Kognity Qu 7

$f(x) = x^2 - 3x$

$f'(x) = 2x - 3$

at $x = 1$, slope of tangent line:

$f'(1) = 2(1) - 3$

$= -\frac{1}{1}$

$\therefore$ slope of normal is 1

Equation of normal:

$y = mx + b$

$y = x + b$

Point on normal $(1, f(1)) = (1, -2)$

$f(1) = 1^2 - 3(1)$

$= -2$

sub: $-2 = 1 + b$

$b = -3$

$\boxed{y = x - 3}$ Normal at $x = 1$.

x-intercept: $y = 0$, sub:

$0 = x - 3$

$x = 3$, crosses at

$0 = x - 3$

$x = 3$, crosses at $(3, 0)$

$\therefore b = 3$

Product Rule

If $F(x) = f(x).g(x)$

Then $F'(x) = f(x).g'(x) + g(x).f'(x)$

$y = u.v$ $\quad (u = u(x),\ v = v(x))$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Ex: Differentiate $f(x) = \underbrace{x^3}\times\underbrace{e^{\overbrace{2x-7}^{\text{chain}}}}$

$$f'(x) = x^3.\frac{d}{dx}e^{2x-7} + e^{2x-7}.\frac{d}{dx}x^3$$

$$= x^3 e^{2x-7}.\frac{d}{dx}(2x-7) + e^{2x-7}.3x^2$$

$$= x^3 e^{2x-7}.2 + e^{2x-7}.3x^2$$

$$= x^2 e^{2x-7}(2x+3)$$

$= x^2 e^{2x-7}(2x+3)$

check on calculator?

wabbitemu
TI 83+

$$\frac{d}{dx}x^2 = 2x$$

MATH
↳ nDeriv

$$\left(\overset{\overset{f(x)}{\downarrow}}{x^2},\ \overset{\overset{\text{w.r.t.}}{\downarrow}}{x},\ \overset{\overset{\text{value.}}{\downarrow}}{3}\right) = \underline{\underline{6}}.$$

$$\frac{d}{dx}x^2\Big|_{x=3}$$

$f(x) = x^2$

$f(3)$

$2(3) = \underline{\underline{6}}.$

Quotient Rule.

If $F(x) = \dfrac{f(x)}{g(x)}$

then $F'(x) = g(x).f'(x) - f(x).g'(x)$

then $F'(x) = \dfrac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$

If $y = \dfrac{u}{v}$, $u = u(x)$, $v = v(x)$.

then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$.

Ex:

$y = \dfrac{2x-4}{x^3}$

$u(x) = 2x-4$

$v(x) = x^3$

$$\frac{dy}{dx} = \frac{x^3 \dfrac{d}{dx}(2x-4) - (2x-4) \cdot \dfrac{d}{dx} x^3}{(x^3)^2}$$

$$= \frac{x^3 \cdot 2 - (2x-4) \cdot 3x^2}{x^6}$$

$$= \frac{2x^3 - 6x^3 + 12x^2}{x^6}$$

$$= \frac{-4x^{\cancel{3}} + 12\cancel{x^2}}{x^{\cancel{6}4}}$$

$$= \frac{-4x+12}{x^4}$$