Chapter 3 Practice Test

Multiple Choice

For #1 to #6, choose the best answer.

1. Which function is NOT a quadratic function?

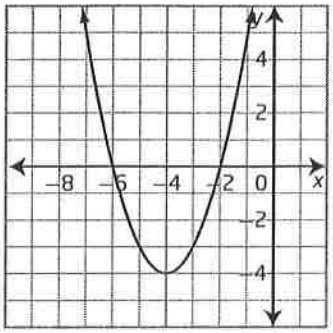
   A $f(x) = 2(x + 1)^2 - 7$

   B $f(x) = (x - 3)(2x + 5)$

   C $f(x) = 5x^2 - 20$

   D $f(x) = 3(x - 9) + 6$

2. Which quadratic function represents the parabola shown?



   A $y = (x + 4)^2 + 4$

   B $y = (x - 4)^2 + 4$

   C $y = (x + 4)^2 - 4$

   D $y = (x - 4)^2 - 4$

3. Identify the range for the function $y = -6(x - 6)^2 + 6$.

   A $\{y \mid y \leq 6, y \in \mathrm{R}\}$

   B $\{y \mid y \geq 6, y \in \mathrm{R}\}$

   C $\{y \mid y \leq -6, y \in \mathrm{R}\}$

   D $\{y \mid y \geq -6, y \in \mathrm{R}\}$

4. Which quadratic function in vertex form is equivalent to $y = x^2 - 2x - 5$?

   A $y = (x - 2)^2 - 1$

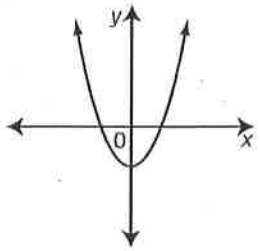
   B $y = (x - 2)^2 - 9$

   C $y = (x - 1)^2 - 4$

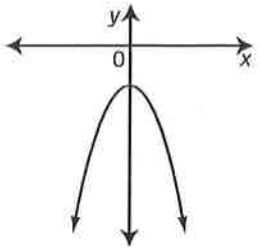
   D $y = (x - 1)^2 - 6$

5. Which graph shows the function $y = 1 + ax^2$ if $a < 0$?

A



B



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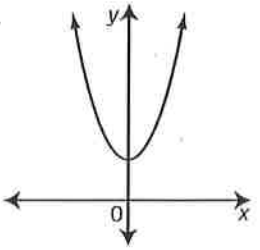
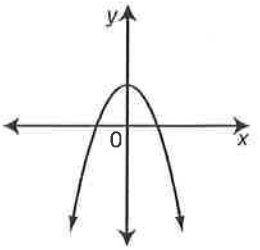


D



6. What conditions on a and q will give the function $f(x) = a(x - p)^2 + q$ no x-intercepts?

   A $a > 0$ and $q > 0$

   B $a < 0$ and $q > 0$

   C $a > 0$ and $q = 0$

   D $a < 0$ and $q = 0$

Short Answer

7. Write each quadratic function in vertex form by completing the square.

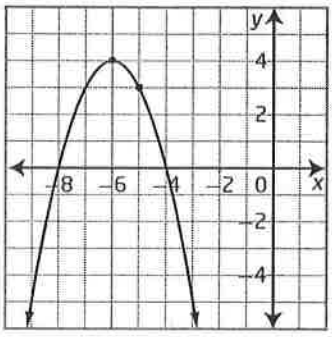
a) $y = x^2 - 18x - 27$

b) $y = 3x^2 + 36x + 13$

c) $y = -10x^2 - 40x$

8. a) For the graph shown, give the coordinates of the vertex, the equation of the axis of symmetry, the minimum or maximum value, the domain and range, and the x-intercepts.

b) Determine a quadratic function in vertex form for the graph.



9. a) Identify the transformation(s) on the graph of $f(x) = x^2$ that could be used to graph each function.

i) $f(x) = 5x^2$

ii) $f(x) = x^2 - 20$

iii) $f(x) = (x + 11)^2$

iv) $f(x) = -\frac{1}{7}x^2$

b) For each function in part a), state which of the following would be different as compared to $f(x) = x^2$ as a result of the transformation(s) involved, and explain why.

i) vertex

ii) axis of symmetry

iii) range

10. Sketch the graph of the function $y = 2(x - 1)^2 - 8$ using transformations. Then, copy and complete the table.

Vertex	
Axis of Symmetry	
Direction of Opening	
Domain	
Range	
x-Intercepts	
y-Intercept	

11. The first three steps in completing the square below contain one or more errors.

$y = 2x^2 - 8x + 9$
$y = 2(x^2 - 8x) + 9$
$y = 2(x^2 - 8x - 64 + 64) + 9$

a) Identify and correct the errors.

b) Complete the process to determine the vertex form of the function.

c) Verify your correct solution in several different ways.

