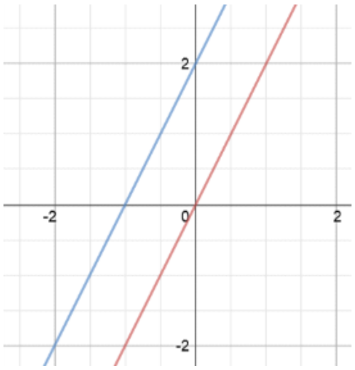
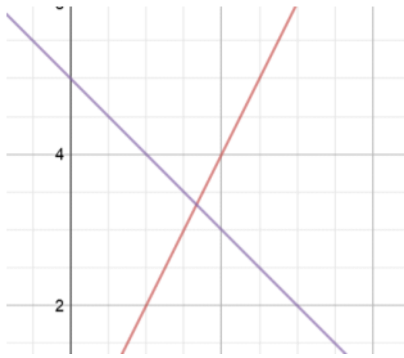
Systems of Equations

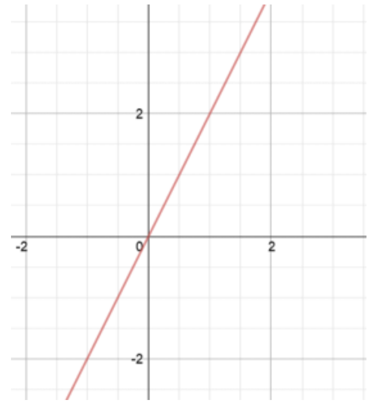
Linear - Linear Systems



0 Solutions (parallel)

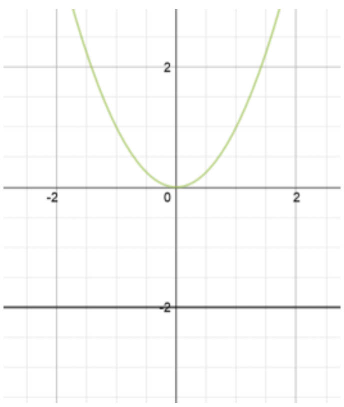


1 Solution (intersecting)

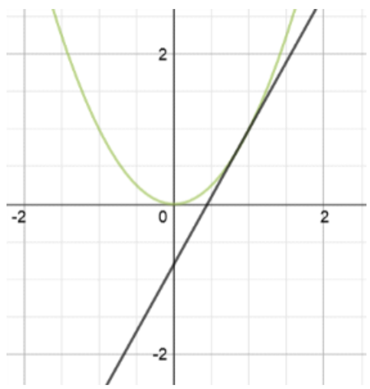


Infinitely many solutions (coinciding)

