{3}{5}$

C.  $\frac{4}{5}$ 

- D. there is no finite sum
- 113. The exponential function  $y = 2^x$  can be used to determine the number of ancestors you have in a previous generation. For example, if x = 2, then y = 4 means that 2 generations ago you have 4 ancestors (your 4 grandparents). Determine an expression that represents the total number of ancestors you have in the last *n* generations.
  - A.  $2(2^{n-1}-1)$  B.  $2^n-1$
- C.  $2^{n+1}-1$  D.  $2(2^n-1)$

# ADDITIONAL QUESTIONS

114. If a person received a 10% salary increase at the end of each year and earned a total of \$91 576.50 during the first five years of work, determine the starting salary.

- 115. If  $\sum_{k=1}^{\infty} (\sin x)^{k-1} = 6$ , determine x to the nearest degree.  $(0^{\circ} \le x \le 90^{\circ})$ 
  - A. 36°

- B. 46°
- C. 56°

- D. 66°
- 116. If 10 is a single geometric mean of x and y, express y in terms of x.
  - A.  $y = \frac{100}{x}$  B.  $y = \frac{x}{100}$  C. y = 20 x D. y = 100 x

- 117. For a certain series,  $S_n = 3^{n+1}$ . Determine  $t_3$ .
  - A. 18

B. 27

C. 54

- D. 81
- 118. Determine a single geometric mean between  $\sec x 1$  and  $\sec x + 1$ .
  - A. -1

B. 1

- C.  $\cos x$
- D. tan x
- 119. For any series, which of the following is equivalent to  $S_{n-1} + S_n$ ?
  - A.  $2S_{n-1} + t_n$
- B.  $2S_{n-1} t_n$  C.  $S_n 1$  D.  $2S_n + 1$
- 120. Determine the sum of the first 12 terms of the geometric series  $\log_b 1 + \log_b 10 + \log_b 100 + \dots$ 
  - A.  $\frac{66}{\log b}$
- B.  $\frac{72}{\log h}$
- C. 66

- 121. Determine an expression for:  $\sum_{n=1}^{5} \log_a n$ 
  - A.  $\log_a 5$
- B.  $\log_a 6$
- C.  $\log_a 15$
- D.  $\log_a 120$

# **SEQUENCES AND SERIES ANSWER KEY**

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	C A A B C C D C 11818.82 A C B C A A D D B A B C D C A B
26 27	B B
28	С
29 30	B C
31	A
32	С
33	С
34 35	D D
36	C
37	Α
38	С
39 40	C D C C C B C
41	C
42	Č
43	В
44	
45	В

51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 89 89 89 89 89 89 89 89 89 89 89	A C C C B C B C B A A C C C C B A D D A D B D D C A B C C A A C B D D B C B C D D C B
94	D
0.5	

95

```
101
      n = 7
102
      3, 12, 48
103
104
      D
105
      С
106
      С
      511 students
107
108
109
      В
110
      D
111
      a) 0.648 m b) 19.30
      c) 20 m
112
      D
113
      D
      $15,000
114
115
      С
116
      Α
117
      С
118
      D
119
      Α
120
      Α
121
      D
```