12. The fuel consumption for a vehicle is related to the speed that it is driven and is usually given in litres per one hundred kilometres. Engines are generally more efficient at higher speeds than at lower speeds. For a particular type of car driving at a constant speed, the fuel consumption, C, in litres per one hundred kilometres, is related to the average driving speed, v, in kilometres per hour, by the function $C(v) = 0.004v^2 - 0.62v + 30$.

a) Without graphing, determine the most efficient speed at which this car should be driven. Explain/show the strategy you use.

b) Describe any characteristics of the graph that you can identify without actually graphing, and explain how you know.

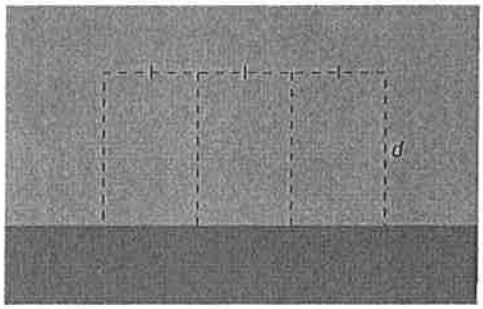
13. The height, h, in metres, of a flare t seconds after it is fired into the air can be modelled by the function $h(t) = -4.9t^2 + 61.25t$.

a) At what height is the flare at its maximum? How many seconds after being shot does this occur?

b) Verify your solution both with and without technology.

Extended Response

14. Three rectangular areas are being enclosed along the side of a building, as shown. There is enough material to make 24 m of fencing.



a) Write the function that represents the total area in terms of the distance from the wall.

b) Show that the function fits the definition of a quadratic function.

c) Graph the function. Explain the strategy you used.

d) What are the coordinates of the vertex? What do they represent?

e) What domain and range does the function have in this situation? Explain.

f) Does the function have a maximum value? Does it have a minimum value? Explain.

g) What assumptions are made in using this quadratic function model?

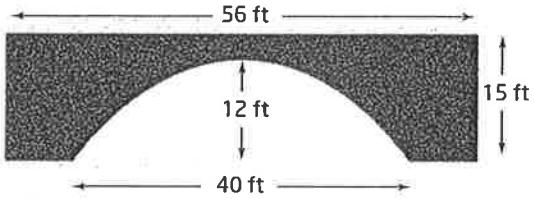
15. A stone bridge has the shape of a parabolic arch, as shown. Determine a quadratic function to represent the shape of the arch if the origin

a) is at the top of the opening under the bridge

b) is on the ground at the midpoint of the opening

c) is at the base of the bridge on the right side of the opening

d) is on the left side at the top surface of the bridge



16. A store sells energy bars for \$2.25. At this price, the store sold an average of 120 bars per month last year. The manager has been told that for every 5¢ decrease in price, he can expect the store to sell eight more bars monthly.

a) What quadratic function can you use to model this situation?

b) Use an algebraic method to determine the maximum revenue the manager can expect the store to achieve. What price will give that maximum?

c) What assumptions are made in this situation?

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1. D
2. C
3. A
4. D
5. D
6. A
7. a) $y = (x - 9)^2 - 108$
b) $y = 3(x + 6)^2 - 95$
c) $y = -10(x + 2)^2 + 40$
8. a) vertex: $(-6, 4)$, axis of symmetry: $x = -6$, maximum value of 4, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 4, y \in \mathrm{R}\}$, x-intercepts occur at $(-8, 0)$ and $(-4, 0)$
b) $y = -(x + 6)^2 + 4$

i) change in width by a multiplication of the y-v[illegible]s by a factor of 5

ii) vertica[illegible]islation of 20 units down

iii) horizontal translation of 11 units to the left

iv) change in width by a multiplication of the y-values by a factor of $\frac{1}{7}$ and a reflection in the x-axis