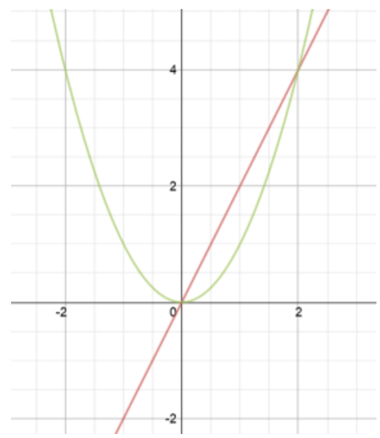
Linear - Quadratic Systems



0 Solutions

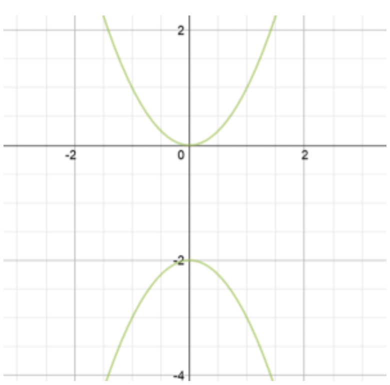


1 Solution

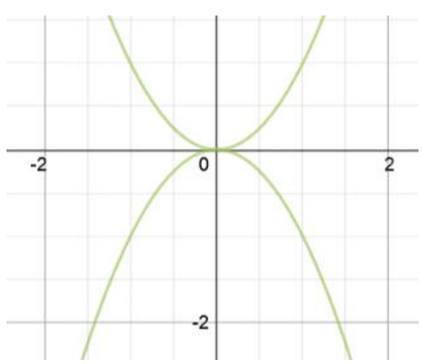


2 Solutions

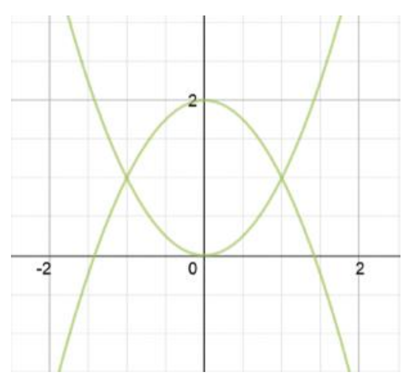
Quadratic - Quadratic Systems



0 Solutions



1 Solution



2 Solutions

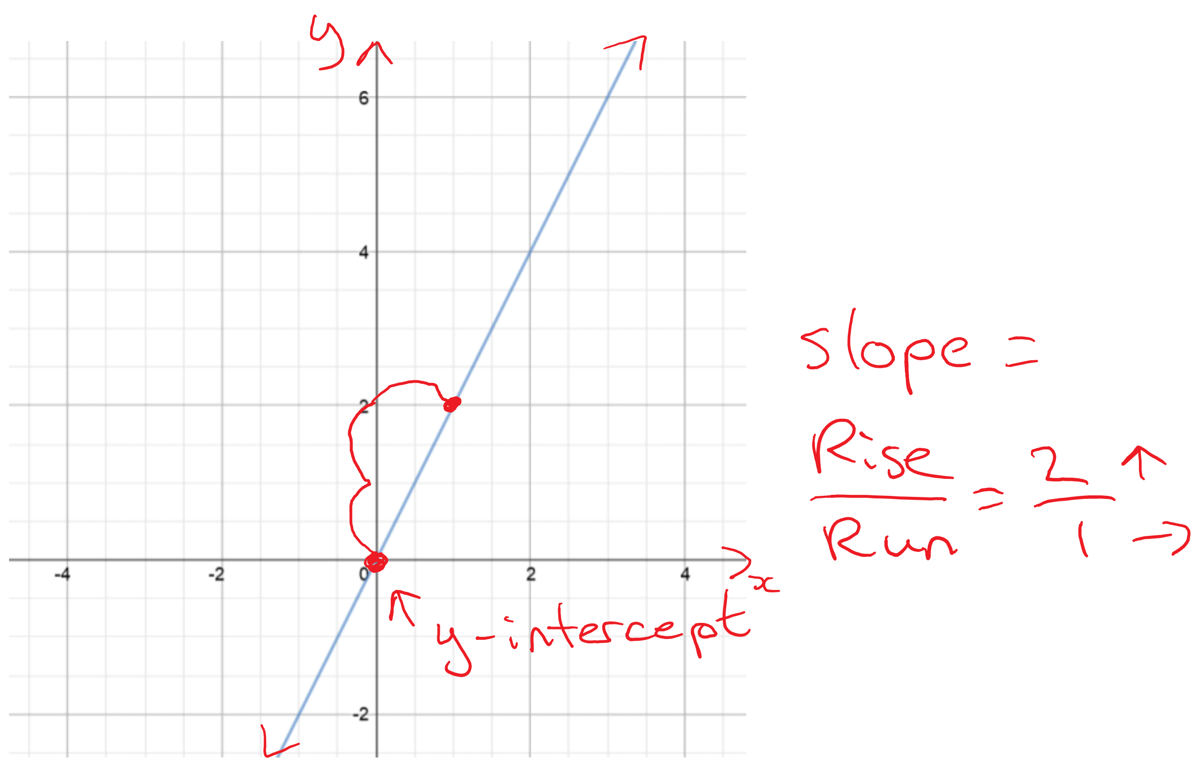
Methods of solving a system of equations:

- Graphing *(find the points of intersection)*
- Algebraically *(using substitution or elimination*

Example: Solve the following system of equations graphically:

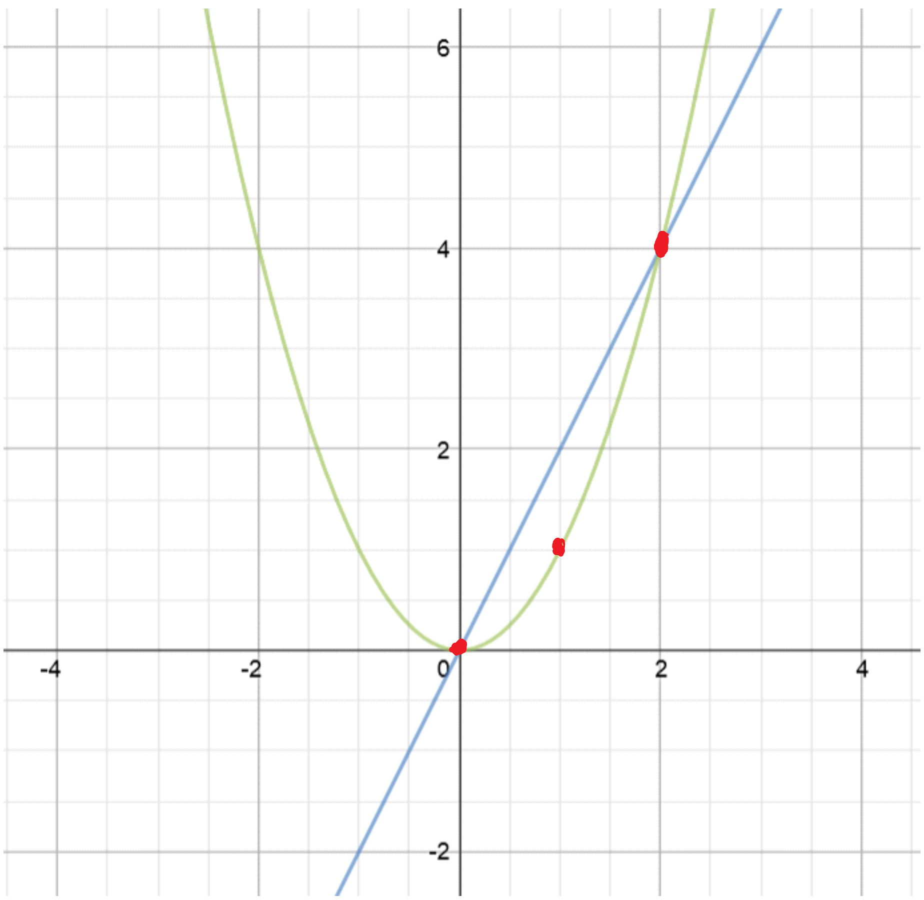
$$y = 2x$$
$$y = x^2$$

First we will sketch $y = 2x$. Comparing this to the slope-intercept form of a straight line $y = mx + b$, we see that the y-intercept is at 0 and the slope of the line is 2:



Now sketch $y = x^2$ on the same graph.

Comparing to the vertex/standard form of a quadratic function $y = a(x - p)^2 + q$ we see that the direction of this graph is up and the vertex is at the origin:



x	y
0	0
1	1
2	4

We can see that our graphs intersect at $(0,0)$ and $(2,4)$, thus these two points are the solutions of the given system of equations.

Example: Solve the following system of equations algebraically:

$$y = 2x \quad - 1$$
$$y = x^2 \quad - 2$$

Using substitution:

Substitute equation 1 into equation 2:

$2x = x^2$ Now solve this quadratic.

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0, or\ x = 2$$

From equation 1:

When $x = 0$:

$y = 2 \times 0 = 0$ So $(0, 0)$ is a possible solution.

When $x = 2$:

$y = 2 \times 2 = 4$ So $(2, 4)$ is a possible solution.

We must now check that $(0,0)$ and $(2,4)$ do lie on both lines and are actual solutions:

Substituting $(0,0)$ into equation 2 we obtain:

LHS	RHS
0	$0^2 = 0$

LHS=RHS thus $(0,0)$ lies on the line $y = x^2$.

Substituting $(2,4)$ into equation 2 we obtain:

LHS	RHS
4	$2^2 = 4$

LHS=RHS thus $(2,4)$ lies on the line $y = x^2$.

We have already verified the points satisfy equation 1 thus $(0,0)$ and $(2,4)$ are both solutions of the given system of equations.

Example: Solve the following system of equations using the method of elimination:

$$y = -2x^2 + 10 \quad — ①$$
$$x - 2y = -15 \quad — ②$$

Note: x and x^2 are not like terms, thus it is only possible to eliminate the y term.

Multiply equation 1 by 2 then add to equation 2:

$2 \times ①$: $2y = -4x^2 + 20$ *Now add like terms with equation 2.*

$$2y = -4x^2 + 20$$
$$x - 2y = -15$$

$x = -4x^2 + 5$ *Solve this quadratic.*

$4x^2 + x - 5 = 0$

$(4x + 5)(x - 1) = 0$

$x = -\dfrac{5}{4} \quad or\ x = 1$

Substituting into equation 1:

When $x = -\frac{5}{4}$:

$y = -2\left(-\dfrac{5}{4}\right)^2 + 10$

$= \frac{55}{8}$ So $\left(-\frac{5}{4}, \frac{55}{8}\right)$ is a possible solution.

When $x = 1$:

$y = -2(1)^2 + 10$

$= 8$ So $(1, 8)$ is a possible solution.

Now check that both points do indeed lie on both lines and are therefore both solutions of the given system.