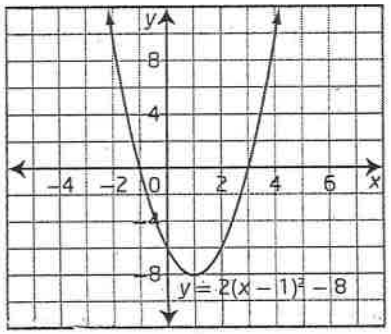
b) Examples:

i) The vertex of the functions in part a) ii) and iii) will be different as compared to $f(x) = x^2$ because the entire graph is translated. Instead of a vertex of (0, 0), the graph of the function in part a) ii) will be located at (0, −20) and the vertex of the graph of the function in part a) iii) will be located at (−11, 0).

ii) The axis of symmetry of the function in part a) iii) will be different as compared to $f(x) = x^2$ because the entire graph is translated horizontally. Instead of an axis of symmetry of $x = 0$, the graph of the function in part a) iii) will have an axis of symmetry of $x = -11$.

iii) The range of the functions in part a) ii) and iv) will be different as compared to $f(x) = x^2$ because the entire graph is either translated vertically or reflected in the x-axis. Instead of a range of $\{y \mid y \geq 0, y \in R\}$, the function in part a) ii) will have a range of $\{y \mid y \geq -20, y \in R\}$ and the function in part a) iv) will have a range of $\{y \mid y \leq 0, y \in R\}$.

10.



Vertex	(1, −8)
Axis of Symmetry	$x = 1$
Direction of Opening	upward
Domain	$\{x \mid x \in R\}$
Range	$\{y \mid y \geq -8, y \in R\}$
x-Intercepts	−1 and 3
y-Intercept	−6

11. a) In the second line, the 2 was not factored out of the second term. In the third line, you need to add and subtract the square of half the coefficient of the x-term. The first three steps should be

$y = 2x^2 - 8x + 9$

$y = 2(x^2 - 4x) + 9$

$y = 2(x^2 - 4x + 4 - 4) + 9$

b) The rest of the process is shown.

$y = 2[(x^2 - 4x + 4) - 4] + 9$

$y = 2(x - 2)^2 - 8 + 9$

$y = 2(x - 2)^2 + 1$

c) The solution can be verified by expanding the vertex form to standard form or by graphing both functions to see that they coincide.

12. Examples:

a) The vertex form of the function $C(v) = 0.004v^2 - 0.62v + 30$ is $C(v) = 0.004(v - 77.5)^2 + 5.975$. The most efficient speed would be 77.5 km/h and will produce a fuel consumption of 5.975 L/100 km.

b) By completing the square and determining the vertex of the function, you can determine the most efficient fuel consumption and at what speed it occurs.

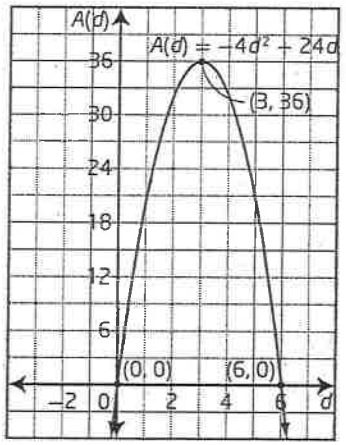
13. a) The maximum height of the flare is 191.406 25 m, 6.25 s after being shot.

b) Example: Complete the square to produce the vertex form and use the value of q to determine the maximum height and the value of p to determine when it occurs, or use the fact that the x-coordinate of the vertex of a quadratic function in standard form is $x = -\frac{b}{2a}$ and substitute this value into the function to find the corresponding y-coordinate, or graph the function to find the vertex.

14. a) $A(d) = -4d^2 + 24d$

b) Since the function is a polynomial of degree two, it satisfies the definition of a quadratic function.

c)



Example: By completing the square, determine the vertex, find the y-intercept and its corresponding point, plot the three points, and join them with a smooth curve.

d) (3, 36); the maximum area of 36 m² happens when the fence is extended to 3 m from the building.

e) domain: $\{d \mid 0 \leq d \leq 6, d \in R\}$, range: $\{A \mid 0 \leq A \leq 36, A \in R\}$; negative distance and area do not have meaning in this situation.

f) Yes; the maximum value is 36 when d is 3, and the minimum value is 0 when d is 0 or 6.

g) Example: Assume that any real-number distance can be used to build the fence.

15. a) $f(x) = -0.03x^2$

b) $f(x) = -0.03x^2 + 12$

c) $f(x) = -0.03(x + 20)^2 + 12$

d) $f(x) = -0.03(x - 28)^2 - 3$

16. a) $R = (2.25 - 0.05x)(120 + 8x)$

b) Expand and complete the square to get the vertex form of the function. A price of $1.50 gives the maximum revenue of $360.

c) Example: Assume that any whole number of price decreases can occur.